Solutions: Energy, Work, and Rotation of a Rigid Body

Use $g = 9.80\,\text{m/s}^2$.

Chapter 10

1. *(Ch. 10)* (a) Spring energy becomes kinetic energy: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = x\sqrt{k/m} = (0.20)\sqrt{400/0.50} = 5.66\,\text{m/s}$.
(b) $\frac{1}{2}kx^2 = 8.0\,\text{J} = mgh \Rightarrow h = \dfrac{8.0}{(0.50)(9.80)} = 1.63\,\text{m}$. Distance along the incline: $d = h/\sin 30^\circ = 3.27\,\text{m}$.

2. *(Ch. 10)* For an elastic collision with m_2 at rest: $v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2}v_{1i} = \dfrac{2.0}{4.0}(4.0) = 2.0\,\text{m/s}$, $v_{2f} = \dfrac{2m_1}{m_1 + m_2}v_{1i} = \dfrac{6.0}{4.0}(4.0) = 6.0\,\text{m/s}$. Check: $K_i = \frac{1}{2}(3.0)(4.0)^2 = 24\,\text{J}$; $K_f = \frac{1}{2}(3.0)(2.0)^2 + \frac{1}{2}(1.0)(6.0)^2 = 6 + 18 = 24\,\text{J}$. ✓

3. *(Ch. 10)* (a) $dU/dx = U_0\left(4x^3/a^4 - 4x/a^2\right) = 0 \Rightarrow x = 0,\ x = \pm a = \pm 0.50\,\text{m}$. $U(0) = 0$ is a local maximum ($d^2U/dx^2 = -4U_0/a^2 < 0$): **unstable**. $U(\pm a) = -U_0 = -4.0\,\text{J}$ are minima ($d^2U/dx^2 = 8U_0/a^2 > 0$): **stable**.
(b) Turning points where $U = E$. With $u = (x/a)^2$: $u^2 - 2u = -0.5 \Rightarrow u = 1 \pm \sqrt{0.5} = 0.293,\ 1.707$. So $x = a\sqrt{u} = 0.271\,\text{m}$ and $0.653\,\text{m}$. The particle oscillates in the well around $x = +a$.
(c) The speed is greatest at the bottom of the well, $U = -4.0\,\text{J}$: $K_{\max} = E - U_{\min} = -2.0 - (-4.0) = 2.0\,\text{J}$, so $v_{\max} = \sqrt{2K/m} = \sqrt{2(2.0)/0.20} = 4.47\,\text{m/s}$.

4. *(Ch. 10)* Let x be the compression; the block falls $h + x$ in total and starts and ends at rest: $mg(h + x) = \frac{1}{2}kx^2 \Rightarrow 750x^2 - 19.6x - 9.8 = 0$. $x = \dfrac{19.6 + \sqrt{19.6^2 + 4(750)(9.8)}}{2(750)} = \dfrac{19.6 + 172.6}{1500} = 0.128\,\text{m}$.

Chapter 11

5. *(Ch. 11)* $W = \int_0^3 (6.0x - 2.0x^2)\,dx = \left[3.0x^2 - \frac{2}{3}x^3\right]_0^3 = 27 - 18 = 9.0\,\text{J}$. Work–energy theorem: $\frac{1}{2}(2.0)v^2 = \frac{1}{2}(2.0)(1.0)^2 + 9.0 \Rightarrow v^2 = 10 \Rightarrow v = 3.16\,\text{m/s}$.

6. *(Ch. 11)* (a) Rope: $W = Fd\cos 30^\circ = (80)(5.0)(0.866) = 346\,\text{J}$. Gravity and normal force are perpendicular to the displacement: $W = 0\,\text{J}$ each. Normal force: $n = mg - F\sin 30^\circ = 196 - 40 = 156\,\text{N}$, so $f_k = 0.25(156) = 39\,\text{N}$ and $W_f = -(39)(5.0) = -195\,\text{J}$.
(b) $W_{\text{net}} = 346 - 195 = 151\,\text{J} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2(151.4)/20} = 3.89\,\text{m/s}$.

7. *(Ch. 11)* Gravity does $mgd\sin\theta = (5.0)(9.80)(4.0)(0.50) = 98.0\,\text{J}$. Friction: $f_k = \mu mg\cos\theta = 0.20(5.0)(9.80)(0.866) = 8.49\,\text{N}$; thermal energy $\Delta E_{th} = f_k d = 33.9\,\text{J}$. $K_f = 98.0 - 33.9 = 64.1\,\text{J} \Rightarrow v = \sqrt{2(64.1)/5.0} = 5.06\,\text{m/s}$.

8. *(Ch. 11)* (a) $F_x = -dU/dx = -(6x^2 - 6) = 6 - 6x^2$ (newtons).
(b) $F_x = 0 \Rightarrow x = \pm 1.0\,\text{m}$. $d^2U/dx^2 = 12x$: positive at $x = +1$ (**stable**), negative at $x = -1$ (**unstable**).

(c) $E = U(0) = 0$. $U(1.0) = 2 - 6 = -4.0\,\text{J}$, so $K = 4.0\,\text{J}$ and $v = \sqrt{2(4.0)/0.50} = 4.0\,\text{m/s}$.

9. *(Ch. 11)* (a) $P_{\text{avg}} = \Delta K/\Delta t = \dfrac{\frac{1}{2}(1500)(25)^2}{8.0} = 58.6\,\text{kW}$ ($\approx 78.6\,\text{hp}$).
(b) Constant speed, so the engine force balances gravity's component plus drag: $F = mg\sin 5.0^\circ + 400 = 1281 + 400 = 1681\,\text{N}$. $P = Fv = (1681)(20) = 33.6\,\text{kW}$.

10. *(Ch. 11)* (a) Energy: $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu_k mg\,x$, with $\frac{1}{2}mv^2 = 10.0\,\text{J}$ and $\mu_k mg = 3.136\,\text{N}$: $250x^2 + 3.136x - 10.0 = 0 \Rightarrow x = \dfrac{-3.136 + \sqrt{3.136^2 + 4(250)(10.0)}}{500} = 0.194\,\text{m}$.
(b) At maximum compression $kx = 96.9\,\text{N} \gg \mu_s mg$ (a few newtons), so the block rebounds. Energy stored: $\frac{1}{2}kx^2 = 9.39\,\text{J}$. Friction on the way back dissipates $\mu_k mg\,x = 0.608\,\text{J}$. $K = 9.39 - 0.61 = 8.78\,\text{J} \Rightarrow v = \sqrt{2(8.78)/0.80} = 4.69\,\text{m/s}$.

11. *(Ch. 11)* $W = \vec{F}\cdot\vec{d} = (3.0)(4.0) + (-2.0)(5.0) = 12 - 10 = 2.0\,\text{J}$. $\cos\theta = \dfrac{W}{|F||d|} = \dfrac{2.0}{\sqrt{13}\sqrt{41}} = 0.0866 \Rightarrow \theta = 85.0^\circ$.

12. *(Ch. 11)* System = both blocks. The hanging block loses $m_2 gh = (2.0)(9.80)(1.5) = 29.4\,\text{J}$ of gravitational PE. Friction on the table block: $\mu m_1 g\,h = 0.20(4.0)(9.80)(1.5) = 11.8\,\text{J}$ of thermal energy. $\frac{1}{2}(m_1 + m_2)v^2 = 29.4 - 11.8 = 17.6\,\text{J} \Rightarrow v^2 = \dfrac{2(17.64)}{6.0} = 5.88 \Rightarrow v = 2.42\,\text{m/s}$.

Chapter 12

13. *(Ch. 12)* (a) $\omega_f = 1200\left(\dfrac{2\pi}{60}\right) = 125.7\,\text{rad/s}$, so $\alpha = \omega_f/t = 15.7\,\text{rad/s}^2$.
(b) $\Delta\theta = \frac{1}{2}\omega_f t = \frac{1}{2}(125.7)(8.0) = 503\,\text{rad} = 80\,\text{rev}$.
(c) $v = \omega r = (125.7)(0.15) = 18.8\,\text{m/s}$; $a_c = \omega^2 r = 2.37\times 10^3\,\text{m/s}^2$; $a_t = \alpha r = 2.36\,\text{m/s}^2$.

14. *(Ch. 12)* (a) $I_{\text{end}} = \frac{1}{3}ML^2 + mL^2 = \frac{1}{3}(2.0)(1.50)^2 + (0.50)(1.50)^2 = 1.500 + 1.125 = 2.625\,\text{kg m}^2$.
(b) Center of mass from the pivot end: $x_{cm} = \dfrac{(2.0)(0.75) + (0.50)(1.50)}{2.5} = 0.90\,\text{m}$. Parallel-axis theorem: $I_{\text{end}} = I_{cm} + Md^2 \Rightarrow I_{cm} = 2.625 - (2.5)(0.90)^2 = 0.600\,\text{kg m}^2$.

15. *(Ch. 12)* (a) Block: $mg - T = ma$. Disk: $TR = I\alpha = \frac{1}{2}MR^2(a/R) \Rightarrow T = \frac{1}{2}Ma$. Then $a = \dfrac{mg}{m + M/2} = \dfrac{(2.0)(9.80)}{2.0 + 2.0} = 4.90\,\text{m/s}^2$, $\quad T = \frac{1}{2}(4.0)(4.90) = 9.80\,\text{N}$.
(b) $v = \sqrt{2ad} = \sqrt{2(4.90)(2.0)} = 4.43\,\text{m/s}$. Energy check: $mgd = 39.2\,\text{J} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(m + \frac{1}{2}M)v^2 = \frac{1}{2}(4.0)v^2 \Rightarrow v = 4.43\,\text{m/s}$. ✓

16. *(Ch. 12)* (a) Torques about the hinge (magnitudes): the cable's vertical component acts at $L = 4.0\,\text{m}$: $T\sin 30^\circ(4.0) = (25)(9.80)(2.0) + (50)(9.80)(3.0) = 490 + 1470 = 1960\,\text{N m}$, so $T = 980\,\text{N}$.
(b) $\sum F_x = 0$: $H_x = T\cos 30^\circ = 849\,\text{N}$, directed *away from the wall*. $\sum F_y = 0$: $H_y + T\sin 30^\circ = (75)(9.80) = 735 \Rightarrow H_y = 735 - 490 = 245\,\text{N}$ upward. $|H| = \sqrt{849^2 + 245^2} = 883\,\text{N}$, at 16.1° above the horizontal.

17. *(Ch. 12)* (a) Energy: $mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left(1 + \frac{2}{5}\right) = \frac{7}{10}Mv^2$, so $v = \sqrt{\frac{10}{7}gh} =$

$\sqrt{\frac{10}{7}(9.80)(1.5)} = 4.58\,\text{m/s}$.
(b) $a = \dfrac{g\sin\theta}{1 + I/MR^2} = \dfrac{(9.80)(0.50)}{1.4} = 3.50\,\text{m/s}^2$. (Check: incline length $= h/\sin\theta = 3.0\,\text{m}$; $v^2 = 2ad = 2(3.5)(3.0) = 21.0$, $v = 4.58\,\text{m/s}$. ✓)
(c) Friction provides the torque: $f_s R = I\alpha = \frac{2}{5}MR^2(a/R) \Rightarrow f_s = \frac{2}{5}Ma = 1.4M$ (newtons per kg). Also $n = Mg\cos 30° = 8.49M$. Hence $\mu_s \geq f_s/n = 0.165$ (equivalently $\frac{2}{7}\tan\theta$).

18. *(Ch. 12)* $I = \frac{1}{3}ML^2$.
(a) $\tau = Mg(L/2)$, so $\alpha = \dfrac{MgL/2}{ML^2/3} = \dfrac{3g}{2L} = 14.7\,\text{rad/s}^2$.
(b) Energy: the center of mass drops $L/2$: $Mg\dfrac{L}{2} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \Rightarrow \omega = \sqrt{3g/L} = 5.42\,\text{rad/s}$.
(c) $v_{\text{tip}} = \omega L = 5.42\,\text{m/s}$.

19. *(Ch. 12)* (a) No external torque about the axis, so L is conserved. $I_i = 2.5 + 2(3.0)(0.80)^2 = 6.34\,\text{kg m}^2$, $\quad I_f = 2.5 + 2(3.0)(0.20)^2 = 2.74\,\text{kg m}^2$. $\omega_f = \dfrac{I_i}{I_f}\omega_i = \dfrac{6.34}{2.74}(1.5) = 3.47\,\text{rad/s}$.
(b) $K_i = \frac{1}{2}(6.34)(1.5)^2 = 7.13\,\text{J}$; $K_f = \frac{1}{2}(2.74)(3.47)^2 = 16.5\,\text{J}$; $\Delta K = 9.4\,\text{J}$. The student's muscles do positive work pulling the dumbbells inward against the (apparent) outward tendency; the energy comes from the student's chemical energy.

20. *(Ch. 12)* (a) Angular momentum about the pivot is conserved in the collision (the pivot force exerts no torque about the pivot): $mvL = I\omega$, with $I = \frac{1}{3}ML^2 + mL^2 = \frac{1}{3}(0.80)(1.44) + (0.10)(1.44) = 0.528\,\text{kg m}^2$. $\omega = \dfrac{(0.10)(6.0)(1.2)}{0.528} = 1.36\,\text{rad/s}$.
(b) After the collision energy is conserved: $K = \frac{1}{2}I\omega^2 = 0.491\,\text{J}$. The rod's center rises $\frac{L}{2}(1 - \cos\theta)$ and the clay rises $L(1 - \cos\theta)$: $\left[Mg\frac{L}{2} + mgL\right](1 - \cos\theta) = (4.704 + 1.176)(1 - \cos\theta) = 0.491 \Rightarrow \cos\theta = 0.9165 \Rightarrow \theta = 23.6°$.

Note: linear momentum and mechanical energy are not conserved in problem 20's collision (it is perfectly inelastic); only angular momentum about the pivot is.