PHY 121 – Take-Home Quiz

Dynamics II, Work & Kinetic Energy, Interactions & Potential Energy, Impulse & Momentum

Name: ________________________________ **Date:** ______________

Rules for this take-home quiz:

- This is to be completed **without use of the internet or any AI tool** of any kind.
- You *may* use your textbook and your own course notes.
- You may work together with another student in class, but when it comes time to write up solutions, this needs to be done independently.
- Write **legibly**. Illegible work cannot be graded.
- For every problem, in addition to your final answer, include a short discussion (a few sentences) of *how you are thinking about the problem* – what principle(s) apply, why you set it up the way you did, and any assumptions you are making. A correct number with no reasoning will receive little credit; clear reasoning with a minor arithmetic slip will receive most of the credit.
- Show all work: define variables, write the governing equation(s) before substituting numbers, and box your final answer with correct units. Where a diagram is requested, it must be **drawn by hand** with your own labels – not just described in words.
- Your solutions for each question should be preceded by each statement of the question. Staple your final work together.
- You handwritten solutions will be collected at the beginning of class on Tuesday, 23 June 2026. Late submissions will not be accepted.

Problem 1 – Nonuniform Circular Motion (Ch. 8)

A small ball of mass $m = 0.200$ kg is tied to a string of length $R = 0.80$ m and swung in a *vertical* circle. At the instant the ball is at the bottom of the circle, its speed is $v_b = 6.0$ m/s and it is speeding up at a tangential rate of $a_t = 3.0$ m/s^2 (i.e., the speed is increasing as it passes through the bottom).

(a) **Draw a free-body diagram by hand** of the ball at the instant it is at the bottom of the circle. Label the tension $\vec{T}$, the weight $\vec{W}$, and sketch (with a separate arrow, clearly distinguished) the directions of the centripetal (radial) and tangential accelerations at that instant.

(b) Explain, using your diagram, why the net force on the ball at this instant has *two* non-zero perpendicular components, and what physical role each one plays in the ball's motion (i.e., what each component changes about the velocity).

(c) Compute the radial (centripetal) acceleration of the ball at the bottom of the circle.

(d) Using Newton's second law applied separately to the radial direction, find the tension in the string at this instant. (The tangential direction does not affect this calculation directly, but explain briefly why not.)

(e) Compute the magnitude of the *total* acceleration of the ball (radial and tangential components combined) at this instant.

Problem 2 – Work Done by a Variable Force (Ch. 9)

A particle moves along the x-axis from $x_i = 0$ to $x_f = 4.0$ m under the influence of a force whose x-component varies with position as

$$F_x(x) = (6.0\ \text{N/m})x - (0.75\ \text{N/m}^2)x^2.$$

(a) Sketch F_x versus x over the interval $0 \le x \le 4.0$ m (by hand, roughly to scale), and shade the region whose area represents the work done on the particle.

(b) Set up the integral $W = \int_{x_i}^{x_f} F_x(x)\,dx$ for this force, and explain in words why this integral – rather than simply $W = Fd$ – is the correct way to compute work when the force is not constant.

(c) Evaluate the integral to find the work done on the particle as it moves from x_i to x_f.

(d) If the particle has mass $m = 1.5$ kg and starts from rest at $x_i = 0$, use the work–kinetic energy theorem to find its speed at $x_f = 4.0$ m.

Problem 3 – Dot Product and Work at an Angle (Ch. 9)

A force $\vec{F} = (25.0\,\hat{\imath} - 15.0\,\hat{\jmath})$ N acts on a block as it undergoes a displacement $\vec{d} = (8.0\,\hat{\imath} + 3.0\,\hat{\jmath})$ m.

(a) Compute the work done by $\vec{F}$ using the component form of the dot product, $W = \vec{F} \cdot \vec{d} = F_x d_x + F_y d_y$.

(b) Separately, compute the magnitudes $|\vec{F}|$ and $|\vec{d}|$, and the angle θ between the two vectors using $\cos\theta = \dfrac{\vec{F} \cdot \vec{d}}{|\vec{F}||\vec{d}|}$.

(c) Using this angle θ, recompute the work as $W = |\vec{F}||\vec{d}|\cos\theta$, and confirm it agrees with part (a).

(d) This force has a negative y-component while the displacement has a positive y-component. Explain, physically and in terms of the angle θ you found, why the work done by $\vec{F}$ is nonetheless positive overall.

Problem 4 – Potential Energy Curves and Equilibrium (Ch. 10)

A particle of mass $m = 0.50$ kg moves along the x-axis under a conservative force with potential energy function

$$U(x) = (2.0\ \mathrm{J/m^4})x^4 - (8.0\ \mathrm{J/m^2})x^2,$$

for x in meters.

(a) Find $F_x(x)$ from $U(x)$ using $F_x = -\dfrac{dU}{dx}$, and explain in words the physical meaning of the minus sign – i.e., why force points toward *decreasing* potential energy.

(b) Find all values of x where $F_x = 0$ (the equilibrium points), and classify each as stable or unstable by examining the curvature of $U(x)$ (or the sign of $\dfrac{dF_x}{dx}$) at that point. Briefly justify your classification physically: what does the system do if displaced slightly from a stable point versus an unstable one?

(c) Sketch $U(x)$ by hand over the range $-2.5 \le x \le 2.5$ m, marking the equilibrium points you found.

(d) If the particle has total mechanical energy $E = 0$ J, sketch a horizontal line at $U = 0$ on your graph and shade the regions where the particle's motion is allowed (i.e., where $E \ge U(x)$). Use this to describe, in words, the possible motion of the particle.

Problem 5 – Spring (Hooke's Law) and Energy Conservation with Friction (Ch. 10)

A 1.20 kg block is pressed against a horizontal spring of spring constant $k = 850$ N/m, compressing it $x_0 = 0.120$ m from its natural length, and is then released. The block slides across a rough surface (coefficient of kinetic friction $\mu_k = 0.180$) before leaving the spring's influence and continuing on to a frictionless region. The rough surface extends from the initial compression to the equilibrium, after which the surface is frictionless.

(a) Write the elastic potential energy stored in the spring at maximum compression, and explain why this quantity depends on x_0^2 rather than x_0 – i.e., give a physical/geometric reason (related to how spring force varies with displacement) that the potential energy is quadratic.

(b) Using an energy equation of the form $U_{s,i} + K_i + W_{nc} = U_{s,f} + K_f$ over the region where the block is in contact with the spring and friction acts simultaneously (spring contact length $= x_0$), find the block's speed at the instant it leaves the spring (i.e., when the spring returns to natural length).

(c) Explain why, in this problem, you cannot simply set the initial elastic PE equal to the final KE the way you could in a completely frictionless version of this kind of problem.

Problem 6 – Elastic Collision, General Case (Ch. 10)

Derive, from first principles, the general formulas for the final velocities in a 1D *elastic* collision between mass m_1 (initial velocity v_{1i}) and mass m_2 (initially at rest).

(a) Write the two governing conservation equations (momentum and kinetic energy) symbolically, with $v_{2i} = 0$.

(b) Show the key algebraic step: factor the kinetic energy equation using the difference of squares, divide by the momentum equation, and combine the result with momentum conservation to solve for v_{1f} and v_{2f} in terms of m_1, m_2, and v_{1i} only. Show enough steps that someone reading your work could follow the logic without already knowing the answer.

(c) Using your general formulas, evaluate v_{1f} and v_{2f} for the specific case $m_1 = 3.0$ kg, $m_2 = 1.0$ kg, $v_{1i} = 4.0$ m/s.

(d) Check, by direct substitution of your numerical results, that both momentum and kinetic energy are conserved.

(e) Comment on what your general formula predicts in the limiting case $m_2 \gg m_1$ (a light object striking a much heavier, effectively immovable one), and explain whether this matches your physical intuition.

Problem 7 – Two-Dimensional Momentum: An Explosion (Ch. 11)

A firework shell of total mass $M = 2.50$ kg is moving horizontally with velocity $v_0 = 12.0$ m/s (in the $+x$ direction) when it explodes into two fragments of equal mass. Immediately after the explosion, fragment 1 moves with velocity $\vec{v}_1 = (8.0\,\hat{\imath} + 15.0\,\hat{\jmath})$ m/s.

(a) Explain why momentum is conserved *as a vector* during this explosion even though kinetic energy is not, and why this means you must conserve the x- and y-components *separately*.

(b) Using conservation of momentum in both the x- and y-directions, find the velocity components (and hence the magnitude and direction) of fragment 2 immediately after the explosion.

(c) Compute the total kinetic energy of the system just before and just after the explosion, and state where the difference in kinetic energy came from physically.

Problem 8 – Impulse from a Time-Varying Force (Ch. 11)

A 0.060 kg ball is struck by a bat. The force exerted by the bat on the ball as a function of time is modeled as

$$F(t) = F_{\max} \sin\left(\frac{\pi t}{T}\right), \qquad 0 \le t \le T,$$

with $F_{\max} = 1500$ N and contact time $T = 4.0$ ms, with $F(t) = 0$ outside this interval. Before contact, the ball moves toward the bat at 20.0 m/s; treat the direction toward the bat initially as the negative direction.

(a) Sketch $F(t)$ versus t by hand over $0 \le t \le T$, and explain (referring to your sketch) what the *area* under this curve represents physically.

(b) Set up the integral $J = \int_0^T F(t)\,dt$ and evaluate it to find the impulse delivered to the ball. (You may use $\int_0^T \sin(\pi t/T)\,dt = \frac{2T}{\pi}$.)

(c) Using the impulse–momentum theorem, find the ball's velocity immediately after being struck. State your sign convention explicitly and interpret the sign of your answer physically.

(d) Compute the *average* force on the ball during contact (using $\bar{F} = J/T$), and compare it to $F_{\max}$. Explain why the average force is smaller than the peak force, referring to the shape of your sketch in part (a).

End of quiz. Please check that all problems are complete and that your reasoning discussion is included for each.