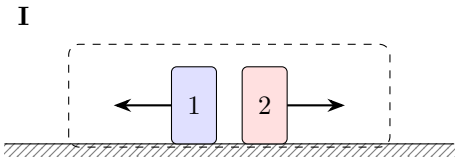
PHY 121 Tutorial: Momentum Conservation

Work in your group. Draw diagrams, explain your reasoning in words, and check units and reasonableness for every numerical answer. Items marked ★ are extensions for groups that finish early.

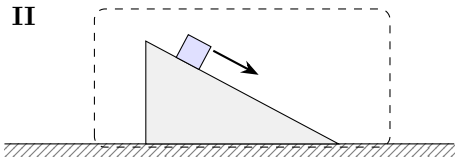
Part A: When is momentum conserved?

Reminder. The total momentum of a system is conserved in a given direction if the net *external* force on the system has no component in that direction. Take $+x$ to the right and $+y$ up.

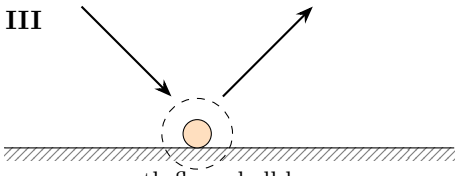
A1. Each situation below has a dashed boundary that shows the system. (I) Two skaters, initially at rest on frictionless ice, push off each other. (II) A block slides down a frictionless wedge; the wedge sits on a frictionless table and is free to slide. (III) A ball bounces off a smooth, frictionless floor. (IV) A block slides across a rough table and slows down.



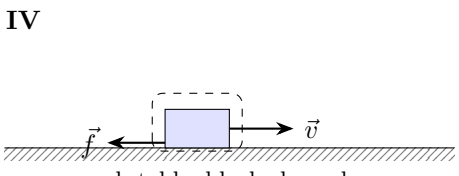
frictionless ice; skaters push apart



frictionless table; wedge free to slide



smooth floor; ball bounces

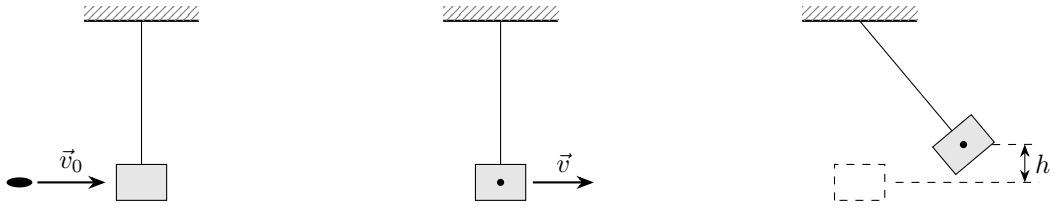


rough table; block slows down

For each situation decide whether the total momentum of the system is conserved in the x-direction and in the y-direction. Give a reason in terms of the external forces on the system.

	p_x conserved?	p_y conserved?	Reason (external forces on the system)
I			
II			
III			
IV			

A2. A bullet is fired horizontally into a block of wood hanging at rest from a string. The bullet embeds in the block, and the bullet-plus-block swings up to a maximum height h. Take the system to be the bullet and the block together, and consider the *horizontal* component of momentum.



1: just before 2: just after 3: highest point

(a) For each interval, 1→2 (the bullet embeds; this takes a very short time) and 2→3 (the swing), choose the best statement and name the forces that support your choice: **(A)** the total horizontal momentum and the mechanical energy are both conserved; **(B)** the momentum is conserved but the mechanical energy is not; **(C)** the mechanical energy is conserved but the momentum is not; **(D)** neither is conserved.

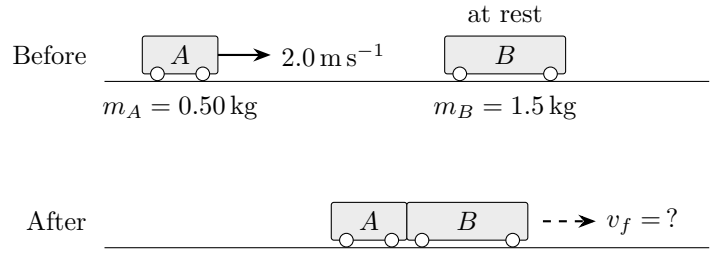
(b) During the swing (2→3), which external force changes the horizontal momentum of the system? Which way does it point?

A3. A 0.50 kg ball is released from rest and falls 1.0 m. **Student 1:** "Momentum isn't conserved when a ball falls, because the ball's momentum keeps increasing." **Student 2:** "Momentum is conserved. You just have to include the Earth." (a) For the ball alone, what is the net external force? Is the ball's momentum conserved? (b) Which system would you choose so that momentum *is* conserved? Which forces are internal to that system? (c) ★ Estimate the Earth's speed at the instant the ball reaches the bottom ($m_E = 6.0 \times 10^{24}$ kg). Which student is right?

★A4. Two cars (total mass about 2500 kg, total momentum about $3 \times 10^4\,\mathrm{kg\,m\,s^{-1}}$) collide on a dry road. The collision lasts about 0.10 s, and road friction on the tires is at most about $\mu_k Mg$ with $\mu_k \approx 0.7$. Estimate the impulse of friction during the collision and compare it with the total momentum. Is it reasonable to treat the momentum of the two-car system as conserved *during* the collision? Would it be reasonable to treat it as conserved during the skid that follows?

Part B: Completely inelastic collisions in one dimension

B1. Two carts on a nearly frictionless air track collide and stick together on Velcro bumpers. Cart A ($m_A = 0.50\,\text{kg}$) moves at $2.0\,\text{m s}^{-1}$ toward cart B ($m_B = 1.5\,\text{kg}$), which is initially at rest.



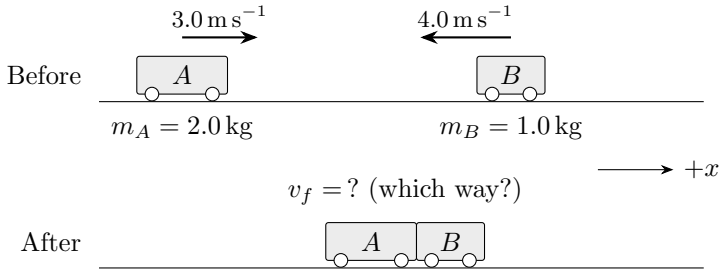
(a) Is the total momentum of the two carts conserved during the collision? Explain using external forces.

(b) Find the velocity of the carts just after the collision.

(c) Compare the total kinetic energy before and after. What fraction of the initial kinetic energy is lost? Where did it go?

(d) **Student 1:** "Kinetic energy isn't conserved in this collision, so we can't use conservation of momentum." **Student 2:** "Whether momentum is conserved depends on the external forces, not on whether kinetic energy is lost." Which student is right?

B2. Now cart A ($m_A = 2.0\,\text{kg}$) moves right at $3.0\,\text{m s}^{-1}$ and cart B ($m_B = 1.0\,\text{kg}$) moves left at $4.0\,\text{m s}^{-1}$. They collide and stick together.



(a) Write down the momentum of each cart *with its sign*. What is the total momentum?

(b) Before calculating, predict whether the carts move right, move left, or stop after the collision. Explain your prediction.

(c) Find the final velocity (magnitude and direction).

(d) How much kinetic energy is lost?

★B3. For a completely inelastic collision in which the target is initially at rest, show that the fraction of the initial kinetic energy that is lost is $m_B/(m_A + m_B)$. Check it against B1. Use it to explain why a bullet that embeds in a much more massive block loses almost all of its kinetic energy in the collision (connect this to the ballistic pendulum).

★B4. In B2, what speed would cart B need for the carts to be at rest after they stick? How much kinetic energy is lost in that case?

Part C: Guided practice: a two-dimensional completely inelastic collision

At an icy intersection (assume negligible friction between the tires and the road), a 1500 kg SUV traveling east at $15\,\mathrm{m\,s^{-1}}$ collides with a 1000 kg compact car traveling north at $20\,\mathrm{m\,s^{-1}}$. The vehicles lock together. We will find the velocity of the wreckage just after the collision one step at a time. Take $+x$ east and $+y$ north.



C1. Compute the momentum magnitude $p = mv$ of each vehicle. Which vehicle has the larger speed? Which has the larger momentum? On the Before picture, draw each vehicle's momentum as an arrow whose length reflects its size.

C2. Take the system to be *both vehicles*. List the external forces on the system. Is the x-component of the total momentum conserved? Is the y-component conserved? Explain each in a sentence.

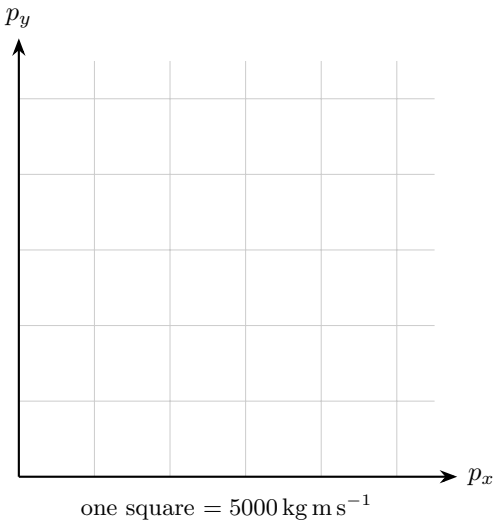
C3. Fill in the table of initial velocity and momentum components.

	m (kg)	v_x (m/s)	v_y (m/s)	p_x (kg·m/s)	p_y (kg·m/s)
SUV	1500				
Compact car	1000				
Total (system)	2500				

C4. Apply conservation of momentum separately in each direction. The wreckage has total mass $m_{\mathrm{SUV}} + m_{\mathrm{car}}$ and velocity components $(v_{f,x},\, v_{f,y})$.
x: _____ $= (m_{\mathrm{SUV}} + m_{\mathrm{car}})\, v_{f,x}$ $\quad \Rightarrow v_{f,x} =$ _____
y: _____ $= (m_{\mathrm{SUV}} + m_{\mathrm{car}})\, v_{f,y}$ $\quad \Rightarrow v_{f,y} =$ _____

C5. Find the speed and direction of the wreckage (give the angle above the $+x$ axis, that is, north of east). Then check that your answer is reasonable: Should the angle be closer to east or to north? Why? Should the final speed be larger or smaller than $15\,\mathrm{m\,s^{-1}}$? Why?

C6. Momentum is a vector, so you can check your answer with a picture. Using the grid below (one square = $5000\,\mathrm{kg\,m\,s^{-1}}$), draw $\vec{p}_{\mathrm{SUV}}$ starting at the origin, then draw $\vec{p}_{\mathrm{car}}$ starting at the *tip* of $\vec{p}_{\mathrm{SUV}}$. Draw the vector from the origin to the final tip. Measure its length and angle. Does this match $M\vec{v}_f$ from C5?



C7. Compute the total kinetic energy before and after the collision. What fraction of the initial kinetic energy was lost? Momentum was conserved, so where did the energy go?

★C8. Suppose the SUV's speed is unchanged. How fast would the compact car have to be traveling for the wreckage to head exactly northeast (45° north of east)? What would the speed of the wreckage be?

Before you leave. In one sentence, write the first thing you should check before using conservation of momentum on a problem.