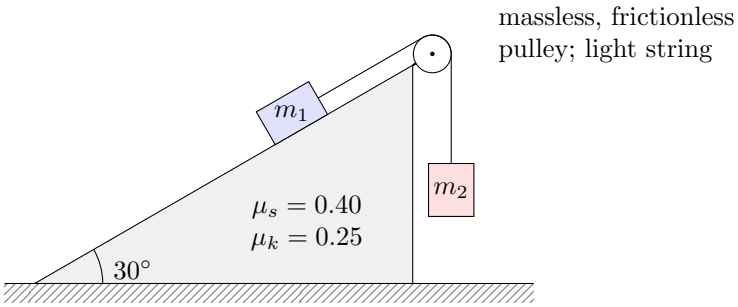
PHY 121 Homework: Newton's Laws and Energy Conservation

Show your work. Draw a diagram for every problem, explain your reasoning in words, and check units and reasonableness for every numerical answer. Items marked ★ are optional extensions.

Part A: Newton's second law with two blocks, a string, and friction

A1. Block 1 ($m_1 = 4.0\,\text{kg}$) sits on a rough incline that makes an angle $\theta = 30^\circ$ with the horizontal ($\mu_s = 0.40$, $\mu_k = 0.25$). A light string runs parallel to the incline, over a massless, frictionless pulley at the top, and is attached to a hanging block 2 ($m_2 = 5.0\,\text{kg}$). Both blocks are released from rest.



(a) Draw a free-body diagram for each block. Label every force with the object that exerts it (for example, "$\vec{N}$ from the incline on block 1").

(b) Will the blocks start to move when released? Decide using forces, not a hunch, and say what you compared. If they move, which way does each block go?

(c) Choose a positive direction for each block that follows the string. Why can you use the same acceleration a for both blocks? Write Newton's second law along your chosen direction for each block. (What does Newton's second law say about the direction perpendicular to the incline?)

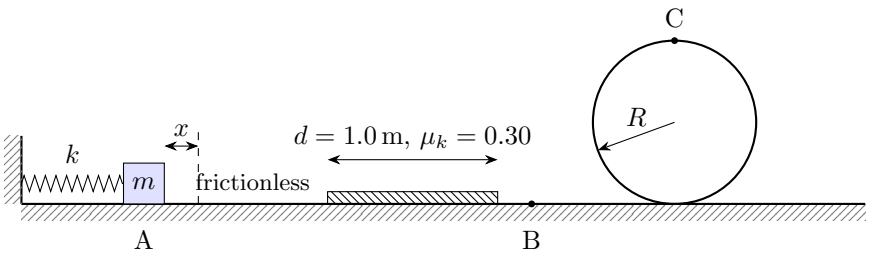
(d) Solve for the acceleration of the blocks and the tension in the string.

(e) **Student 1:** "Block 2 is hanging from the string, so the tension equals its weight: $T = m_2 g$." **Student 2:** "If $T = m_2 g$, block 2 would not accelerate. The tension has to be smaller than $m_2 g$." Which student do you agree with? Rank T, $m_2 g$, and $m_1 g \sin\theta$ from largest to smallest and justify the ranking with your free-body diagrams.

★A2. Keep m_1 and the incline the same but change m_2. What is the *largest* value of m_2 for which the blocks stay at rest when released? Which way does friction on block 1 point in that case? Is your answer consistent with what you found in A1(b)?

Part B: Energy conservation with a spring, a rough patch, and a loop

B1. A block ($m = 0.50\,\text{kg}$) is pressed against a spring ($k = 1000\,\text{N m}^{-1}$), compressing it by $x = 0.15\,\text{m}$, and released from rest at A. The track is frictionless except for a rough patch of length $d = 1.0\,\text{m}$ with $\mu_k = 0.30$. After the patch the block enters a vertical circular loop of radius $R = 0.40\,\text{m}$. Use $y = 0$ at the level of the track.



(a) Consider three stages: (i) from release until the block leaves the spring, (ii) across the rough patch, (iii) from the bottom of the loop to the top (C). For each stage, which forces do work on the block, and is that work positive, negative, or zero?

(b) Draw energy bar charts (K, U_g, U_s, and thermal energy E_{th}) for the block at A, at B (just after the patch), and at C (top of the loop). Put numbers on the bars.

B2. What is the speed of the block just before the rough patch? What is its speed at B? What happened to the kinetic energy that is "missing"?

B3. **Student 1:** "At the very top of the loop the block is at its highest point, so the track isn't pushing on it: $N = 0$." **Student 2:** "The block is still moving quickly at the top, so the track has to push down on it." (a) Find the speed of the block at C. (b) Draw a free-body diagram of the block at C and write Newton's second law for the radial direction (toward the center of the circle). (c) Find the normal force the track exerts at C. Which student is right?

B4. What is the *smallest* spring compression for which the block just stays on the track all the way around the loop? (Think about what is true of N at the critical speed.) Compare your answer to the actual compression of 0.15 m.

★B5. Suppose the spring is compressed by 0.30 m instead of 0.15 m. Before you calculate, predict whether the normal force at C doubles. Then check your prediction.