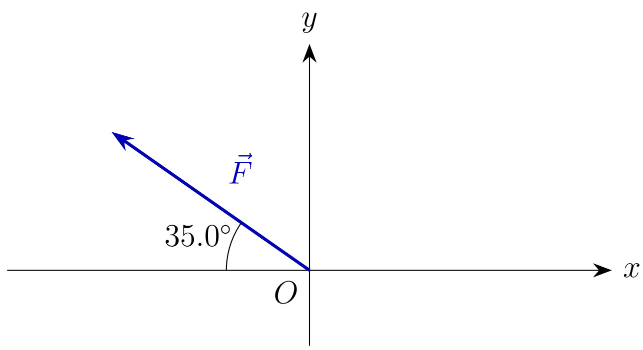
Two-Dimensional Vectors

Components, Magnitude, Direction, and Unit Vectors

Report all numerical answers to three significant figures. Measure angles counterclockwise from the $+x$ axis unless the problem says otherwise.

Question 1: Magnitude and Angle → Components

A rope pulls on a crate with a force of magnitude $F = 25.0\,\mathrm{N}$. The rope makes an angle of $35.0°$ above the $-x$ axis, as shown.



(a) Find the x- and y-components of $\vec{F}$. Be careful with signs.

(b) Write $\vec{F}$ in unit-vector notation, $\vec{F} = F_x\,\hat{\imath} + F_y\,\hat{\jmath}$.

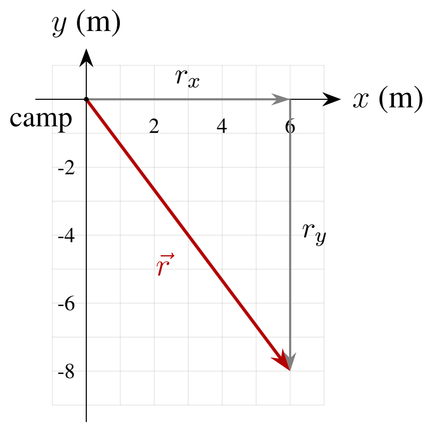
(c) What is the direction of $\vec{F}$ measured counterclockwise from the $+x$ axis?

(d) Find the unit vector $\hat{F}$ that points in the direction of $\vec{F}$. Verify that its magnitude is 1.

Question 2: Components $\rightarrow$ Magnitude and Angle

A hiker's displacement from camp is

$$\vec{r} = (6.00\,\mathrm{m})\,\hat{\imath} + (-8.00\,\mathrm{m})\,\hat{\jmath}.$$



(a) How far is the hiker from camp? (Find $|\vec{r}|$.)

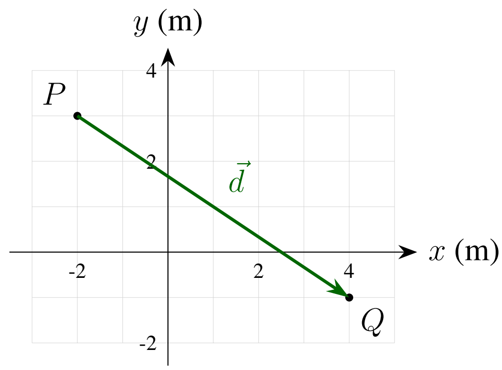
(b) Find the angle of $\vec{r}$ measured counterclockwise from the $+x$ axis. Also express the direction as an angle below the $+x$ axis.

(c) Find the unit vector $\hat{r}$ in the direction of $\vec{r}$.

(d) A second hiker walks in exactly the same direction but covers a distance of $25.0\,\mathrm{m}$. Use $\hat{r}$ to write the second hiker's displacement in unit-vector notation.

Question 3: Vector Between Two Points

A robot moves in a straight line from point $P = (-2.00\,\mathrm{m},\ 3.00\,\mathrm{m})$ to point $Q = (4.00\,\mathrm{m},\ -1.00\,\mathrm{m})$. Let $\vec{d}$ be its displacement.



(a) Find the components d_x and d_y of the displacement, and write $\vec{d}$ in unit-vector notation.

(b) Find the magnitude of $\vec{d}$.

(c) Find the direction of $\vec{d}$, measured counterclockwise from the $+x$ axis.

(d) Find the unit vector $\hat{d}$ in the direction of the motion.

(e) The robot then returns from Q back to P. Write the return displacement in unit-vector notation and state its magnitude and direction. How is it related to $\vec{d}$?

Question 4: Conceptual understanding of accelerated motion.

Consider the following familiar (hopefully) kinematic equation:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

In your own words, give a physical interpretation of each of the three terms on the right side of the equation.

Question 5: 1D & 2D kinematics

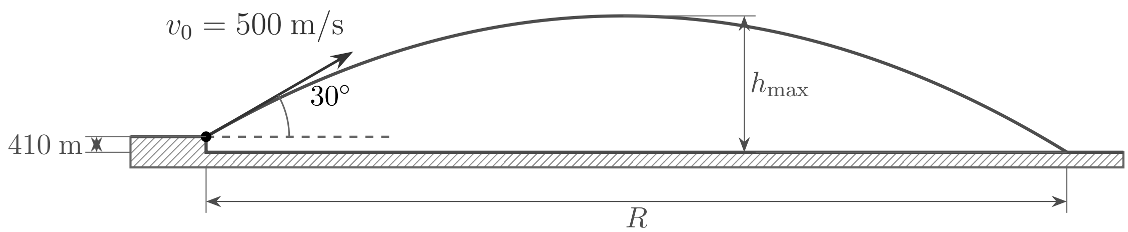
Consider a bullet shot from a gun with a 1 m long barrel. The speed of the bullet as it leaves the barrel is $400\ \frac{\text{m}}{\text{s}}$.

1. Calculate the acceleration, assumed constant, of the bullet while it travels through the barrel.
2. If the gun was angled parallel to the ground, calculate the distance the bullet traveled before hitting the ground. Assume the person is holding the gun 1.5 m above the ground.

Question 6: Projectile motion

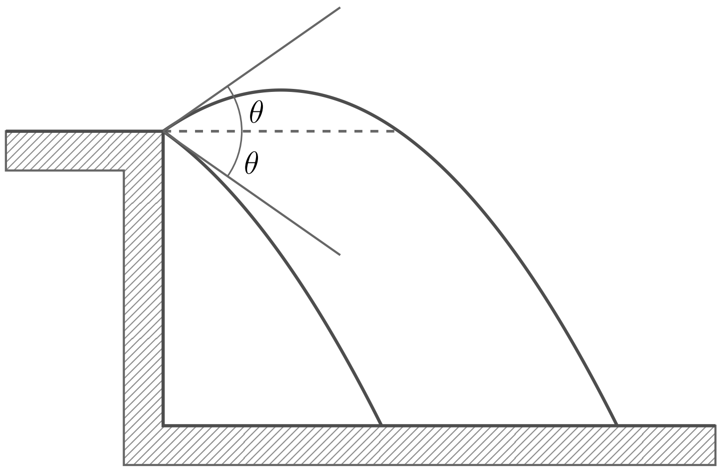
A cannonball is fired from the top of half dome (height = 410 m) at an initial angle of $30°$ and initial speed of 500 m/s.

1. How long after launch does the cannonball reach it's maximum height above the ground.
2. What is this maximum height h_{max} that the cannonball achieves during its flight?
3. How long is it until the cannonball impacts with the ground.
4. What is the maximum distance R that the cannonball travels?



Question 7: Symbolic 2D projectile question

Two balls are thrown with the same speed v_0 from the top of a cliff. The angles of their initial velocities are θ above and below the horizontal, as shown in the figure below. How much farther along the ground does the top ball hit than the bottom ball? Hint: The two trajectories have a part in common.



Solutions

Question 1

(a) The vector lies in the second quadrant, so $F_x < 0$ and $F_y > 0$. Using the $35.0°$ angle measured from the $-x$ axis:

$$F_x = -F\cos 35.0° = -(25.0\,\mathrm{N})(0.8192) = -20.5\,\mathrm{N},$$
$$F_y = +F\sin 35.0° = (25.0\,\mathrm{N})(0.5736) = 14.3\,\mathrm{N}.$$

(b) $\vec{F} = (-20.5\,\mathrm{N})\,\hat{\imath} + (14.3\,\mathrm{N})\,\hat{\jmath}$.

(c) Counterclockwise from $+x$: $\theta = 180° - 35.0° = 145°$.
(Check: $\tan^{-1}(14.3/(-20.5)) = -35.0°$; adding $180°$ because $F_x < 0$ gives $145°$.)

(d)

$$\hat{F} = \frac{\vec{F}}{|\vec{F}|} = \frac{(-20.5\,\mathrm{N})\,\hat{\imath} + (14.3\,\mathrm{N})\,\hat{\jmath}}{25.0\,\mathrm{N}} = -0.819\,\hat{\imath} + 0.574\,\hat{\jmath}.$$

Equivalently $\hat{F} = \cos 145°\,\hat{\imath} + \sin 145°\,\hat{\jmath}$. Check: $\sqrt{0.819^2 + 0.574^2} = \sqrt{0.671 + 0.329} = 1.00$. ✓

Question 2

(a) $|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(6.00)^2 + (-8.00)^2}\ \mathrm{m} = \sqrt{100}\ \mathrm{m} = 10.0\,\mathrm{m}$.

(b) The reference angle is $\tan^{-1}\left(\dfrac{8.00}{6.00}\right) = 53.1°$. Since $r_x > 0$ and $r_y < 0$ the vector is in the fourth quadrant, so

$$\theta = 360° - 53.1° = 307° \quad \text{(counterclockwise from } +x\text{)},$$

or equivalently $53.1°$ below the $+x$ axis.

(c)

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{6.00\,\hat{\imath} - 8.00\,\hat{\jmath}}{10.0} = 0.600\,\hat{\imath} - 0.800\,\hat{\jmath}.$$

(d) $\vec{r}_2 = (25.0\,\mathrm{m})\,\hat{r} = (15.0\,\mathrm{m})\,\hat{\imath} + (-20.0\,\mathrm{m})\,\hat{\jmath}$.
(Check: $\sqrt{15.0^2 + 20.0^2} = 25.0\,\mathrm{m}$. ✓)

Question 3

(a) Components are (final) − (initial):

$$d_x = Q_x - P_x = 4.00 - (-2.00) = 6.00\,\mathrm{m},$$
$$d_y = Q_y - P_y = -1.00 - 3.00 = -4.00\,\mathrm{m},$$

so $\vec{d} = (6.00\,\mathrm{m})\,\hat{\imath} + (-4.00\,\mathrm{m})\,\hat{\jmath}$.

(b) $|\vec{d}| = \sqrt{6.00^2 + (-4.00)^2}\ \mathrm{m} = \sqrt{52.0}\ \mathrm{m} = 7.21\,\mathrm{m}$.

(c) Reference angle $\tan^{-1}(4.00/6.00) = 33.7°$. The vector is in the fourth quadrant ($d_x > 0$, $d_y < 0$), so

$$\theta = 360° - 33.7° = 326°,$$

i.e. $33.7°$ below the $+x$ axis.

(d)

$$\hat{d} = \frac{6.00\,\hat{\imath} - 4.00\,\hat{\jmath}}{7.21} = 0.832\,\hat{\imath} - 0.555\,\hat{\jmath}.$$

(e) The return trip is $\vec{d'} = \vec{P} - \vec{Q} = (-6.00\,\mathrm{m})\,\hat{\imath} + (4.00\,\mathrm{m})\,\hat{\jmath} = -\vec{d}$. Its magnitude is the same, $7.21\,\mathrm{m}$, and its direction is $326° - 180° = 146°$ (second quadrant). Reversing a vector keeps its magnitude and adds $180°$ to its direction; the unit vector is $-\hat{d}$.