Work, Kinetic Energy, and Potential Energy

PHY 121 Group Tutorial

Names: _____ _____ _____ _____

Part	Min.	Topic
A	10	Work and the dot product
B	12	The work–kinetic energy theorem
C	12	Gravitational potential energy: $U_g = mgy$
D	12	The spring force and spring potential energy
E	12	Synthesis: a spring-launched block on an incline
	3	Wrap-up

Conventions. Name every work as "work done *by* X *on* Y," for example $W_{\text{rope on crate}}$. For a *constant* force $\vec{F}$ acting on an object that undergoes a displacement $\Delta\vec{r}$,

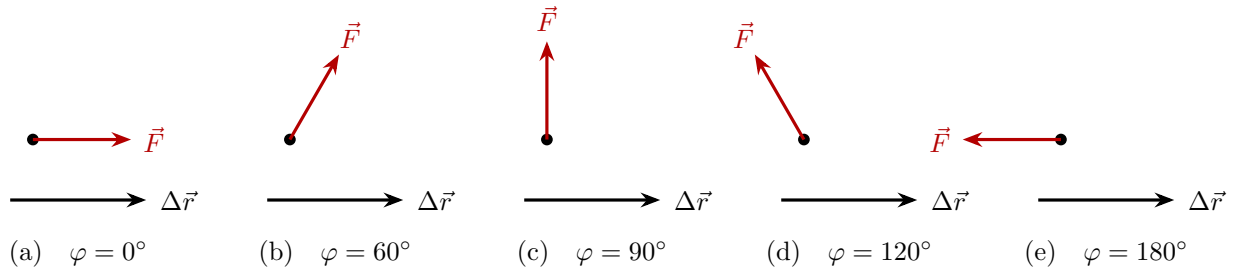
$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r \cos\varphi = F_x \Delta x + F_y \Delta y,$$

where φ is the angle between $\vec{F}$ and $\Delta\vec{r}$ when they are drawn tail to tail. Work is a scalar; its SI unit is the joule ($1\,\text{J} = 1\,\text{N m}$). Use $g = 9.8\,\text{m s}^{-2}$ throughout. Springs are ideal and massless; strings and surfaces are frictionless unless stated.

Part A: Work and the dot product

(about 10 min)

Five students each push a block through the same displacement $\Delta\vec{r}$ of magnitude $\Delta r = 2.0\,\text{m}$, directed to the right. In every case the applied force has the same magnitude, $F = 10\,\text{N}$, but a different direction. The angle φ between $\vec{F}$ and $\Delta\vec{r}$ is given under each panel.



A1. Predict (before any calculation): Rank the five situations by the work done by the applied force, from most positive to most negative. Indicate ties. Explain the reasoning behind your ranking in a sentence.

A2. Calculate the work done by the applied force in each of the five situations. Does your ranking from A1 agree? In terms of the force component along the displacement, what does a *negative* work mean physically?

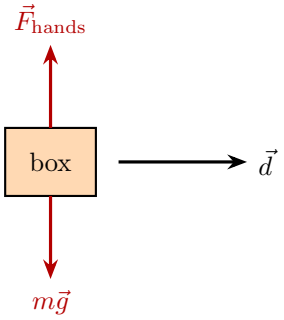
A3. The dot product can also be evaluated from components. A single force $\vec{F} = (6.0\,\hat{\imath} + 2.0\,\hat{\jmath})\,\text{N}$ acts on an object while it undergoes the displacement $\Delta\vec{r} = (3.0\,\hat{\imath} - 1.0\,\hat{\jmath})\,\text{m}$.

(a) Find the work done by $\vec{F}$ using the component form.

(b) Find the magnitudes F and Δr, and then use $W = F\Delta r\cos\varphi$ to find the angle φ between $\vec{F}$ and $\Delta\vec{r}$.

(c) Is the sign of W consistent with the angle you found? Why?

A4. A student carries a box of mass m across a room, holding it at constant height and walking at *constant velocity* through a displacement d to the right.



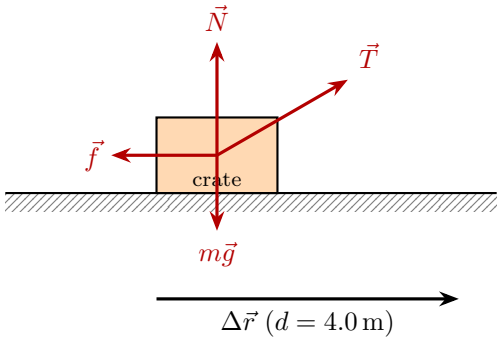
Student 1: "My hands exert an upward force mg and the box moves a distance d, so I do work mgd on the box."
Student 2: "The box doesn't change its height, so the work is zero."

(a) Which student do you agree with? Use the definition of work, including the angle, to justify your answer.

(b) What is the work done by gravity on the box? What is the *net* work on the box? Is that consistent with the box moving at constant velocity?

(c) Last week, a puck moved at constant speed on a circle on a frictionless table. What is the work done by the string on the puck during one quarter of a turn? Explain using φ.

A5. A crate of mass 20 kg is pulled along a rough floor by a rope. The rope exerts a tension $T = 50\,\text{N}$ at 30° above the horizontal, and the floor exerts a kinetic friction force $f = 15\,\text{N}$. The crate moves a distance $d = 4.0\,\text{m}$ to the right.



(a) Complete the table.

Force on crate	Magnitude (N)	Angle φ with $\Delta\vec{r}$	Work (J)
Rope ($\vec{T}$)			
Friction ($\vec{f}$)			
Gravity ($m\vec{g}$)			
Normal ($\vec{N}$)			
Net work (sum)			

(b) Which forces do positive work, negative work, and zero work? Why does the work done by the normal force and by gravity vanish here?

Group checkpoint

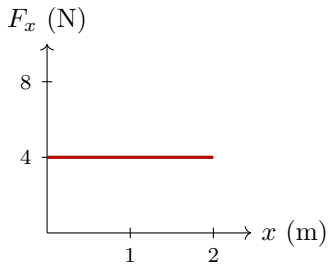
Does the sign of the work depend on whether a force is "helping" or "opposing" the motion? State in one sentence what determines the sign of the work done by a force. Show your table to an LA before moving on.

★ **A6.** For the crate in A5, find the *net force* vector (components) and compute $\vec{F}_{\text{net}} \cdot \Delta\vec{r}$. Compare with the sum of the individual works in your table.

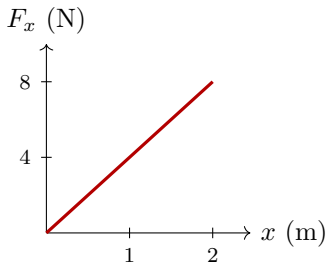
★ **A7.** For $\vec{F} = (6.0\,\hat{\imath} + 2.0\,\hat{\jmath})\,\mathrm{N}$, find a displacement $\Delta\vec{r}$ (give components) for which the force does zero work. Is the answer unique?

Part B: The work–kinetic energy theorem *(about 12 min)*

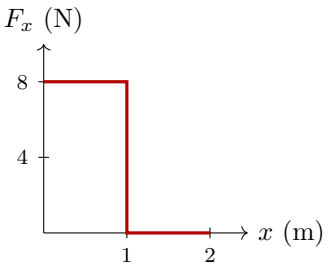
Three identical carts of mass $m = 1.0\,\text{kg}$ start from rest on a frictionless horizontal track. Each cart is pushed along the track by a force F_x that depends on position as shown below. Each push ends at $x = 2.0\,\text{m}$.



Cart 1



Cart 2



Cart 3

B1. Predict (before any calculation): After the push ends at $x = 2.0\,\text{m}$, which cart has the greatest speed? Rank the three, and indicate ties. Write your reasoning.

B2. On the graphs, shade the region that represents the work done by the applied force for each cart. Why does the area under an F_x vs. x graph give the work? Find the work done on each cart. Does your ranking in B1 survive?

B3. The work–kinetic energy theorem states that the net work done on an object equals its change in kinetic energy, $W_{\text{net}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.

(a) Use it to find the final speed of each cart at $x = 2.0\,\text{m}$.

(b) Does the work–kinetic energy theorem care *how* the force varies along the way, or only about the net work? Explain.

(c) In the case of cart 3, what is the cart's speed between $x = 1.0\,\text{m}$ and $x = 2.0\,\text{m}$?

B4. A car skids to a stop with its wheels locked. The kinetic friction force is (approximately) constant. When the car's initial speed is $20\,\text{m}\,\text{s}^{-1}$ the skid is 40 m long.

> **Student 1:** "If the car is going twice as fast, the skid should be twice as long, because the speed is twice as large."
> **Student 2:** "I think it will be four times as long. Kinetic energy depends on the speed *squared*."

(a) Which student do you agree with? Write the work–kinetic energy theorem for the skid and use it to support your answer. What is the sign of the work done by friction? What is the sign of ΔK?

(b) Find the skid length when the initial speed is $40\,\mathrm{m\,s^{-1}}$.

(c) Find the kinetic friction force in terms of the car's mass m, then the coefficient of kinetic friction μ_k. Does the answer depend on the mass of the car?

B5. Return to the crate in A5 ($m = 20\,\mathrm{kg}$). Suppose it starts from rest. Use the net work from your table in A5 to find the crate's speed after being pulled 4.0 m.

Group checkpoint

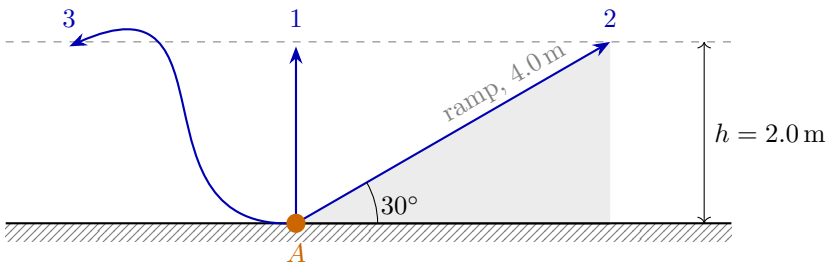
In B2, why did three very different force histories lead to the same final speed? In one sentence, say which quantity the final speed depends on, and which quantity it does *not* depend on. Show your answer to an LA.

★ **B6.** Check your answer to B5 a different way: find the net horizontal force on the crate, the acceleration, and then use kinematics to get the speed after 4.0 m. Which method was easier? When would the energy method be much easier?

★ **B7.** A ball is tossed straight up and caught at the height where it was released, so the net work done on it is zero. Is the ball's speed constant throughout the toss? What does "zero net work" guarantee about the speed, and what does it *not* guarantee?

Part C: Gravitational potential energy: $U_g = mgy$ *(about 12 min)*

A ball of mass $m = 0.50\,\mathrm{kg}$ is moved *slowly* from point A on the ground to a height $h = 2.0\,\mathrm{m}$ along three different paths: (1) straight up; (2) up a smooth 30° ramp of length 4.0 m; and (3) along a curved track.



C1. Predict (before any calculation): Rank the three paths by the magnitude of the work done by gravity on the ball. Explain your reasoning.

Student 1: "The ramp path is longer, so gravity does more work on the ball there."
Student 2: "Gravity only pulls vertically. All three paths have the same change in height, so I think the work is the same."

C2. Calculate the work done by gravity on the ball for (a) path 1 and (b) path 2 using $W = \vec{F} \cdot \Delta\vec{r}$ with the angle between $m\vec{g}$ and $\Delta\vec{r}$. (c) Without a new calculation, explain why the curved path 3 must give the same answer. Who in the dialogue was right?

C3. The result in C2 says that the work done by gravity depends only on the starting and ending *heights*. We use this to define the gravitational potential energy through $\Delta U_g = -W_{\text{gravity}}$.

(a) Show that $\Delta U_g = mg(y_f - y_i)$, so that we can take $U_g = mgy$.

(b) Why does the formula $U_g = mgy$ require us to choose where $y = 0$?

C4. A book of mass 2.0 kg sits on a table whose top is 0.80 m above the floor. It is then lifted to a shelf 1.50 m above the floor. Three students choose different locations for $y = 0$ and calculate the book's gravitational potential energy.

Student's origin ($y = 0$ at…)	U_g on table (J)	U_g on shelf (J)	ΔU_g (J)
the floor			
the table top			
the ceiling, 2.5 m above the floor			

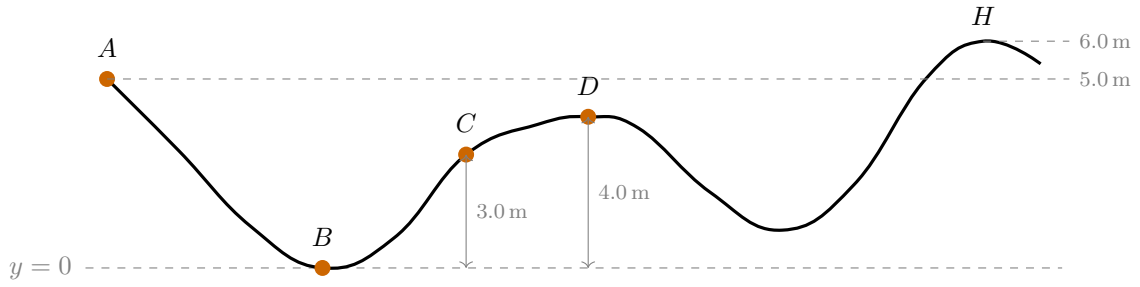
Student 1: "My answer for the book on the table is 0 J."
Student 2: "Mine is 15.7 J." **Student 3:** "Mine is negative!"

Who is right? Which quantity in the table is the same for every choice, and which is the one with physical consequences?

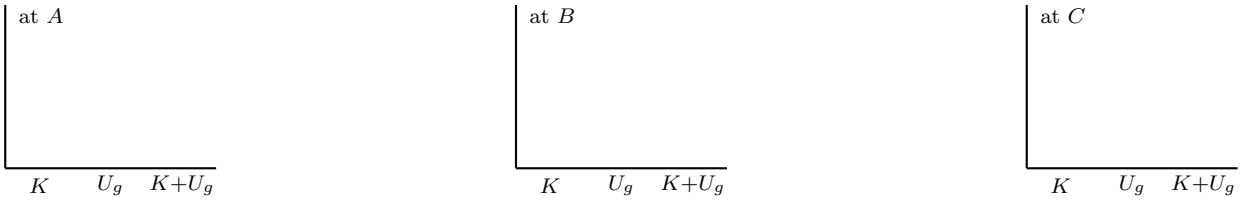
Group checkpoint

Compare your table in C4 with a neighboring group. Agree on a one-sentence statement of why the choice of $y = 0$ never affects the physics. Show it to an LA.

A small ball of mass $m = 1.0$ kg is released from rest at point A on a frictionless track. The vertical heights above $y = 0$ are marked. Neglect air resistance.



C5. In the bar charts below, show the ball's kinetic energy K, its gravitational potential energy U_g (with $y = 0$ at point B), and the total $K + U_g$ at points A, B, and C. Use the same vertical scale in all three charts, and write the value of each bar in joules.



C6. Rank the *speeds* of the ball at A, B, C, and D. Then find the speed at B, at C, and at D.

C7. Will the ball reach the top of hill H? If not, describe what the ball does, and at what height it first stops momentarily. Use energy.

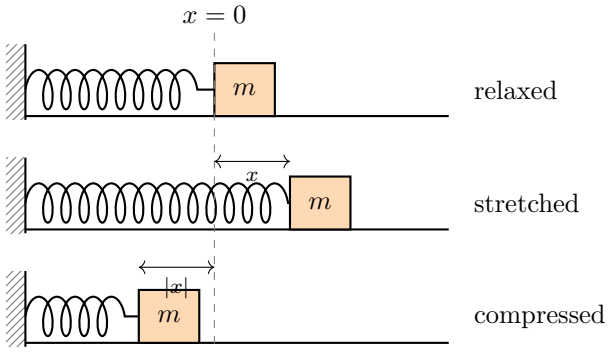
Group checkpoint

Suppose the track had friction. Which of your statements above would fail, and why? Why does the idea of a potential energy U_g require that the work done by gravity be independent of path? Show your answer to an LA.

★ **C8.** Three identical balls are thrown from the same cliff edge with the *same speed* v_0: one straight up, one horizontally, and one straight down. Rank the speeds of the balls just before they hit the ground far below. Justify your answer using energy, then explain why the *times* of flight differ even though the final speeds do not.

Part D: The spring force and spring potential energy *(about 12 min)*

A block of mass m rests on a frictionless table and is attached to one end of an ideal spring of spring constant k. The other end of the spring is fixed to a wall. The x-axis points to the right, and $x = 0$ is the block's position when the spring is relaxed.

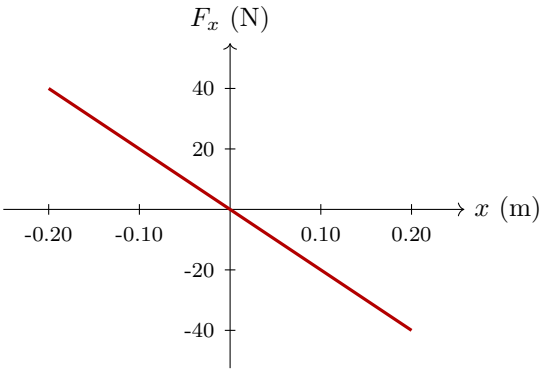


D1. Predict (before any calculation): The block starts at $x = 0$ and is moved slowly to $x = +d$ (stretched) and then back to $x = 0$.

(a) In each of the three pictures, draw an arrow on the block for the force exerted by the spring. Does the sign of F_x (spring on block) match the sign of x, or is it opposite? Write Hooke's law with the correct sign.

(b) Is the work done *by the spring* on the block positive, negative, or zero (i) while the block moves from $x = 0$ to $x = +d$, and (ii) while it moves from $x = +d$ back to $x = 0$? Explain with the angle φ.

D2. The spring force changes as the block moves, so $W = F\Delta x \cos\varphi$ cannot be used directly. The graph shows the force exerted *by the spring on the block* versus x for a spring with $k = 200\,\mathrm{N\,m^{-1}}$.



(a) Shade the area between the graph and the x-axis for the stretch from $x = 0$ to $x = 0.10\,\mathrm{m}$ (call it region 1) and from $x = 0.10\,\mathrm{m}$ to $x = 0.20\,\mathrm{m}$ (region 2).

(b) **Predict** (before calculating): is the work done by the spring in region 2 less than, equal to, or greater than the work in region 1?

(c) Find the work done by the spring on the block in region 1, then in region 2. Check the signs against your answer in D1.

(d) Find the work that an outside agent (a hand moving the block slowly) must do in each region. Which region required more effort?

D3. Generalize: show that the work done *by the spring* as the block moves from $x = 0$ to a position x is $W_{\text{spring}} = -\frac{1}{2}kx^2$. Use the area of a triangle on the graph of F_x vs. x. Then explain why defining $\Delta U_s = -W_{\text{spring}}$ leads to the spring potential energy $U_s = \frac{1}{2}kx^2$ (with $U_s = 0$ at $x = 0$).

D4. Two students discuss a spring compressed by 0.10 m with $k = 200\,\mathrm{N\,m^{-1}}$.

> **Student 1:** "The spring is compressed, so $x = -0.10\,\mathrm{m}$, which means the spring potential energy is negative, $U_s = -1.0\,\mathrm{J}$."
> **Student 2:** "Hold on. x is squared in the formula. Is a compressed spring really lower in energy than a relaxed one?"

(a) Who is right? Compare U_s for compression and stretch by the same amount. Can $U_s = \frac{1}{2}kx^2$ ever be negative?

(b) By what factor does U_s change if the stretch is doubled? Compare with what you found in D2.

Group checkpoint

Compare your answers to D2(b) and D2(c) with a neighboring group. In your own words, why does it cost three times as much work to stretch the spring from 0.10 m to 0.20 m as from 0 to 0.10 m? Show your answer to an LA.

D5. A block of mass $m = 0.50\,\mathrm{kg}$ is pressed against the spring ($k = 200\,\mathrm{N\,m^{-1}}$), compressing it by 0.10 m, and is released from rest on the frictionless table. The block leaves the spring when the spring reaches its relaxed length.

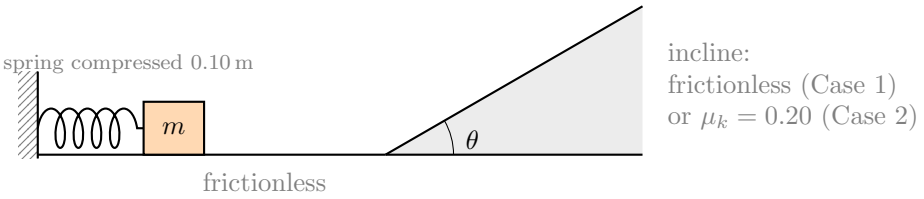
(a) Use energy to find the block's speed as it leaves the spring.

(b) Is there any work done by gravity or the normal force on the block? Why do they not appear in your equation?

★ **D6.** (a) What is the block's speed if the spring is compressed by 0.20 m instead? (b) What if the mass of the block is four times larger (2.0 kg), with the compression still 0.10 m? (c) Explain why you could not solve D5 by applying the constant-acceleration equations from earlier in the semester.

Part E: Synthesis: a spring-launched block on an incline *(about 12 min)*

The block from D5 ($m = 0.50\,\mathrm{kg}$, $k = 200\,\mathrm{N\,m^{-1}}$, compression 0.10 m) is released from rest. It leaves the spring, slides along a frictionless horizontal table, and then slides up an incline of angle $\theta = 30°$. There are two cases: in **Case 1** the incline is frictionless, and in **Case 2** the incline is rough with $\mu_k = 0.20$. Let d be the distance along the incline that the block travels before it stops momentarily.

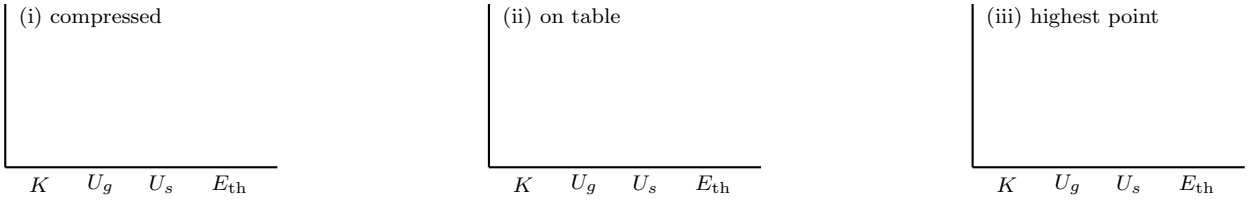


E1. Predict (before any calculation).

(a) In which case does the block climb to the greater distance d along the incline: Case 1, Case 2, or the same? Where does the energy go in Case 2?

(b) In which case is the block moving faster when it later comes back to the bottom of the incline (assuming it slides back down in Case 2)?

E2. For Case 1, complete the energy bar charts (with $y = 0$ at the table) for the three instants, using the same vertical scale for all three: (i) the spring compressed and the block at rest, (ii) the block moving on the table after leaving the spring, and (iii) the block at its highest point on the incline.



E3. Case 1. Write the energy equation from (i) to (iii) and find the maximum height h above the table and the distance d along the incline.

E4. Two students discuss Case 2.

> **Student 1:** "With friction, mechanical energy isn't conserved. So we can't use energy methods for this problem."
> **Student 2:** "We can still use energy. Friction does negative work on the block, and the energy appears as thermal energy, E_{th}."

Which student do you agree with? Write the work–energy statement

$$K_i + U_{g,i} + U_{s,i} + W_{\text{friction}} = K_f + U_{g,f} + U_{s,f}$$

for Case 2 (from the instant the spring is compressed and the block is at rest to the highest point on the incline). Express W_{friction} in terms of μ_k, m, g, θ, and d (use the normal force on the incline), and find d. Compare with Case 1.

Group checkpoint

Check your Case 2 value of d with a neighboring group. How much of the original spring energy ends up as thermal energy during the ascent? Is d smaller or larger than in Case 1, as you predicted in E1(a)? Show your reasoning to an LA.

★ **E5.** In Case 2, the block comes to rest momentarily. The coefficient of static friction between the block and incline is $\mu_s = 0.40$. Will the block stay at rest or slide back down? Justify using forces. If it slides back, use energy to find its speed when it returns to the bottom of the incline.

★ **E6.** Suppose the incline in Case 2 was replaced by a long horizontal rough surface with the same $\mu_k = 0.20$. How far would the block slide before stopping? Compare with the distance along the incline.

Wrap-up (3 min). Each person writes, in their own words, the two most useful ideas from today that they did not fully believe or understand at the start.