

Key to 1 & 2

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0.1 Introduction

It was brought to my attention that on last Fridays class, I was misleading particularly in the differentiation between net force and contact force in problems 1 & 2. I sincerely apologize to those of you who I confused or mislead. Below I have dissected the difference between net force and contact force to clear any waters I may have muddled. Additionally, I have written solution guides for problem 1 & 2 which can also be found below.

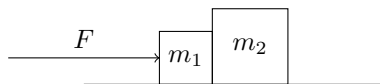
0.2 Net force and contact force

Before solving the problems, I'd like to take a moment to examine both the contact force and the net force. Newton's second law formally states that the sum of all forces acting on an object is equal to the mass of the object times its acceleration ¹, i.e.

$$\sum_{i=0}^N F_i = ma.$$

This sum of forces is the net force which I will represent with F_{net} . The contact force, on the other hand, is the force which keeps solid objects from falling through each other. This arises from the electron clouds around each atom that repel one another. The normal force is in essence the contact force that counteracts the force of gravity when an object is at rest on a surface.

1. Two blocks (m_1 and m_2) are pushed along a horizontal frictionless surface by some force F directed horizontally to the right. (Let $m_1 = 2$ kg, $m_2 = 4$ kg, and $F = 18$ N).



- (a) Draw a free body diagram for each block (only horizontal forces). What is the acceleration of each block?

When considering the two masses as a conglomerate, with mass $M = m_1 + m_2$ and applying Newton's second law, one arrives at

$$F = Ma$$

where a is the acceleration of the system. Thus, we see that

$$a = \frac{F}{M} = \frac{F}{m_1 + m_2}.$$

This is critical in that we now have the accelerations of both blocks. That is,

$$a_1 = a_2 = a = \frac{F}{m_1 + m_2} = \frac{18 \text{ N}}{2 \text{ kg} + 4 \text{ kg}} = 3 \frac{\text{m}}{\text{s}^2}. \quad (1)$$

¹In true formality, Newton's law states that the sum of all forces acting on an object is equal to the first time derivative of momentum. That is, $\sum F_i = \frac{dp}{dt} = \frac{d}{dt}mv = m\frac{dv}{dt} + v\frac{dm}{dt}$. Since we rarely consider scenarios where the mass of an object changes, the time derivative of m is usually 0 which reduces the right side to just ma .

- (b) What is the *magnitude* of the contact force between the two blocks?

Since we have the acceleration of both blocks, we have the net force on blocks 1 and 2. More precisely

$$F_{\text{net}1} = m_1 a \text{ and } F_{\text{net}2} = m_2 a.$$

When considering the system as a conglomerate mass, we ignored any contact forces between the blocks. Since the contact force between any two objects is equal in magnitude and opposite in direction, we are justified in doing so to correctly arrive at the acceleration of both blocks. Now, however, as we consider each block individually, we must reconsider the contact force. Consider

$$F_{\text{net}1} = m_1 a.$$

As discussed above, the net force is the sum of all forces acting on an object. Thus, in the case of m_1 we have

$$F_{\text{net}1} = F + F_{c1} = m_1 a$$

where F_{c1} is the contact force of m_2 acting on m_1 . Solving for F_{c1} gives

$$F_{c1} = m_1 a - F.$$

Since contact forces are equal and opposite, we also have that

$$F_{c2} = -F_{c1} = F - m_1 a.$$

Rewriting in a terms of F , m_1 , and m_2 using (1) and plugging it into the formula for F_{c1} yields

$$F_{c1} = m_1 \left(\frac{F}{m_1 + m_2} \right) - F = F \left(\frac{m_1}{m_1 + m_2} - 1 \right) = F \left(\frac{m_1}{m_1 + m_2} - \frac{m_1 + m_2}{m_1 + m_2} \right) = \frac{-F m_2}{m_1 + m_2}.$$

To find the magnitude, we simply take the absolute value of F_{c1} or F_{c2} , however, as we established $F_{c2} = -F_{c1}$, and thus their absolute values are equal.

Plugging in numbers, one gets that

$$F_{c1} = \frac{-18 \text{ N} \cdot 4 \text{ kg}}{(2 + 4) \text{ kg}} = -12 \text{ N}.$$

Now we take the absolute value to arrive at 12 N.

- (c) What is the net force of the first block m_1 ? Is it larger or smaller than F , and why?.

As established above, the net force on m_1 is given by

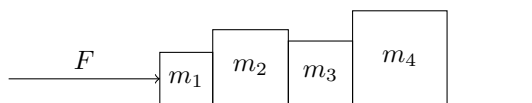
$$F_{\text{net}1} = F + F_{c1} = m_1 a = \frac{F m_1}{m_1 + m_2}.$$

Note that the ratio of m_1 divided by the sum $m_1 + m_2$ is always less than 1. It follows that $F_{\text{net}1} < F$. Thinking about this physically, it should make sense that this is the case since if we applied the 18 N force to just the little block, its acceleration would be $18/2 = 9 \text{ m/s}^2$. Since its actual acceleration is less than 9 we know the net force must be less than F .

- (d) How does the force of the 2 kg on the 4 kg block compare to the force of the 4 kg block on the 2 kg block? Which of Newton's laws tells you this?

From Newton's third law, the force on m_1 by m_2 is equal in magnitude and opposite in direction of the force on m_2 by m_1 . I.e., as we stated, $F_{c1} = -F_{c2}$.

2. Four blocks (m_1, m_2, m_3 , and m_4) are pushed to the right along a horizontal frictionless surface by a force F . (Let $m_1 = 2 \text{ kg}, m_2 = 4 \text{ kg}, m_3 = 3 \text{ kg}, m_4 = 5 \text{ kg}$, and $F = 42 \text{ N}$.)



- (a) What is the acceleration of each block?

In a similar manner to above we will treat the masses as a conglomerate with mass $M = m_1 + m_2 + m_3 + m_4$. Then, applying Newton's second law one has that

$$F = Ma \implies a = \frac{F}{M} = \frac{F}{m_1 + m_2 + m_3 + m_4}.$$

Plugging in numbers we get

$$a = \frac{42 \text{ N}}{(2 + 4 + 3 + 5) \text{ kg}} = 3 \frac{\text{m}}{\text{s}^2}.$$

- (b) What is the *magnitude* contact force between m_3 and m_2 ?

In order to find this, we must first find the contact force between m_1 and m_2 . Consider the forces acting on m_1 ; we have two forces in total one being the 42N force, the other is the contact force F_{c21} of m_2 acting on m_1 . Now we set up Newton's law again for m_1 which gives

$$F_{net1} = F + F_{c21} = m_1 a \implies F_{c21} = m_1 a - F.$$

Rewriting a in terms of F and the masses we have that

$$\begin{aligned} F_{c21} &= m_1 \left(\frac{F}{m_1 + m_2 + m_3 + m_4} \right) - F = F \left(\frac{m_1}{m_1 + m_2 + m_3 + m_4} - 1 \right) = \\ &= F \left(\frac{m_1}{m_1 + m_2 + m_3 + m_4} - \frac{m_1 + m_2 + m_3 + m_4}{m_1 + m_2 + m_3 + m_4} \right) = \frac{-F(m_2 + m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}. \end{aligned}$$

Additionally, like the last question, we also now have $F_{c21} = -F_{c12}$, however, now we have a third block that is pushing back against m_2 . Consider the forces on m_2 , we have the contact force from m_1 pushing it to the right (F_{c12}) and another contact force from m_3 pushing it to the left (F_{c32}). Once again setting up Newton's law yields

$$F_{net2} = F_{c12} + F_{c32} = m_2 a \implies F_{c32} = m_2 a - F_{c12} = m_2 a + F_{c21}.$$

Replacing a like we did before then gives

$$\begin{aligned} F_{c32} &= m_2 \left(\frac{F}{m_1 + m_2 + m_3 + m_4} \right) - \frac{F(m_2 + m_3 + m_4)}{m_1 + m_2 + m_3 + m_4} = F \left(\frac{m_2}{m_1 + m_2 + m_3 + m_4} - \frac{m_2 + m_3 + m_4}{m_1 + m_2 + m_3 + m_4} \right) = \\ &= \frac{-F(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}. \end{aligned}$$

Since we have the contact force from m_3 acting on m_2 given by F_{c32} above, we have that contact from m_2 acting on m_3 . The magnitude of the contact force between m_2 and m_3 is then simply $|F_{c32}| = |-F_{c32}| = |F_{c23}|$.

Plugging in numbers one has

$$|F_{c32}| = \left| \frac{-42(3 + 5) \text{ N} \cdot \text{kg}}{(2 + 4 + 3 + 5) \text{ kg}} \right| = \frac{336}{14} \text{ N} = 24 \text{ N}.$$

- (c) What is the net force on the 5 kg block?

Since we know the blocks acceleration and mass we immediately have that the net force since in general $F_{net} = ma$. This the net force on the 5 kg block is $5 \text{ kg} \cdot 3 \text{ m/s}^2 = 15 \text{ N}$.

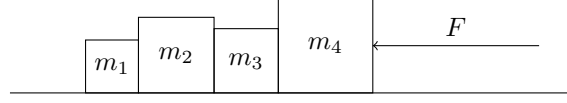
- (d) **New situation:** The 42 N force is now applied to the *right-hand* side of the 5 kg block, pushing the stack to the *left*. Without recalculating from scratch, predict: is the contact force between 3 kg and 4 kg larger, smaller or the same as in (b)? Then compute it.

When the force was applied from the left hand side, the contact force from m_2 acting on m_3 is accelerating the right most two blocks. More precisely it is applying a force to m_3 which in turn pushes on m_4 . Thus, the contact force should from m_2 on m_3 should be sufficient to accelerate of both m_3 and m_4 to exactly 3 m/s^2 . That is to say, if I've done everything right then $F_{c23}/(m_3 + m_4)$ should yield 3 m/s^2 . Indeed we see this is the case as

$$\frac{F_{c23}}{m_3 + m_4} = \frac{24 \text{ N}}{(3 + 5) \text{ kg}} = \frac{24 \text{ N}}{8 \text{ kg}} = 3 \frac{\text{m}}{\text{s}^2}.$$

Since $m_1 + m_2 < m_3 + m_4$ we can reason that if the force were to be applied from the right, then the contact force between blocks 2 and 3 would be smaller since the force on m_2 from m_3 has to accelerate less mass.

To verify let's calculate it.



Note before beginning the calculation, since the magnitude of the applied force is the same, the blocks still have the same acceleration. Additionally, I will consider the applied force to be in the positive direction. In the same fashion as we treated m_1 , we will set up Newton's law for m_4 which gives that

$$F_{net4} = F + F_{c34} = m_4 a \implies F_{c34} = m_4 a - F.$$

Then we have that

$$\begin{aligned} F_{c34} &= m_4 \left(\frac{F}{m_1 + m_2 + m_3 + m_4} \right) - F = F \left(\frac{m_4}{m_1 + m_2 + m_3 + m_4} - 1 \right) = \\ &= F \left(\frac{m_4}{m_1 + m_2 + m_3 + m_4} - \frac{m_1 + m_2 + m_3 + m_4}{m_1 + m_2 + m_3 + m_4} \right) = \frac{-F(m_1 + m_2 + m_3)}{m_1 + m_2 + m_3 + m_4}. \end{aligned}$$

Now that we have the force of m_3 pushing on m_4 we can set up Newton's law for m_3 . The two forces on m_3 are the contact force from m_4 which is $F_{c43} = -F_{c34}$, and the force from m_2 pushing back on m_3 i.e.,

$$F_{net3} = F_{c43} + F_{c23} = F_{c23} - F_{c34} = m_3 a \implies F_{c23} = m_3 a + F_{c34}$$

Thus,

$$F_{c23} = m_3 \left(\frac{F}{m_1 + m_2 + m_3 + m_4} \right) - \frac{F(m_1 + m_2 + m_3)}{m_1 + m_2 + m_3 + m_4} = \frac{-F(m_1 + m_2)}{m_1 + m_2 + m_3 + m_4}.$$

Plugging in numbers and taking the absolute value, one then has

$$|F_{c23}| = \left| \frac{-42(2+4)}{2+4+3+5} \right| \frac{\text{N kg}}{\text{kg}} = 18 \text{ N}.$$

As predicted, the contact magnitude of the force between m_2 and m_3 is less now that the applied forces pushes the blocks to the right.