Practice Exam: Energy, Work, and Rotation of a Rigid Body

Chapters 10 (Energy), 11 (Work), and 12 (Rotation of a Rigid Body)
Use $g = 9.80\,\text{m/s}^2$. Show all work. Problems 1–4 cover Ch. 10, 5–12 cover Ch. 11, and 13–20 cover Ch. 12.

1. *(Ch. 10)* A 0.50 kg block is pressed against a spring ($k = 400\,\text{N/m}$) and compressed 0.20 m. It is released on a frictionless horizontal surface, leaves the spring, and slides up a frictionless 30° incline. (a) What is the block's speed as it leaves the spring? (b) How far along the incline does it travel before stopping momentarily?

2. *(Ch. 10)* A 3.0 kg cart moving at 4.0 m/s collides elastically with a stationary 1.0 kg cart on a frictionless track. Find the velocity of each cart after the collision, and verify that kinetic energy is conserved.

3. *(Ch. 10)* A 0.20 kg particle moves along the x-axis with potential energy $U(x) = U_0\big[(x/a)^4 - 2(x/a)^2\big]$, where $U_0 = 4.0\,\text{J}$ and $a = 0.50\,\text{m}$. (a) Find the equilibrium positions and classify each as stable or unstable. (b) If the particle's total mechanical energy is $E = -2.0\,\text{J}$, find the turning points of the motion for $x > 0$. (c) What is the particle's maximum speed for this energy?

4. *(Ch. 10)* A 2.0 kg block is dropped from rest from 0.50 m above a vertical spring ($k = 1500\,\text{N/m}$) that stands on the floor. What is the maximum compression of the spring?

5. *(Ch. 11)* A 2.0 kg object moving along x has speed 1.0 m/s at $x = 0$. It is acted on by a force $F_x = (6.0x - 2.0x^2)$ N (x in meters). What is its speed at $x = 3.0\,\text{m}$?

6. *(Ch. 11)* A 20 kg crate is pulled 5.0 m across a horizontal floor by a 80 N rope that makes a 30° angle above the horizontal. The coefficient of kinetic friction is 0.25. The crate starts from rest. (a) Find the work done by the rope, by gravity, by the normal force, and by friction. (b) What is the crate's speed after 5.0 m?

7. *(Ch. 11)* A 5.0 kg block slides from rest down a 4.0 m-long incline tilted at 30°. The coefficient of kinetic friction is 0.20. Find the block's speed at the bottom and the thermal energy generated.

8. *(Ch. 11)* A 0.50 kg particle moves along x in a region where $U(x) = 2x^3 - 6x$ (SI units, U in joules). (a) Find the force $F_x(x)$ on the particle. (b) Locate the equilibrium points and classify their stability. (c) The particle is released from rest at $x = 0$. What is its speed when it reaches $x = 1.0\,\text{m}$?

9. *(Ch. 11)* (a) A 1500 kg car accelerates from rest to 25 m/s in 8.0 s on level ground. What average power must the engine deliver to the wheels (ignore drag)? (b) The same car climbs a 5.0° hill at a constant 20 m/s while a 400 N drag force opposes the motion. What power is required?

10. *(Ch. 11)* A 0.80 kg block slides at 5.0 m/s along a rough horizontal surface ($\mu_k = 0.40$) and runs into the end of a horizontal spring ($k = 500\,\text{N/m}$, unstretched, fixed at the far end). The surface is rough everywhere. (a) What is the maximum compression of the spring? (b) Assuming the spring force at maximum compression exceeds the maximum static friction, the block rebounds. What is its speed as it leaves the spring at its relaxed length?

11. *(Ch. 11)* A particle moves through displacement $\vec{d} = (4.0\,\hat{\imath} + 5.0\,\hat{\jmath})$ m while acted on by a constant force $\vec{F} = (3.0\,\hat{\imath} - 2.0\,\hat{\jmath})$ N. Find the work done by the force and the angle between $\vec{F}$ and $\vec{d}$.

12. *(Ch. 11)* A 4.0 kg block rests on a table ($\mu_k = 0.20$) and is connected by a light string over a frictionless, massless pulley to a hanging 2.0 kg block. The system is released from rest. Use energy methods to find the speed of the blocks after the hanging block has dropped 1.5 m.

13. *(Ch. 12)* A flywheel starts from rest and speeds up uniformly to 1200 rpm in 8.0 s. (a) Find its angular acceleration. (b) How many revolutions does it make in this time? (c) At the end of the 8.0 s, what are the speed and the centripetal and tangential accelerations of a point 0.15 m from the axis?

14. *(Ch. 12)* A uniform rod of mass 2.0 kg and length 1.50 m has a small 0.50 kg ball attached to one end (treat the ball as a point mass). (a) Find the moment of inertia about an axis perpendicular to the rod through the end *without* the ball. (b) Find the moment of inertia about a parallel axis through the system's center of mass.

15. *(Ch. 12)* A 2.0 kg block hangs from a light string wrapped around the rim of a solid disk pulley ($M = 4.0$ kg, $R = 0.20$ m) that rotates on a frictionless horizontal axle. The block is released from rest. (a) Find the block's acceleration and the string tension. (b) Find the block's speed after it has fallen 2.0 m, and check it using energy conservation.

16. *(Ch. 12)* A uniform beam (25 kg, 4.0 m) is attached to a vertical wall by a frictionless hinge at one end and is held horizontal by a cable attached to the far end. The cable makes a 30° angle with the beam and is attached to the wall above the hinge. A 50 kg sign hangs from the beam 3.0 m from the wall. (a) Find the tension in the cable. (b) Find the horizontal and vertical components of the force the hinge exerts on the beam, and the magnitude of that force.

17. *(Ch. 12)* A solid sphere ($I = \frac{2}{5}MR^2$) is released from rest and rolls without slipping down a 30° incline from a vertical height of 1.5 m. (a) What is its speed at the bottom? (b) What is the acceleration of its center of mass along the incline? (c) What is the minimum coefficient of static friction needed to prevent slipping?

18. *(Ch. 12)* A uniform rod ($M = 1.5$ kg, $L = 1.0$ m) is pivoted at one end on a frictionless horizontal pin and released from rest in the horizontal position. (a) What is its angular acceleration at the instant of release? (b) What is its angular speed when it passes through the vertical position? (c) What is the speed of the free end at that instant?

19. *(Ch. 12)* A student sits on a frictionless rotating stool holding a 3.0 kg dumbbell in each hand, 0.80 m from the axis. The student, stool, and arms alone have a moment of inertia of 2.5 kg m^2, and the system rotates at 1.5 rad/s. The student pulls the dumbbells in to 0.20 m from the axis (treat them as point masses). (a) Find the final angular speed. (b) Find the change in kinetic energy, and explain where the energy comes from.

20. *(Ch. 12)* A uniform rod ($M = 0.80$ kg, $L = 1.2$ m) hangs vertically from a frictionless pivot at its upper end. A 0.10 kg lump of clay moving horizontally at 6.0 m/s strikes the lower end of the

rod and sticks to it. (a) Find the angular speed of the rod–clay system just after the collision. (b) Through what maximum angle does the rod swing from the vertical?