Problem Set #2

Question 1

Consider a pencil that stands upright on its tip and then falls over. Let's idealize this pencil as a mass m sitting on the end of a massless rod of length l.

(i) Assume that the pencil makes an initial (small) angle θ_0 with the vertical, and that it's initial angular speed is ω_0. The angle will eventually become large, but while it is small, (so that $\sin\theta \approx \theta$), what is θ as a function of time?

(ii) You might think that it should be possible (theoretically, at least) to make the pencil balance for an arbitrarily long time, by making the initial θ_0 and ω_0 sufficiently small. However, it turns out that due to the Heisenberg uncertainty principle — which says that the product of the uncertainy in position (Δx and momentum (Δp) must be less greater than or equal to $\hbar$ —- it is impossible to balance the pencil for more than a certain amount of time. The goal of this part of the problem is to be quantitative, and the time limit is sure to surprise you. So, using the Heisenberg Uncertainty principle:

$$\Delta x \Delta p \geq \hbar,$$

where $\hbar = 1.05 \times 10^{-34}$ J·s. this means that the initial conditions satisfy

$$(l\theta)(ml\omega_0) \geq \hbar.$$

With this constraint, your task is to find the maximum time it can take your $\theta(t)$ solution in part (i) to become of order 1. In other words, determine (roughly) the maximum time the pencil can balance. Assume that $m = 0.01$ kg, and $l = 0.1$ m.

Question 2 A ball is thrown at speed v from zero height on level ground on the moon. At what angle should it be thrown so that the area under the trajectory is maximum?

Question 3

Taylor, Problem 2.5

Question 4

Taylor, Problem 2.8

Question 5

Taylor, Problem 2.10