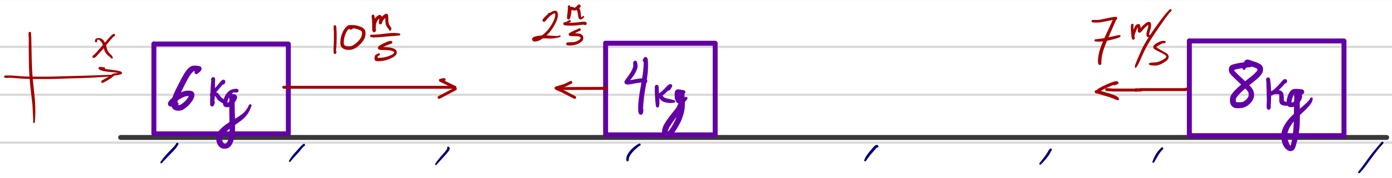
Week 7 Momentum, inelastic Collisions, & impulse

Question 1: Consider the system of blocks shown below. They each slide without friction, and when they collide, they stick. Answer the following questions.



(a) What is the speed of the 10 kg block that results when the 6 kg & 4 kg blocks collide?

Since there are no net external forces, the momentum of the 4kg / 6kg system will be conserved:

$$\vec{P}_i = \vec{P}_f$$

$$6kg \cdot 10\tfrac{m}{s} + 4kg \cdot \left(-2\tfrac{m}{s}\right) = 10kg \cdot V_f \quad ; \quad \text{I chose } +x \text{ to right.}$$

$$52\,\tfrac{kg \cdot m}{s} = 10\,kg \cdot V_f$$

thus,

$$V_f = 5.2\,m/s$$

(b) Next, this 10 kg block collides & sticks with the 8 kg block. What is the final speed of the resulting 18 kg object?

Same idea — momentum is conserved:

$$\vec{P}_i = \vec{P}_f$$

$$10kg \cdot 5.2\tfrac{m}{s} + 8\,kg \cdot \left(-7\tfrac{m}{s}\right) = 18\,kg \cdot V_f$$

$$52\,\tfrac{kg \cdot m}{s} - 56\,\tfrac{kg \cdot m}{s} = 18\,kg \cdot V_f$$

$$-4\,kg \cdot m/s = 18\,kg \cdot V_f$$

$$-\frac{2}{9}\,\frac{m}{s} = V_f$$

(c) Suppose all three blocks collided simultaneously. What would the final speed of the 18 kg object be in this case?

there will be no difference:

$$\vec{P}_i = \vec{P}_f$$

$$6kg \cdot 10\tfrac{m}{s} + 4kg\left(-2\tfrac{m}{s}\right) + 8kg\left(-7\tfrac{m}{s}\right) = 18\,kg \cdot V_f$$

$$(60 - 8 - 56)\,\tfrac{kg \cdot m}{s} = 18kg \cdot V_f$$

$$-4\,\tfrac{kg \cdot m}{s} = 18\,kg \cdot V_f$$

$$-\frac{2}{9}\,\frac{m}{s} = V_f$$

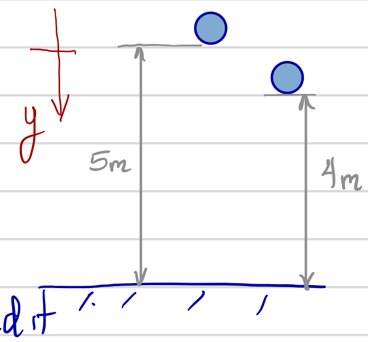
Question 2: A ball of mass 50g is dropped from a height of 5.0m & rebounds to a height of 4.0m.

(a) Using your knowledge of kinematics, calculate the velocity with which the ball first impacts the ground.

$$v^2 = v_0^2 + 2a\Delta y \quad (v_0 \to 0)$$

$$v^2 = 2g \cdot 5m$$

$$v = \sqrt{2(9.8\tfrac{m}{s^2})5m} = 9.899\ \tfrac{m}{s}$$



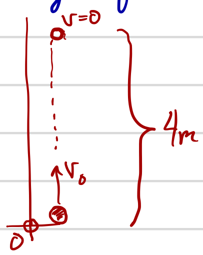
(b) The ball rebounds to a maximum height of 4.0 m. With what velocity did it leave the surface?

$$v^2 = v_0^2 + 2a\Delta y \quad (v \to 0)$$

$$0 = v_0^2 + 2(-g)\,4m$$

$$\therefore \quad v_0^2 = 2g \cdot 4m$$

$$v_0 = 8.854\ m/s$$



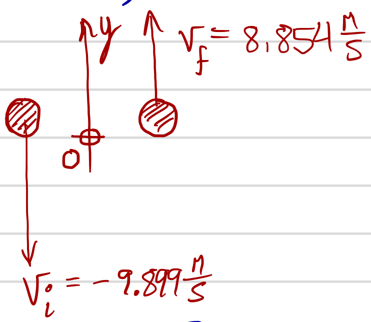
(c) If the ball was in contact with the ground for 0.0005 sec, what was the average force acting on the ball during the collision?

Now we find the average force via

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{\vec{P}_f - \vec{P}_i}{\Delta t}$$

$$\bar{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = \frac{0.05\,kg\left(8.854\tfrac{m}{s} - (-9.899\tfrac{m}{s})\right)}{0.0005\ s}$$

$$\bar{F} = 1875\ N$$



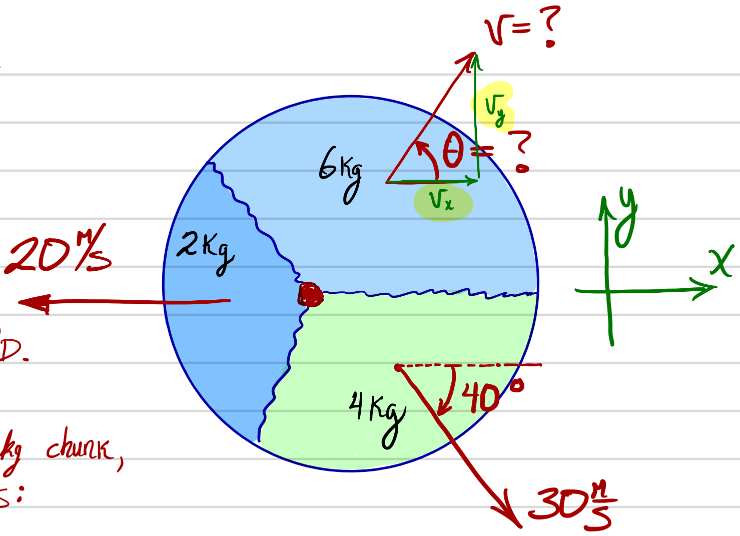
(d) What was the ball's average acceleration during the collision?

Via $F = ma$,

$$a = \frac{1875\,N}{.05\,kg} = 37{,}500\ \tfrac{m}{s^2}$$

A huge acceleration!

Question 3 A 12 kg disk is initially at rest on a frictionless surface. It explodes into three chunks as shown.
Compute the speed and the angle at which the 6 kg chunk travels after the explosion.



Another conservation of momentum question; this time in 2D.
I've picked +x to right & +y upward.
In terms of the unknown velocity components v_x & v_y for the 6 kg chunk, we can write conservation of momentum in the x & y directions:

— x —

$$2\,kg\cdot(-20\tfrac{m}{s}) + 4\,kg\cdot 30\tfrac{m}{s}\cos 40^\circ + 6\,kg\cdot v_x = 0$$

$$-40\,\tfrac{kg\cdot m}{s} + 91.925\,\tfrac{kg\cdot m}{s} + 6\,kg\, v_x = 0 \quad \text{(initial momentum} = 0\text{)}$$

$$\therefore \quad v_x = -8.654\,\tfrac{m}{s}$$

— y —

$$2\,kg\cdot 0\tfrac{m}{s} + 4\,kg\left(-30\tfrac{m}{s}\cdot\sin 40^\circ\right) + 6\,kg\cdot v_y = 0$$

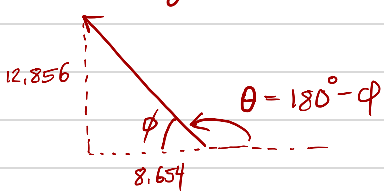
$$-77.135\,\tfrac{kg\cdot m}{s} + 6\,kg\cdot v_y = 0$$

$$v_y = +12.856\,\tfrac{m}{s}$$

① Now that we have the components of $\vec{v}_f$, we use the Pythagorean theorem to find the speed:

$$v = \sqrt{v_x^2 + v_y^2} = 15.5\,\tfrac{m}{s}$$

② the angle is clearly greater than 90° since v_x is negative:



$$\tan\phi = \frac{12.856}{8.654} = 56.1^\circ$$

$$\therefore \quad \theta = 123.9^\circ$$