Rotational Dynamics

Physics 114

Fall 2025

The goal of this lab is to apply the principles of rotational dynamics to measure the moment of inertia of a rotating system.

Part 1: Introduction

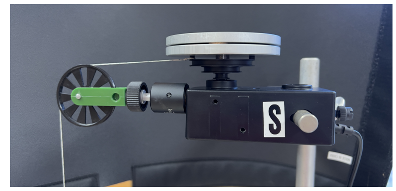

Figure 1: The apparatus.

Figure 1 shows the apparatus you will use. A string is wound around one of three pulleys oriented horizontally under the silver disks, which are free to rotate about the vertical axis. These pulleys have radii R of 24.0 mm, 14.5 mm, and 5.0 mm. The string runs over a second pulley oriented vertically, and a varying mass hangs from the free end of the string (not shown in the figure).

Taking the vertical pulley to have negligible mass and the string to be ideal, the tension T in the string will be uniform. We can then apply Newton's 2nd Law to the hanging mass m:

$$mg - T = ma \tag{1}$$

The string exerts a torque RT on the horizontal disks, and, assuming there is some friction in the apparatus that creates a frictional torque τ_f, we can apply Newton's 2nd Law for rotation to the horizontal disks:

$$RT - \tau_f = I\alpha \tag{2}$$

where I is the moment of inertia of the rotating disks. If the string doesn't slip from the pulleys, the acceleration a of the hanging mass will equal $R\alpha$. Applying this constraint, and solving the two equations above to eliminate the tension T, we find:

$$m(gR - \alpha R^2) = I\alpha + \tau_f \tag{3}$$

We will use this equation in two different ways to measure I. First, if the linear acceleration $a = \alpha R$ is small compared to g, then the second term on the left hand side can be omitted because it is negligible compared to the first term. In this approximation, the equation can be rearranged to solve for α:

$$\alpha \approx \left(\frac{gR}{I}\right) m - \frac{\tau_f}{I} \tag{4}$$

This approximate relationship shows that α is linear in m, so by plotting α vs. m, we expect a linear trend line with slope equal to gR/I.

The other way to use this equation is to not use the approximation, instead plotting $m(gR - \alpha R^2)$ vs. α. This should give a linear trend line with slope I. While exact, the issue with this method is that the two plotted quantities are not independent (they are both calculated from α), and thus the uncertainties determined with a least-squares fit, such as that done by the LINEST function, will not be accurate.

Part 2: Procedure

Verify that the rotary motion sensor is connected to the computer via the Vernier LabQuest interface box, then open the Vernier Graphical Analysis software. You should see two empty plots in the window, but you will only need the plot of angular velocity (in radians / second) vs. time. Click the "Mode" button and set the data collection to start and end manually. Also, verify that the Mode is set to "Time Based" and the collection rate is at least 20 samples per second.

Wind the string around the lowest horizontal pulley (the one with the smallest radius, $R = 5$ mm), and hang the empty hanger from the other end. Press the COLLECT button, release the hanger, and then end data collection when the hanger finishes falling. Measure the angular acceleration by selecting the linear region of the angular velocity vs. time plot and performing a linear fit. The absolute value of the slope of your fit is the angular acceleration. It's a good idea to take several trials and average your angular accelerations over the trials.

Repeat the above procedure for seven more masses, recording the mass and mean angular acceleration for each. For each mass, verify that αR is less than 0.5 m/s^2: use less mass if you exceed this value (this ensures that the approximation used in Equation 4 is valid). Make two graphs, one plotting α vs. m, and the other plotting $mgR - mR^2\alpha$ vs. α, and determine the moment of inertia I from the slope of each plot, as described in the discussion of Equations 3 and 4.

Question 1: Are the measured values of I consistent? As you answer this, keep in mind that the uncertainties returned by LINEST for the slope of the plot of $mgR - mR^2\alpha$ vs. α are not necessarily accurate.

Repeat the above analysis taking data with the horizontal pulley with the largest radius ($R = 24$ mm). Again use eight different hanging masses such that αR is less than 0.5 m/s^2 (this will be a bit more difficult because of the increased torque). Make the two different graphs as before and measure the moment of inertia from the slopes.

Question 2: Are the measured values of I, when using the largest radius pulley, consistent? Explain.

Question 3: Compare the moments of inertia you measured with the larger radius pulley to those measured with the smaller radius pulley. Are they consistent?

Informal Lab Report Guidelines

Include the following in your informal lab report:

1. Two data tables, one for each pulley radius. Each data table should contain the 8 values of the hanging mass and the average angular acceleration for each mass.

2. Four plots (two for each radius times two for each analysis method) as described in the procedure. The title of the plot should indicate the radius of the pulley.

3. Report the four measured values with uncertainties of the moment of inertia I. Clearly indicate which combination of pulley radius and analysis method corresponds to each reported measurement. Explain how you determined the uncertainty on I for each analysis method.

4. Answer the three questions in the procedure.