RC Low Pass Filter

Paul Nakroshis

Department of Physics

University of Southern Maine

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An RC low pass filter looks like

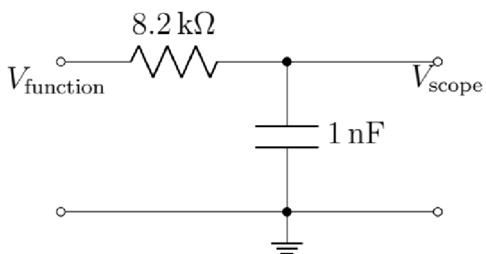

Figure 1: A Low pass filter.

and the output voltage will be given by

$$V_{\text{out}} = \frac{1 / i \omega C}{R + 1 / i \omega C} V_{\text{in}}.$$

In class, we worked out the magnitude of the voltage gain and found that

$$G = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

We can use Mathematica to save us some algebra in computing the magnitude of the voltage gain.

First, we tell Mathematica that R, C, and ω are all positive real numbers:

```
In[1]:= Assumptions[R | C | ω ∈ PositiveReals;];
```

Now we define a function for the complex impedance of a capacitor:

```
In[2]:= Zc[C_, ω_] := 1/(ⅉ ω C);
```

In addition, we define a function to compute the magnitude of the voltage gain:

```
In[3]:= RCLowPassGain[R_, C_, ω_] := Abs[Zc[C, ω]/(R + Zc[C, ω])]
```

Mathematica can simplify this expression algebraically, and you'll see we get the result derived in class (in order to get Mathematica to fully simplify the result, we have to remind it that R, C, and ω are positive):

In[4]:=
```
Simplify[ComplexExpand[RCLowPassGain[R, C, ω]], Assumptions → {C > 0, R > 0, ω > 0}]
```

Out[4]= $$\frac{1}{\sqrt{1 + C^2 R^2 \omega^2}}$$

Now I will make a plot of the power passed by this filter by computing 20 log (Gain):

In[5]:=
```
LogLinearPlot[{20 * Log10[RCLowPassGain[8200, 1.0 * 10^-9, ω]], -3}, {ω, 1, 10^7},
 Frame → True, FrameLabel → {Style["ω", 28], Style["dB", Bold, 16]},
 PlotRange → Full, PlotLabel → "RC Low Pass Filter: 8.2kΩ, C=1nF",
 LabelStyle → {FontFamily → "Times", FontSize → 16},
 PlotStyle → {Thickness[0.005], Opacity[0.8]},
 GridLines → Automatic, PlotTheme → "Scientific"]
```

Out[5]=