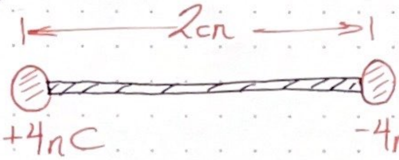
PHYSICS 123 — Week 2 Solutions

Q1 (part a) An electric dipole is shown below:



$\ell = 2\text{cm}$ is the (cm) separation distance between the charges

Center of Mass

the two charges attract one another, and therefore, they COMPRESS the stick. The tension in the rod is a compressive stress of magnitude

$$|\vec{F}| = \frac{kq^2}{\ell^2} = \frac{\left(8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)\left(4\times10^{-9}\,C\right)^2}{(0.02m)^2}$$

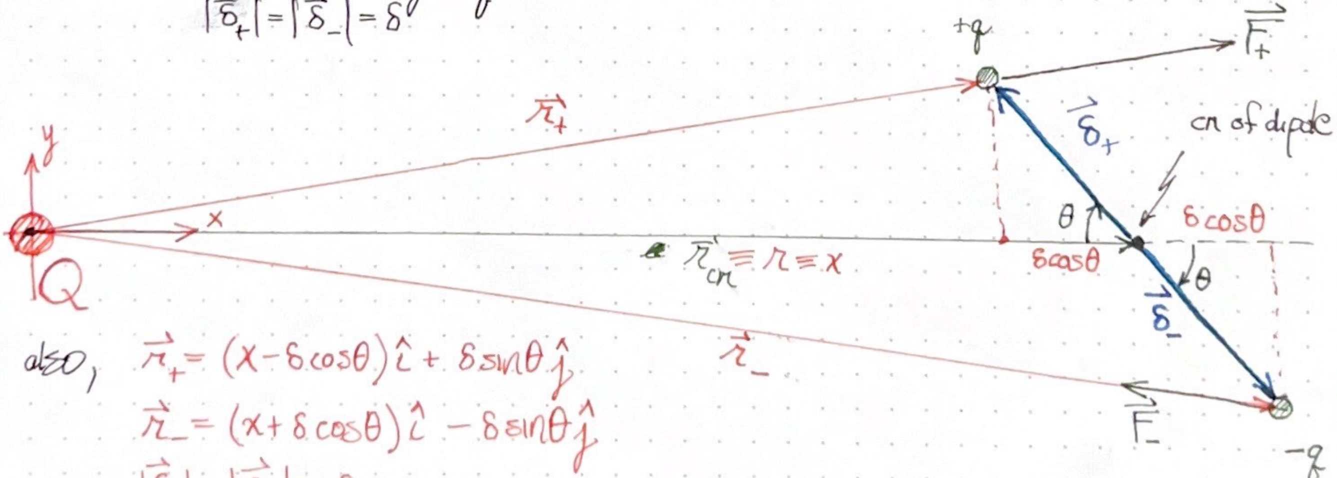
or

$$\boxed{|\vec{F}| = 3.6\times10^{-4}\ \text{Newtons}}\quad (\text{Compressive Stress})$$

(part b) This portion is more involved, and there's an ambiguity in my figure drawing — I didn't indicate the angular orientation of the 10 cm position vector which extends from Q to the cm of the dipole.

So, without loss in generality, I'll choose this vector to lie along the $+\hat{x}$ direction

in this figure, $\vec{\delta}_+$ is a vector from the dipole's center of mass to the charge $+q$.
$\vec{\delta}_-$ " to the charge $-q$.
$|\vec{\delta}_+| = |\vec{\delta}_-| = \delta$



also, $\vec{r}_+ = (x - \delta\cos\theta)\hat{i} + \delta\sin\theta\,\hat{j}$

$\vec{r}_- = (x + \delta\cos\theta)\hat{i} - \delta\sin\theta\,\hat{j}$

$|\vec{\delta}_+| = |\vec{\delta}_-| \equiv \delta$

Q1 (b) (Continued)

Looking at the figure on the previous page, notice that I haven't used the given quantities (dipole spacing $= 2\text{cm} = 2\delta$, $|\vec{r}_{cm}| \equiv x = 10\text{cm}$, $Q = +400\,nC$, $q = 4nC$) but I've elected to express everything symbolically. But, realize that $\delta, x, Q, \& \, q$ are all known quantities so that the forces $\vec{F}_+ \,\&\, \vec{F}_-$ are all actually computable quantities: (recall that for any vector, $\hat{r} = \frac{\vec{r}}{r}$)

$$\vec{F}_+ = \frac{kQq}{r_+^2}\hat{r}_+ = \frac{kQq}{r_+^3}\vec{r}_+$$

$$\vec{F}_- = \frac{kQq}{r_-^2}\hat{r}_- = -\frac{kQq}{r_-^3}\vec{r}_-$$

therefore, the net force on the dipole, $\vec{F}_{NET} = \vec{F}_+ + \vec{F}_-$ is given by

$$\boxed{\begin{aligned} \vec{F}_{NET} &= kQq\left\{\frac{\vec{r}_+}{r_+^3} - \frac{\vec{r}_-}{r_-^3}\right\} \\ \vec{r}_+ &= (x - \delta\cos\theta)\hat{i} + \delta\sin\theta\hat{j} \\ \vec{r}_- &= (x + \delta\cos\theta)\hat{i} - \delta\sin\theta\hat{j} \end{aligned}}$$

using the known values,

$kQq = 1.4384 \times 10^{-5}\ N\cdot m^2$

$$\left\{\begin{aligned} \vec{r}_+ &= (10 - 1\cdot\cos45°)\hat{i} + 1\sin45°\hat{j} = (9.293\hat{i} + 0.707\hat{j})\text{cm} \\ \vec{r}_- &= (10 + 1\cdot\cos45°)\hat{i} - 1\sin45°\hat{j} = (10.707\hat{i} - 0.707\hat{j})\text{ cm} \\ r_+ &= |\vec{r}_+| = 9.3198\text{ cm} = 0.093198\text{ m} \\ r_- &= |\vec{r}_-| = 10.7304\text{ cm} = 0.107304\text{ m} \end{aligned}\right.$$

$$\left.\begin{aligned} \frac{\vec{r}_+}{r_+^3} &= (114.798\hat{i} + 8.734\hat{j})\ m^{-2} \\ \frac{\vec{r}_-}{r_-^3} &= (86.660\hat{i} - 5.722\hat{j})\ m^{-2} \end{aligned}\right\} \Rightarrow \boxed{\vec{F}_{NET} = (4.479\hat{i} + 2.079\hat{j}) \times 10^{-4}\,N}$$

$$\left\{\frac{\vec{r}_+}{r_+^3} - \frac{\vec{r}_-}{r_-^3}\right\} = (31.138\hat{i} + 14.456\hat{j})\ m^{-2}$$

$$\left(|\vec{F}_{NET}| = 4.938 \times 10^{-4}\,N\right)$$

$$|\vec{a}| = \frac{|\vec{F}_{NET}|}{m} = 0.988\,\frac{m}{s^2}$$

The net torque on the dipole about the center of mass will be

$$\vec{\tau}_{NET} = \vec{\tau}_+ + \vec{\tau}_-$$
$$= \vec{\delta}_+ \times \vec{F}_+ + \vec{\delta}_- \times \vec{F}_-$$

because I've computed $\vec{\delta}_+$, $\vec{\delta}_-$, $\vec{F}_+$, & $\vec{F}_-$ in Cartesian coordinates, and the δ's & F's exist in the xy plane, you should be able to see that the torque will be in the $-\hat{z}$ direction.

We can compute the torques using the determinant method: $(\delta = 0.01\,m)$

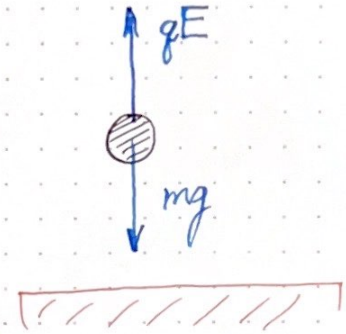
$$\vec{\tau}_+ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\delta\cos\theta & \delta\sin\theta & 0 \\ 1.651\times10^{-3} & 0.1256\times10^{-3} & 0 \end{vmatrix} = -8.883\times10^{-7}\ N\cdot m - 1.1674\times10^{-5} = -1.256\times10^{-5}\ N\cdot m\ \hat{k}$$

$$\vec{\tau}_- = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ +\delta\cos\theta & -\delta\sin\theta & 0 \\ -1.2465\times10^{-3} & +8.23\times10^{-5} & 0 \end{vmatrix} = 5.8199\times10^{-7} - 8.814\times10^{-6} = -8.232\times10^{-6}\ N\cdot m\ \hat{k}$$

The total torque is the sum of these two torques:

$$\boxed{\vec{\tau}_{NET} = -2.079\times10^{-5}\ N\cdot m\ \hat{k}}$$

Q2 | The forces acting on the drop of oil will be the downward force of gravity $\vec{F} = m\vec{g}$ and the upward electrical force $\vec{F} = q\vec{E}$.



the mass of the oil drop is $m = \rho \cdot Vol$ or

$$m = \rho \cdot \frac{4}{3}\pi r^3 = \,?$$ (we need ρ in kg/m^3 !)

the density of the oil is given in kg/l, so we need to either convert it to kg/m^3 (or to kg/cm^3 ; provided we convert r to cm)

$$\rho = 0.917 \frac{kg}{l}\left(\frac{1\, l}{1000\, cm^3}\right)\left(\frac{100\, cm}{1\, m}\right)^3 = 917\, kg/m^3$$

$$r = 4\,\mu m = 4 \times 10^{-6}\, m$$

$$\therefore \quad m = 2.458 \times 10^{-13}\, kg$$

for the drop to be levitated, the electric & gravitational forces must be equal in size, so $qE = mg$ or

$$E = |\vec{E}| = \frac{mg}{q} = \frac{2.458 \times 10^{-13}\, kg \cdot 9.8 \frac{m}{s^2}}{2(1.60 \times 10^{-19}\, C)}$$

$$\boxed{|\vec{E}| = 7.53 \times 10^{6} \frac{N}{C}}$$

Q3 Suppose each person has a mass m. Then, since we're mostly made of H_2O, the number of water molecules is $\frac{m}{0.018} \cdot N_A$ & there are 10 electrons/H_2O

10 electrons/H_2O molecule

$$N_e = \underbrace{\frac{m}{0.018\ kg/mole}}_{\#\text{ moles of } H_2O} \cdot N_A \cdot 10 = \left(3.344 \times 10^{26}\right) m$$

electrons

Avagadro's # $= 6.02 \times 10^{23}\ \frac{\text{molecules}}{\text{mole}}$

If each person had a $\frac{1}{100}\%$ excess of electrons, that would mean each mass m would have $10^{-4} N_e$ excess electrons & thus a charge of

$$q = e \cdot N_e \cdot 10^{-4} = \left(1.6 \times 10^{-19}\ C\right)\left(3.344 \times 10^{26}\ m\right) \cdot 10^{-4}$$

$$q = \left(\underbrace{5350}_{\text{Coulombs/kg}} \cdot m\right) \text{ Coulombs}$$

mass in kg

So, if person 1 has mass m_1 & person 2 has mass m_2, and they are separated by a distance d, the repulsive force will be

$$|\vec{F}| = \frac{k q_1 q_2}{d^2} \cong \frac{k m_1 m_2 \cdot \left(5350\ \frac{\text{Coulombs}}{kg}\right)^2}{d^2}$$

Let $m_1 = 50\ kg$, $m_2 = 80\ kg$, $d = 2\ m$

$$F \sim 2.6 \times 10^{20} \text{ Newtons}$$

An enormous force — slightly more than the gravitational force of attraction between the Earth & Moon!

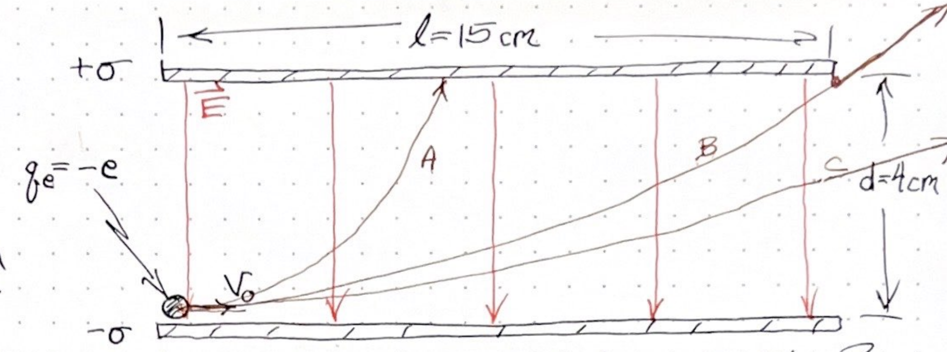
Needless to say — All objects are normally very close to electrically neutral — or you'd notice it!

Q4

We're given

$|\sigma| = 88.5 \frac{nC}{m^2}$

$\vec{V}_0 = V_0 \hat{\imath} = 2\times10^6 \frac{m}{s}\, \hat{\imath}$



a) How long will it take for the electron to leave the region between the plates?

When the electron enters the region between the plates, it feels an electrical force $\vec{F} = q_e \vec{E}$ which will be upwards. The electron will also feel a downward force due to gravity, but this will be negligible in size compared to the upward electrical force.

To the extent that the electrical Field is uniform, the electron will feel a uniform upward force of size $eE = e \cdot \frac{\sigma}{\varepsilon_0}$ and therefore, a uniform y acceleration of

$$\boxed{a_y = \frac{eE}{m_e} = \frac{e\sigma}{\varepsilon_0 m_e}} = 1.756\times10^{15} \frac{m}{s^2} \text{ upward}$$

The electron will therefore follow a parabolic trajectory; but there are 3 possible forms of this trajectory (Labelled A, B, & C in figure)

In scenario A, the electron does not escape the plates & instead it smashes into the upper plate.

In scenarios B & C, it does escape — in these two cases, the time to exit is determined soley on the initial x velocity (which is constant).

$$\Delta t = \frac{l}{V_0} = \frac{0.15\,m}{2\times10^6 \frac{m}{s}} = 7.5\times10^{-8}\,s$$

How far vertically will the electron travel in this time?

$$\Delta y = \frac{1}{2} a_y \Delta t^2 = \frac{1}{2} \frac{e\sigma}{\varepsilon_0 m_e} \cdot \Delta t^2 = \underline{\underline{4.9\,m}}$$

so electron will smash into upper plate!

Now that we know the electron will not exit the plates, we can use its vertical acceleration to determine how long it will take to hit the top plate:

$$d = \frac{1}{2} a_y t^2 \Rightarrow t = \sqrt{\frac{2d}{a_y}} = \sqrt{\frac{2d}{\frac{e\sigma}{\epsilon_0 m_e}}} = \sqrt{\frac{2d\epsilon_0 m_e}{e\sigma}}$$

or,

$$t = \sqrt{\frac{2(0.04\,m)(8.85\times10^{-12}\frac{C^2}{N\cdot m^2})(9.11\times10^{-31}\,kg)}{(1.6\times10^{-19}\,C)(88.5\times10^{-9}\,\frac{C}{m^2})}}$$

$$\underline{t = 6.75\times10^{-9}\,s}$$

In this time, it will have traveled horizontally a distance

$$x = v_0 t = 0.0135\,m = \underline{1.35\,cm}$$

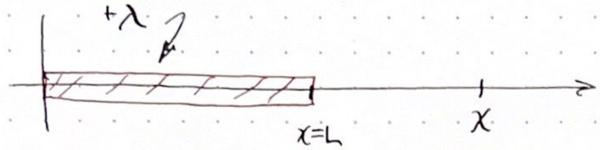
the velocity components at impact will be

$$\vec{v} = v_0\hat{i} + a_y t\hat{j}$$

$$\boxed{\vec{v} = 2\times10^{6}\,\frac{m}{s}\,\hat{i} + 1.19\times10^{7}\,\frac{m}{s}\,\hat{j}}$$

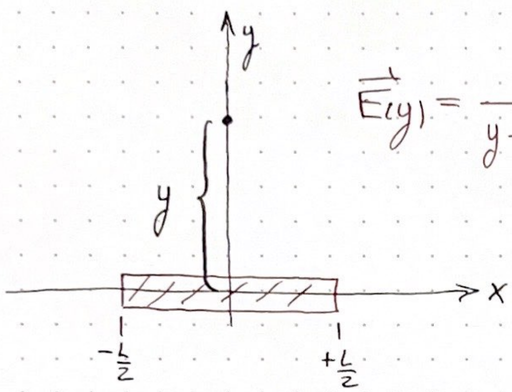
Q5 | In class, we computed the electric field for a thin charged stick in the two situations below:

A)



$$\vec{E}(x) = \frac{kQ}{x(x-L)}\,\hat{\imath}$$

B)



$$\vec{E}(y) = \frac{kQ}{y\sqrt{y^2+\frac{L^2}{4}}}\,\hat{\jmath}$$

In case (A), as x becomes much larger than L, we expect that the electric field approaches kQ/x^2, as we cannot "see" the physical extent of the stick when very far away. Let's look at $\vec{E}(x)$ & factor out x from $(x-L)$ to obtain

$$\vec{E}(x) = \frac{kQ}{x^2\left(1-\frac{L}{x}\right)}\,\hat{\imath}$$

Since $\frac{L}{x} \to 0$ as x becomes $\gg L$, we see that $\vec{E}(x) \longrightarrow \frac{kQ}{x^2}$ ✓ Q.E.D.

In case (B), we expect (similarly) that $\vec{E}(y)$ will approach kQ/y^2 as $y \gg L$:

$$\vec{E}(y) = \frac{kQ}{y^2\sqrt{1+\frac{L^2}{4y^2}}} \xrightarrow{y \gg L} \frac{kQ}{y^2}$$ ✓ Q.E.D.