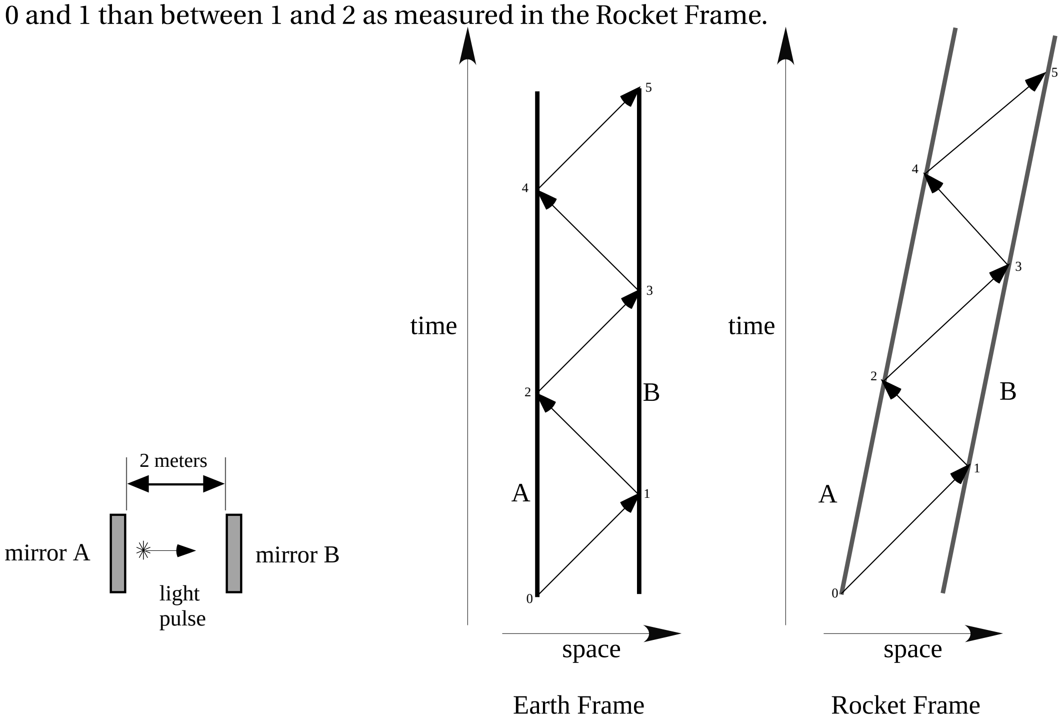
Problem Set #2

Due: 19 Sep 2024 At the beginning of class

September 10, 2024

Q1: The Swindler

A pulse of light is reflected back and forth between two mirrors A and B separated by 2 meters of distance in the x-direction in the Earth Frame, as shown in the figure below. A swindler tells us that this device constitutes a clock that "ticks" every time the pulse arrives at either mirror.
The swindler claims that events 0, 1,2, 3 and 4 create sequential "ticks" of this clock in the Earth Frame and every other frame. However, we notice that the ticking of the clock is uneven in a Rocket Frame moving with speed v_{rel} in the Earth Frame. For example, there is more time between events 0 and 1 than between 1 and 2 as measured in the Rocket Frame.



1. a what is the physical basis for the "bad" behavior of this clock? Use the Lorentz equations to account for the uneven ticking of this clock in the rocket frame. (Hint: think carefully about which direction the rocket must be moving relative to the earth frame to produce a rocket frame diagram as shown).
2. Use some of the same events 0 through 4 to define a "good" clock that ticks evenly in both the Earth Frame and the Rocket Frame. From the spacetime diagrams, show qualitatively that your good clock "runs slow" as observed from the Rocket Frame—as it must, since the clock is in motion with respect to the Rocket Frame.

Q2: Understanding Cherenkov radiation

A flash of light is emitted at an angle ϕ' with respect to the x' -axis of the rocket frame.

1. Show that the angle ϕ the direction of motion of this flash makes with respect to the x-axis of the laboratory frame is given by the equation

$$\cos\phi = \frac{\cos\phi' + v_{\text{rel}}}{1 + v_{\text{rel}}\cos\phi'}$$

   (Hint: you can use the Lorentz Transformation or the addition of velocity formulas from your book).
2. A particle at rest in the rocket frame emits light uniformly in all directions. Consider the 50 percent of light that goes in the forward hemisphere in this rocket frame. Show that in the laboratory frame this light is concentrated in a narrow forward cone of half angle ϕ_o whose axis lies along the direction of motion of the particle. The half angle ϕ_o is defined by the equation:

$$\cos\phi_o = v_{\text{rel}}.$$

3. Draw a picture of the light rays in the forward direction as seen from the rocket frame and the laboratory frame; use 8 equally spaced rays between -90^o and +90^o in the rocket frame, and show how these rays look when viewed from the laboratory frame.

Q3: Spacetime Diagrams

In frame K, event B occurs 2 μs after event A and at Δx = 1.5 km from event A.

1. How fast must an observer be moving along the +x-axis so that events A and B occur simultaneously?
2. Is it possible for event B to precede event A for some observer?
3. Draw a spacetime diagram that illustrates your answers to (a) and (b). You will have to use the method we outlined in class to properly illustrate your answer to (b).
4. Compute the spacetime interval and proper distance between the events.

Q4: Velocity Addition formulas

In class, we derived the velocity addition formula for u'_x. Do the same for u'_y and u'_z