

Forces, Newton's Laws, and Circular Motion

PHY 121 Group Tutorial

Names: _____

How this tutorial works. Work in groups of three or four. Take one role in each part and rotate: **Manager** (keeps the group on time and on task), **Recorder** (writes the group's agreed answer), **Skeptic** (asks "why?" and "how do we know?"), and **Reporter** (explains the group's reasoning to a Learning Assistant or the class).

- Everyone must be able to explain every answer. Reach agreement *before* writing, and write *reasons*, not just answers.
- When you reach a **Group checkpoint**, stop and compare with a neighboring group, or call over an LA. Do not continue until the group agrees.
- Questions marked ★ are extensions. Do them if your group finishes a part early; otherwise skip them.

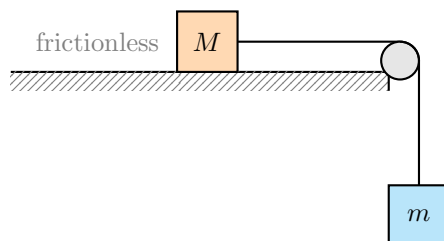
Part	Min.	Topic
A	11	Connected objects: tension and acceleration
B	11	Kinematics of uniform circular motion
C	14	Dynamics of circular motion: flat curve and vertical loop
D	9	Synthesis: the Rotor ride
E	13	Circular motion meets connected objects
	3	Wrap-up

Conventions. Name every force as "force *by* X *on* Y," for example $\vec{F}_{\text{table on book}}$. A free-body diagram (FBD) shows *only* the forces acting *on* the chosen object, drawn as arrows starting from a dot that represents it, with lengths roughly proportional to magnitudes. Use $g = 9.8 \text{ m s}^{-2}$ throughout. Strings are light and inextensible; pulleys are light and frictionless unless stated.

Part A: Connected objects: tension and acceleration

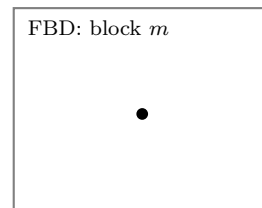
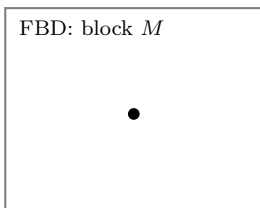
(about 11 min)

A block of mass $M = 3.0 \text{ kg}$ rests on a frictionless table. A light string attached to it passes over a light, frictionless pulley and supports a hanging block of mass $m = 1.0 \text{ kg}$. The system is released from rest.



A1. Predict (before any calculation): Will the blocks accelerate once released? In which directions? How do the magnitudes of the two accelerations compare, and why?

- A2.** Draw an FBD for each block. Use the dot to represent the block, and choose a positive direction for each: to the right for M , downward for m . Why is it acceptable to choose different positive directions for the two blocks?



- A3.** Two students discuss the hanging block after the release.

Student 1: “The string holds up the hanging block, so the tension equals its weight, $T = mg$.”

Student 2: “But if $T = mg$, the net force on it would be zero and it wouldn't accelerate down.”

Decide who is right. While the hanging block is accelerating, is T greater than, less than, or equal to mg ? Explain using Newton's second law applied to the hanging block.

- A4.** Write Newton's second law for each block along its chosen axis. Solve for the acceleration a and the tension T in terms of M , m , and g , then evaluate both for the given masses. Check that $T < mg$.

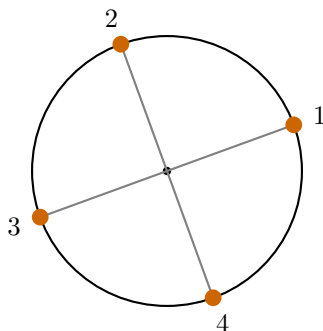
Group checkpoint

Does the tension act the same on both blocks? Why does the same T appear in both equations? Check your results for a and T with a neighboring group.

- ★ **A5.** Test your formulas with two limiting cases. (a) $M \rightarrow \infty$ (the table block cannot move). (b) $M \rightarrow 0$. What do a and T become in each case? Do these agree with what you would expect physically?
- ★ **A6.** Suppose the hanging block is removed and a person pulls the string downward with a constant force equal to $mg = 9.8 \text{ N}$. Compare the acceleration of M to its acceleration in A4: larger, smaller, or the same? Explain without a long calculation.

Part B: Kinematics of uniform circular motion*(about 11 min)*

A small puck is attached to a string (shown in gray) and moves on a frictionless air table in a horizontal circle at *constant speed*. The figure shows a top view; the puck moves counterclockwise. Four positions are marked.



- B1.** (a) At each marked position, draw an arrow for the puck's velocity (use equal lengths, because the speed is constant).
 (b) Is the velocity of the puck constant? Is its speed constant? Explain the difference.
 (c) The string is cut when the puck is at position 2. Sketch the puck's path afterward on the figure.

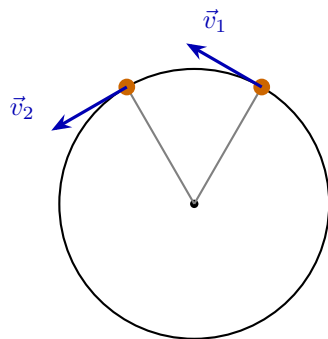
Student 1: "The puck flies straight outward, along a line away from the center."

Student 2: "It keeps curving along the circle for a while, since it was going around."

Student 3: "It goes straight along the direction it was moving at the instant of the cut."

- (d) Which student (if any) do you agree with? Use a law of motion to justify your choice.

- B2.** The figure below shows the velocity vectors $\vec{v}_1$ and $\vec{v}_2$ of the puck at two nearby instants (positions at 60° and 120° from the right).



puck moves counterclockwise

Redraw $\vec{v}_1$ and $\vec{v}_2$ here with their tails together



- (a) In the box, redraw $\vec{v}_1$ and $\vec{v}_2$ with their tails at the dot (keeping lengths and directions). Then draw $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, from the tip of $\vec{v}_1$ to the tip of $\vec{v}_2$.

- (b) Transfer the direction of $\Delta \vec{v}$ to the circle at the point midway between the two positions. How does this direction relate to the circle?
- (c) What is the direction of the puck's average acceleration over this interval? Is it parallel, perpendicular, or neither to the velocity?

B3. Return to the first figure and draw the puck's *acceleration* at positions 1–4 (in a different color). Is the acceleration vector constant? Is its magnitude constant? Explain.

★ **B4. Magnitude.** Suppose the puck moves at speed v on a circle of radius r . In a short time Δt it travels an arc length $s = v\Delta t$, so the radius to the puck sweeps an angle $\Delta\theta = s/r$.

- (a) The vectors $\vec{v}_1$ and $\vec{v}_2$ have equal length v and an angle $\Delta\theta$ between them. For small $\Delta\theta$, show that $|\Delta \vec{v}| \approx v \Delta\theta$.
- (b) Show that the magnitude of the acceleration is $a = v^2/r$.
- (c) Evaluate a for $v = 3.0 \text{ m s}^{-1}$ and $r = 0.50 \text{ m}$. Express your answer as a multiple of g .

Group checkpoint

State in one sentence the direction of the acceleration in uniform circular motion, and why that direction is consistent with the speed staying constant. Show your answer to an LA.

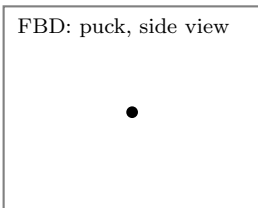
Part C: Dynamics of circular motion

(about 14 min)

C1. Return to the puck on the air table (Part B), still moving at constant speed on a circle.

- (a) What is the direction of the *net* force on the puck? Name every object that exerts a force on the puck in the horizontal plane.
- (b) Draw an FBD for the puck as seen from the side (looking horizontally at the table), with the center of the circle to the left.

FBD: puck, side view

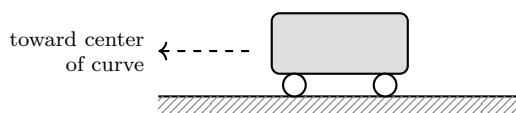


Student 1: “There must be two horizontal forces: the string pulls the puck inward, and an outward “centrifugal” force balances it. That’s why the puck stays on the circle.”

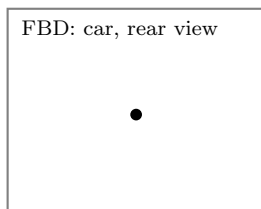
Student 2: “If they balanced, the net force would be zero and the puck would move in a straight line.”

- (c) Which student do you agree with? What does the label “centripetal force” tell you about a force, and what does it *not* tell you?

C2. A car drives at constant speed around a circular, *flat* (unbanked) curve of radius r . The figure shows the car from behind. The center of the curve is to the left.



- (a) Draw an FBD for the car.



- (b) Which force on the car points toward the center? What object exerts it? Is it static or kinetic friction, and why?
- (c) Four cars take curves at constant speed. Rank them by the magnitude of the inward force required, largest to smallest.

Car	A	B	C	D
Mass	m	m	$2m$	m
Speed	v	$2v$	v	v
Radius	r	r	r	$2r$

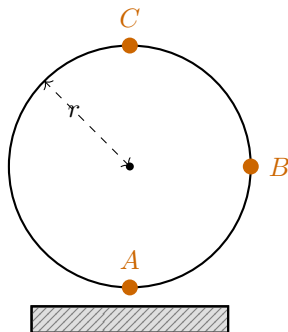
Ranking: _____

- ★ The coefficient of static friction between tires and dry road is $\mu_s = 0.80$ and $r = 50$ m. Find the maximum speed at which the car can take the curve without sliding. How does this change on a wet road with $\mu_s = 0.40$? Does it depend on the mass of the car?

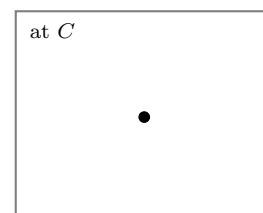
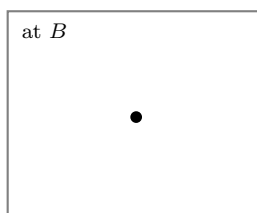
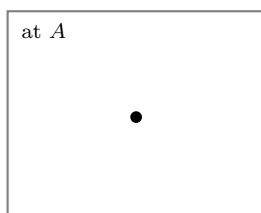
Group checkpoint

In C2(c), compare your ranking with a neighboring group. Did anyone use a general expression for the required inward force? If so, write it down together.

- C3.** A toy car of mass m travels on the *inside* of a vertical circular loop of radius r . Three positions are marked: A (bottom), B (side), and C (top).



- (a) Draw FBDs for the car at A , B , and C .



- (b) Suppose, hypothetically, that the car moves at the *same speed* v at all three positions. Rank the magnitudes of the normal force of the track on the car at A , B , and C . Base your reasoning on Newton's second law in the direction toward the center.
- (c) Two students disagree about the car at the top of the loop.

Student 1: "At the top, the car stays on the track, so the net force on it must be zero."

Student 2: "The net force can't be zero because the car is still moving in a circle."

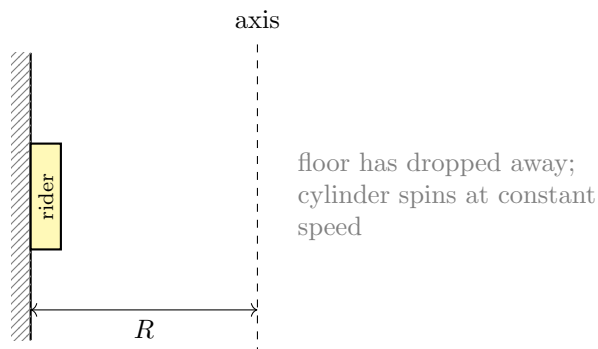
Who is right? What is the direction of the net force at C ?

- ★ What is the smallest speed at C for which the car stays in contact with the track? Evaluate for $r = 5.0$ m.

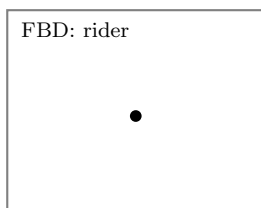
Part D: Synthesis: the Rotor ride

(about 9 min)

In an amusement-park ride called the Rotor, riders stand against the inner wall of a vertical cylinder of radius $R = 2.0$ m. The cylinder spins about its vertical axis; once it is fast enough, the floor drops away and the riders remain pinned to the wall. The coefficient of static friction between riders and wall is $\mu_s = 0.50$.



- D1.** Draw an FBD for a rider (side view, as in the figure).



What is the direction of the rider's acceleration? Which force on the rider points in that direction?

- D2.** A rider describes the experience.

Rider: "I feel pushed outward into the wall. That outward push is what holds me up."

- (a) What actual force presses the rider against the wall, and what object exerts it? What is its third-law partner, and on which object does it act?
- (b) Which force on the rider balances the rider's weight?

D3. Write Newton's second law for the rider in the radial (horizontal) direction and in the vertical direction. Use the condition that the rider just barely does not slide to find the minimum speed $v_{\min}$ in terms of R , μ_s , and g . Evaluate $v_{\min}$ and the corresponding period of rotation for the values above.

- ★ **D4.** (a) Does $v_{\min}$ depend on the rider's mass? Explain.
- (b) The ride spins faster than $v_{\min}$. Does the static friction force on the rider increase, decrease, or stay the same? What about the normal force? Comment on the statement " $f_s = \mu_s N$ always."

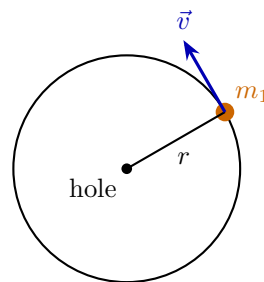
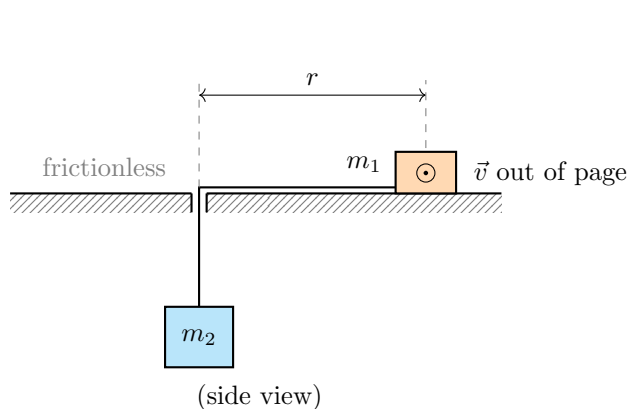
Group checkpoint

Before moving on, have your Reporter explain to an LA: (i) why the rider is not "thrown outward," and (ii) which force supplies the centripetal acceleration and which force supports the rider's weight.

Part E: Circular motion meets connected objects

(about 13 min)

A block of mass m_1 slides on a horizontal, frictionless table, moving in a circle of radius r at constant speed v . A light string attached to m_1 passes through a small, frictionless hole at the center of the circle and is attached to a hanging block of mass m_2 . The hanging block remains at rest.



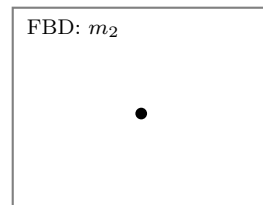
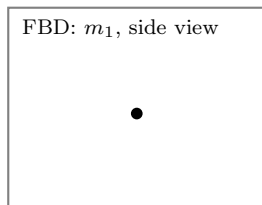
(top view; m_2 is hidden below the table)

E1. Predict (before any calculation).

- (a) Suppose m_1 is simply released from rest with the string taut. What happens to m_2 and to m_1 ?

- (b) Suppose instead m_1 is given a very large speed on the circle. What would you expect to happen to m_2 ?
- (c) What does the statement “ m_2 does not accelerate” tell you about (i) the net force on m_2 and (ii) the radius of m_1 's circle? Argue that there should be one particular speed at which m_2 stays at rest.

E2. Draw an FBD for each block. For m_1 , draw the view from the side with the center of the circle to the left.



E3. Two students discuss the tension in the string.

Student 1: “ m_2 isn't accelerating, so the string pulls up on it with $T = m_2g$. But m_1 *is* accelerating, so the string must pull on m_1 with a bigger force.”

Student 2: “The string is light and the hole is frictionless, so the tension is the same all along the string.”

Which student do you agree with? Explain why this matters for how you write the second law for each block.

E4. Write Newton's second law for m_2 (vertical direction) and for m_1 (in the direction toward the center of the circle). Solve to find the speed v of m_1 for which m_2 does not accelerate, in terms of m_1 , m_2 , r , and g .

Group checkpoint

Check your expression for v with a neighboring group: confirm the units, and decide what your result predicts when $m_2 \rightarrow 0$. Does that make sense physically? Show your reasoning to an LA.

E5. Evaluate your result for $m_1 = 0.50$ kg, $m_2 = 0.20$ kg, and $r = 0.40$ m. Find the tension in the string and check that it gives the correct inward force on m_1 .

- ★ **E6.** (a) Keeping m_2 and r fixed, by what factor must v change if m_1 is doubled? (b) Suppose m_1 is started at a speed slightly *greater* than the value you found. In which direction will m_2 begin to accelerate? Base your answer on the force the string must supply to keep m_1 on a circle compared with the force it can supply while holding m_2 at rest.

Wrap-up (3 min). Each person writes, in their own words, the two most useful ideas from today that they did not fully believe or understand at the start.