Measurement of the Relativistic Time Dilation Using μ-Mesons*

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(Received 14 January 1963)

An experiment has been performed to demonstrate the relativistic time dilation as a large effect, using only comparatively simple equipment. μ-mesons incident on top of Mt. Washington, New Hampshire, were selected to have speeds in the range between 0.9950 c and 0.9954 c. The number of these which survived to reach sea level was measured in Cambridge, Massachusetts. The number expected without time dilation was calculated from the distribution of decay times of these μ-mesons (i.e., the mean life as measured in both this experiment and others) and from the known distance of descent. The observed time dilation factor is 8.8 ± 0.8 to be compared with the effective time dilation factor calculated for mesons of these speeds in our detection geometry $1/(1-v^2/c^2)^{\frac{1}{2}}=8.4\pm2$.

I. INTRODUCTION

ONE of the most startling predictions made by the Theory of Special Relativity[1] is that moving clocks run slow, by a factor $(1-v^2/c^2)^{\frac{1}{2}}$, where v is the speed of the clock relative to an observer and c is the speed of light *in vacuo*. This effect is called the "Einstein Time Dilation."

In Fig. 1(a) three identical clocks are shown. They are all at rest with respect to an observer and set to read the same time. He sees them read the same elapsed time at any later time, as in Fig. 1(b). Suppose, however, one of these clocks is in motion relative to the observer and at a certain instant all clocks read the same, as in Fig. 1(c). When some time has elapsed, as indicated by the changed position of the moving clock in Fig. 1(d), the moving clock will read a shorter elapsed time than the clocks which are at rest. As read by the observer, the clock moving relative to him runs slow.

Because the speed of commonplace objects is much less than the speed of light, v^2/c^2 is a very small number for most objects we observe, and the whole term $(1-v^2/c^2)^{\frac{1}{2}}$ is usually extremely close to unity. Therefore, time dilation is unnoticeable in our everyday experience. For example, as read by an observer at rest on the earth, an ordinary wrist watch on a man walking by the observer loses only about a second every billion years. Even the clock in an astronaut's capsule, at an orbital speed of about 7 km/sec, loses only one second in the typical lifetime of an observer on the earth.

Thus, we need either a very accurate measurement of time, or a relative speed approaching very close to that of light in order to observe a sizeable time dilation effect.

The first of these alternatives, a very accurate fractional measurement of time, provided the means by which the time dilation was first observed. The shift of the frequency of the lines of the spectrum of atoms as the atoms move by the observer, called the "Transverse Doppler Effect," was carried out using very precise measurements[2] of the frequencies of lines emitted by these "atomic clocks."

The other alternative, observation of some sort of clock going at very high speed, is made

* This experiment was the basis for a film *Time Dilation —An Experiment With μ-Mesons* conceived and planned by Francis Friedman, David Frisch, and James Smith to demonstrate the relativistic time dilation as a large effect observable using, only, comparatively simple equipment. The film is part of a series on Special Relativity being developed at the Science Teaching Center of the Massachusetts Institute of Technology, and was made in cooperation with the Commission on College Physics under a grant from the National Science Foundation. The film was produced by Educational Services Incorporated, 47 Galen Street, Watertown, Massachusetts.

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[1] A. Einstein, *Relativity; the Special and the General Theory, a Popular Exposition*, translated by R. W. Lawson (Methuen and Company Ltd., London, 1954), 15th ed. This is Einstein's own popular treatment, and well worth reading. M. H. Shamos, *Great Experiments in Physics* (Henry Holt & Company New York, 1959), p. 315. This is a translation, with commentary, of selections from the original Einstein papers. C. Moller, *The Theory of Relativity* (Clarendon Press, Oxford, England, 1952). An advanced, but thorough and authoritative treatment.

[2] H. Ives, J. Opt. Soc. Am. **28**, 215 (1938). For a more recent measurement using the Mössbauer effect, see H. Hay, J. Schiffer, T. Cranshaw, and P. Egelstaff, Phys. Rev. Letters **4**, 165 (1960).

possible by the use of radioactive particles moving at speeds nearly that of light. As far as we know the probability of the radioactive decay of subatomic particles, and thus the average time they survive before decaying, is set by forces entirely internal to their structure. Therefore, *any dependence of the decay probability of radioactive particles on their speed is an example of a general property of clocks in motion relative to an observer* rather than a property of the speed of these particular particles relative to anything else in the universe. It is irrelevant, for example, that up to the present era the observer has happened to be on the earth.

The characteristic distribution of decay times of a given species of radioactive particle constitutes a clock. The difference between a radioactive decay distribution clock and an ordinary alarm clock is that the characteristic time associated with the radioactive decay must be determined from data on the decay of many such radioactive particles, averaged over many single decay events. By contrast, just two readings of any one of the clocks in Fig. 1 gives a definite time interval. Of course, when we look into the mechanism of an ordinary clock in detail on the atomic level, we find that its behavior, too, is the result of averaging over a large number of microscopic events.

In order to measure the effect of their motion relative to us on the decay rate of radioactive particles, we would most simply try to count their decays per unit time as they move along at one speed, and then change the speed to see whether the decay rates per unit time change. This has been done with many of the new particles created in high energy accelerators, but cannot be done easily with the low intensities of particles available naturally in cosmic rays.

Another way of measuring the effect of the time dilation on the decay of radioactive particles is to make a measurement of the number which have decayed in traversing a known flight path, as a function of the speed with which they moved. In this method one needs to detect the number of radioactive particles arriving at two different places, but there is no need actually to observe their decays.

In the experiment reported here, we used the latter method. We have repeated, with substantial modifications, the first observation of the effect of the time dilation for radioactive particles moving with a high speed relative to us.[3] The radioactive particles we used are μ-mesons which are produced high in the atmosphere and come shooting down toward the earth with a speed greater than $0.99c$. As they come down, some of them disintegrate in flight. The number arriving at medium altitude is, therefore, greater than the number surviving to reach sea level. At a medium altitude on top of Mt. Washington, we counted the mesons in a certain narrow band of speeds, selected out by passage through a thick slab of iron and stopped in a certain thickness of plastic. Then we went down to sea level and counted the μ-mesons that survived the passage down through the rest of the earth's atmosphere to arrive at sea level. The difference of these numbers tells us the number which decayed in flight. In addition (actually simultaneously), we slowed down and stopped a sample of μ-mesons and measured the distribution of their decay times when they were at rest relative to us. Comparison of their rate of decay at rest with their rate of decay in flight showed that the moving mesons decay much more slowly—their "clocks" run slow.

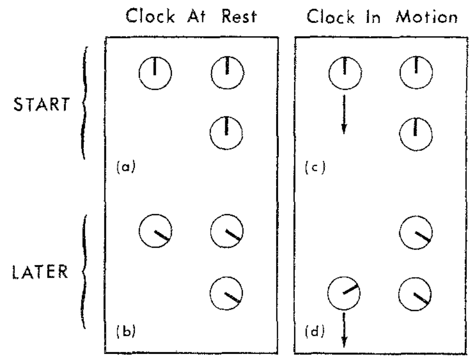


FIG. 1. The behavior of a moving clock. (a) Clocks at rest at time of synchronization; (b) clocks after remaining at rest until a later time; (c) one clock in motion at time of synchronization; (d) the same clock in motion at a later time. Note that the elapsed time read in Fig. 1(d) would be the same even if the moving clocks were started suddenly into motion from rest just after the time of synchronization, and then stopped suddenly just before the time of later observation. The elapsed time as read by the observer depends explicitly on only how long it has been in motion and at what speed relative to the observer, not on its initial or final state of motion, or on any acceleration that it has undergone.

[3] B. Rossi and D. B. Hall, Phys. Rev. 59, 223 (1941).

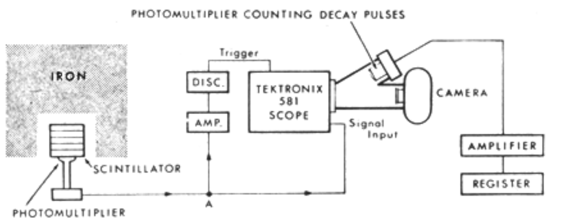


FIG. 2. Scheme of the experiment. The lower parts of the sides of the pile of iron were not as neat as shown, but included wooden supporting beams and some lead bricks in a rather irregular array (see Fig. 3).

II. EQUIPMENT

The setup used is schematized in Fig. 2, and is shown partly assembled in the photograph of Fig. 3. The μ-mesons pass through a "scintillator," matter of molecular structure such that some of the excitation energy given to its molecules by passing charged particles is emitted promptly as light.

The scintillator used was a circular cylinder of doped polystyrene plastic 11 in. high and 11 in. in diameter. It was made by stacking up four disks, wetting their faces with oil so as to provide easy passage of light between them. Most of the μ-mesons in cosmic rays have sufficient energy to pass completely through this scintillator. The many quanta (about 10^5) of light emitted as a meson passes through come out too rapidly to be resolved one from another, and therefore they are detected as a single flash of light.

This light was then detected by a 5-in. RCA photomultiplier (held by Dr. Smith in Fig. 3), giving about 10^3 photoelectrons from the photocathode. These photoelectrons cascaded in the multiplier structure to give an output pulse of about 10^8 electrons.

This electric charge was deposited on the grid of a cathode-follower, and gave an output voltage pulse which traveled down a cable to A, where it branched into two cables. One of these two signals went through an amplifier and discriminator circuit so as to choose pulses larger than a certain voltage and to send them on as large pulses uniform in size. These pulses provided a trigger for the sweep of a Tektronix 581 oscilloscope, so that every time a rapidly moving charged particle entered the scintillator the oscilloscope sweep was started. During the experiment, the discriminator was set to trigger the sweep on even the small pulses from cosmic-ray particles passing through only the corners of the scintillator.

The second of the two identical signals from A went to the vertical amplifier of the oscilloscope through a cable long enough to insure that the sweep had been started by the first signal when the second arrived. Thus, the signal from the passage of the cosmic-ray particle through the scintillator was visible as a deflection on the oscilloscope trace. A photograph of such a trace is seen in Fig. 4(a). The major divisions on the grid, shown in the photograph, were checked with a standard 10-Mc oscillator to be 1 μsec long, within 1%, so that the whole sweep shown here takes about 7.5 μsec.

The duration of the vertical pulse on the oscilloscope trace is about 0.2 μsec. This 0.2-μsec pulse width is determined by the shaping circuit and has nothing to do with the time the particle takes to pass through or stop in the scintillator, or for the scintillation light to be emitted. Such times are only a few nanoseconds (10^{-9} sec), completely negligible on the time scale of Fig. 4(a).

Since the cosmic-ray μ-mesons have various energies, some are moving so slowly that they stop in the plastic scintillator. These mesons—less than 1% of the total—are the ones whose decay we observe.

A μ-meson (μ) is a radioactive particle which decays into a neutrino (ν) and antineutrino ($\bar{\nu}$) and a positive or negative electron (e^+ or e^-) depending on whether the μ-meson is positively

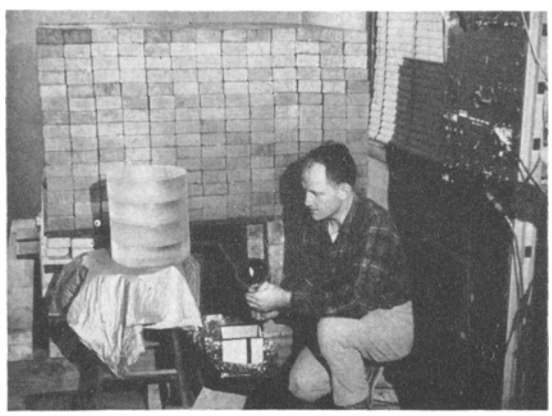


FIG. 3. In this scene taken from the film, Dr. Smith is shown assembling the detector. The iron shielding is in the background, and some of the electronics used can be seen on the right.

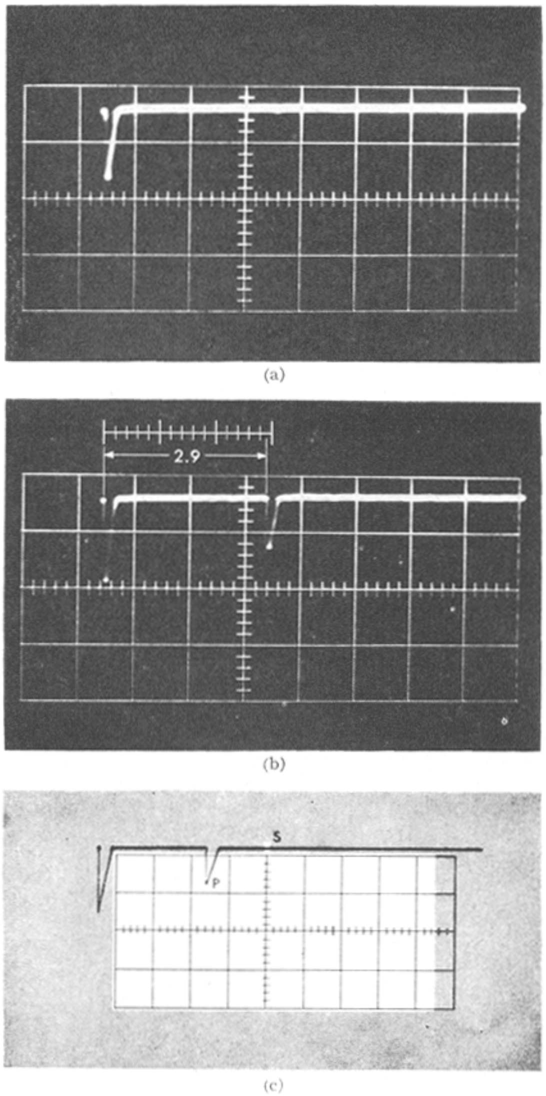


FIG. 4. (a) This photograph shows a single trace caused by a μ-meson traversing the scintillator. Each major division of the time scale is one μsec. (b) This photograph shows a single trace caused by a μ-meson stopping in the scintillator and decaying after 2.9 μsec. The ionization caused by the resultant electron produces the second pulse. (c) The drawing shows the position which the events recorded in 4(b) would have had during the experiment. The undeflected oscilloscope traces were behind the mask (shaded area) during the experiment. Only pulses (like P) from the decay electrons were visible. The small slit (S) in the mask was used to position the oscilloscope trace. A total sweep length of 8.5 μsec was used to detect the decay pulses.

or negatively charged: $\mu^{\pm}\rightarrow e^{\pm}+\nu+\bar{\nu}$. The neutrino and antineutrino are neutral and pass out of the scintillator without being detected. The electron is charged, and so when it shoots through the scintillator—typically for a distance of several inches—it gives out a second flash of light. This gives a second electrical signal on the trace as shown in Fig. 4(b).

The particular meson which gave the signals shown in Fig. 4(b) stopped in the scintillator and remained at rest for 2.9 μsec before decaying. In order to get a statistically significant distribution of decay times, we had to measure a large number of these decay events. To do this, time exposures of the oscilloscope face were made with a Polaroid camera. However, for every particle which decays in the scintillator, hundreds of particles pass straight through without stopping. Thus, a time exposure during which several decay events occurred would be fogged by the light from the

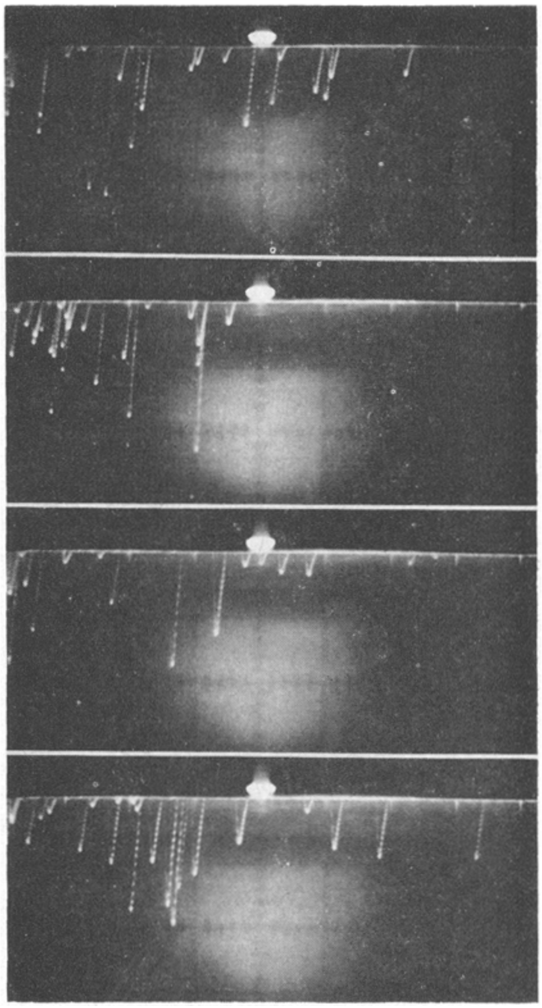


FIG. 5. Time exposures of the decay electrons taken in the manner shown in Fig. 4(c). The bright spot in the top center of each photo is due to the long time exposure through the slit S of the many traces from "straight-throughs."

many traces of the straight-throughs. In order to record only the events where a decay pulse occurred, we masked off the initial pulses and the undeflected traces, both of which are given by the straight-throughs. This mask is shown schematically as the shaded area in Fig. 4(c). Only when a meson stops and decays in the scintillator does a pulse become visible by coming out from behind the mask as at P. Time exposures like those in Fig. 5 can thus show many decay pulses without being fogged by the straight-throughs.

While the photographs were being taken, a small photomultiplier viewed the oscilloscope screen and responded to the light flash given off when each decay pulse showed out from behind the mask. Each electrical signal from this photomultiplier was counted, giving a running total of the pulses photographed. A narrow slot in the mask is shown at S in Fig. 4(c). This slot was used to set the position of the trace and to keep it a constant distance d behind the edge of the mask. The slot was so narrow that the light pulse from a trace passing by it was not sufficiently large to actuate the counter circuits.

To test the time dilation factor predicted by special relativity it was necessary to select mesons with speeds nearly that of light. In order to do this the scintillator on Mt. Washington was placed under a layer of iron $2\frac{1}{2}$ ft thick. (The opening under the pile of iron bars in which the scintillator was placed is visible in Fig. 3.) Mesons coming down vertically with speeds of less than $0.9950c$ were stopped by the iron before they reached the scintillator. Mesons going faster than $0.9954c$ went through both the iron and the plastic. Therefore, a decay signal that was counted indicated that a meson with speed between $0.9950c$ and $0.9954c$ had stopped in the scintillator. This method of selection of meson speeds, and its relation to the assumption that the mesons come in vertically only, is discussed further in Sec. IV-C and IV-I, respectively.

III. RESULTS

The equipment described in the previous section was installed in a laboratory near the peak of Mt. Washington, N. H., at an altitude of 6265 ft. For the film, we took data for one hour and got the distribution of decay times from it. We go over that data first, and do a rough calculation based on it, and then present the rest of the data from other runs. An accurate calculation of the time dilation based on all our data is presented in Sec. V.

During that one-hour run, giving 568 counts, pictures similar to that shown in Fig. 5 were taken, continually, so that all the decay pulses were recorded. The time each meson lived after stopping in the scintillator was plotted as a vertical bar in Fig. 6(a). Note that our apparatus could not detect mesons decaying after 8.5 μsec. This is discussed in Sec. IV-F.

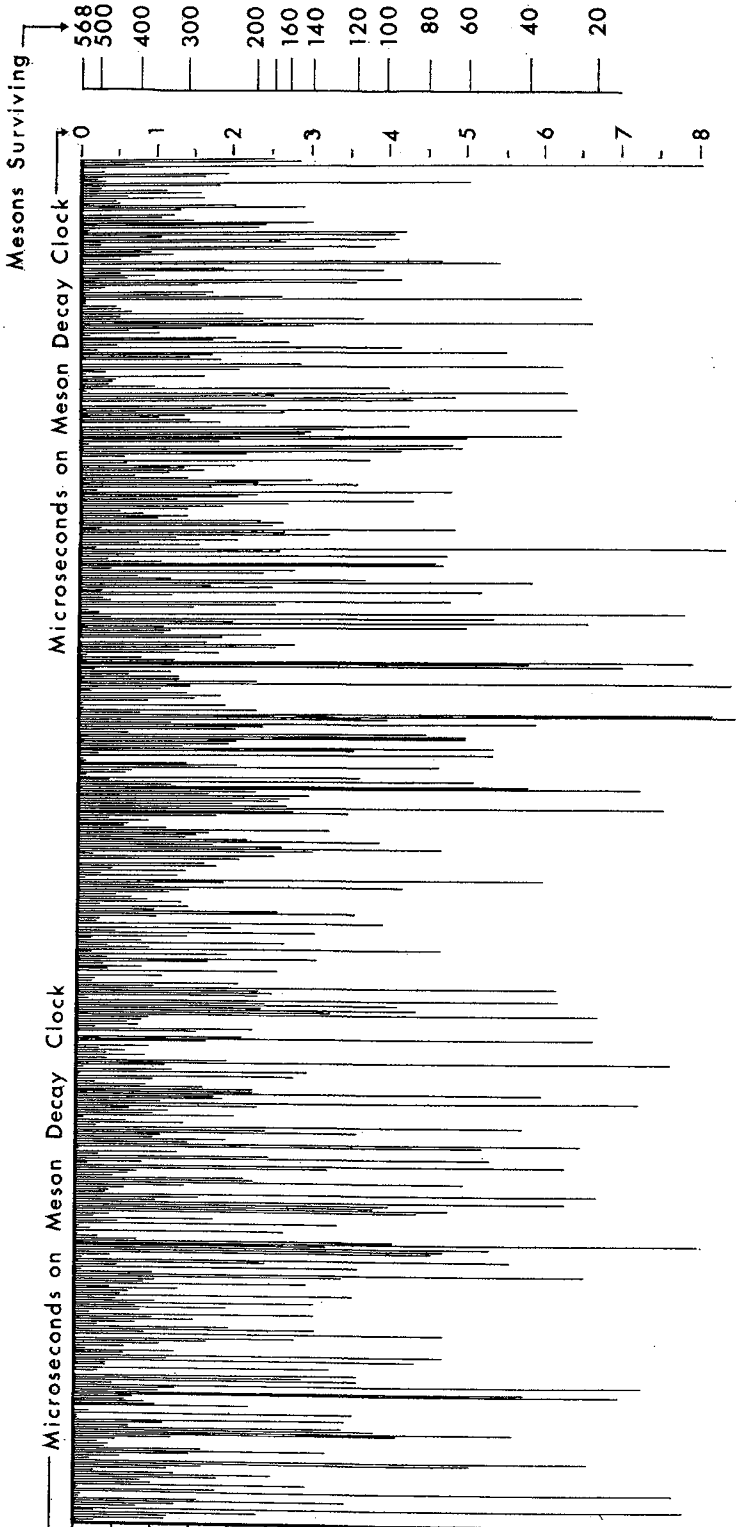
The data of Fig. 6(a) are most directly interpreted as an exponential decay of radioactive particles [Fig. 6(b)] with mean life $2.2\pm0.2 \times10^{-6}$ sec.[4] Although, as we shall see in Sec. IV-D, the exponential decay is not necessary to our argument; the observed exponential form of the decay distribution, and the agreement of our measured mean life with the accepted value, confirm our belief that we were really counting μ-mesons with this simple apparatus.

In the second half of the experiment, we moved the equipment down to 10 ft above sea level in Cambridge, Mass., and again recorded the number of mesons decaying in our counter.

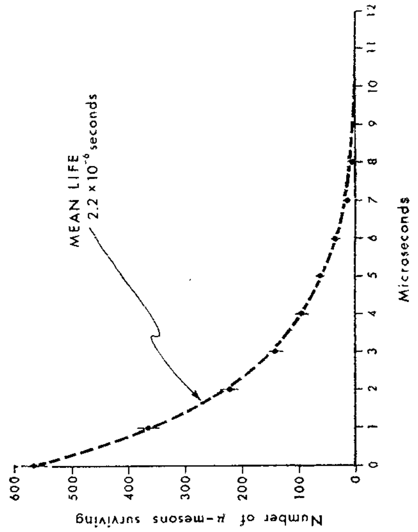
Cosmic-ray intensities are known not to vary appreciably over the difference in latitudes between Mt. Washington and Cambridge. We can assume, then, that there were about 568 mesons per hour—to take the number shown in Fig. 6(a)—descending past the 6265-ft level; but by the time they reached sea level they had been slowed down somewhat by the air. A foot of iron, which is approximately equivalent in slowing-down power to 6265 ft of air was therefore removed from the pile to compensate. This made sure that the mesons counted at sea level also had speeds between $0.9950c$ and $0.9954c$ when they came down past 6265-ft altitude.

In an hour's run at sea level we recorded 412 decays. Although not shown here, decay time distribution data similar to those of Fig. 6(a) were taken during the hour when the 412 decays were counted. The distribution of decay times agrees within statistics with that taken on Mt. Washington, showing that the same type of particles, μ-mesons, were counted at sea level.

[4] R. A. Reiter, T. A. Romanowski, R. B. Sutton, and B. G. Chidley, Phys. Rev. Letters 5, 22 (1960).



(a)



(b)

FIG. 6. (a) This is a reproduction of the chart used, in the film, to display the data from a 1-h run on top of Mt. Washington. The decay time of each μ-meson is represented by a vertical line, the length of which is proportional to the decay time. The lines (decay times) of 568 μ-mesons are displayed side-by-side across the chart. (b) The data are obtained from the chart in Fig. 6(a). The dashed curve is for a theoretical mean life of 2.2 μsec.

TABLE I. Number of μ-meson decay counts in several one-h runs at two altitudes. (The 559 count was inadvertently reported as 570 in the film.)

Run	On Mt. Washington	At Cambridge
1	568	412
2	554	403
3	582	436
4	527	395
5	588	393
6	559	...
Av hourly rate	563 ± 10	408 ± 9

What do we expect for the intensity at sea level? Let us first do the calculation using just the data in Fig. 6, without worrying about statistical fluctuations. In one hour we got 568 counts at the 6265-ft level, and we measured the distribution in time of the decays of these mesons, which were at rest with respect to us. In order to read off from Fig. 6(a) how many mesons we expect to count at 10 ft above sea level, we must first estimate their flight time over a 6255-ft vertical path. The particular speed interval we selected averaged $0.9952c$. To travel 6255 ft (1907 m) at that speed takes: $1907\text{ m}/(0.9952\times3.0\times10^{8}\text{ m/sec})=6.4\times10^{-6}$ sec. Thus, as measured by us, i.e., as read on clocks fixed to the earth, the mesons take 6.4 μsec to travel from the height of the top of Mt. Washington down to sea level.

If we now read off 6.4 μsec on Fig. 6(a), we see that we expect about 27 mesons to be left at sea level. The fact is that we observed 412 instead of 27 mesons left at sea level! We conclude that the mesons decay much more slowly when they are in rapid flight, relative to us, than they do when they are at rest, with respect to us.

If we use the decaying mesons as clocks—and as discussed above, there is every reason to believe that they are just as good as any other clocks—we find, referring to Fig. 6, that 412 surviving mesons corresponds to an elapsed time of only about 0.7 μsec. That is, the moving mesons measure only $0.7/6.4\approx1/9.1$, the time our clocks measure for their trip down. We conclude that *their clocks are running slow by a factor of about nine.*

Actually, we took data during a total of 6 separate hours at Mt. Washington and 5 separate hours at sea level. The data are shown in Table I, and give an average of 563 ± 10 mesons per hour on Mt. Washington, and 408 ± 9 at sea level. We use these more accurate data, and the mean life of μ-mesons accurately known from other experiments, to give a more accurate measure of the time dilation in Sec. V.

IV. DISCUSSION

In this section, we discuss the major assumptions used in this experiment, and the experimental corrections that have been neglected in the previous sections, in order to assure ourselves that we have, indeed, observed the time dilation.

A. Formation of μ-Mesons

The most important body of knowledge necessary to the interpretation of this experiment is the great amount of information about cosmic rays and about their interactions with such materials as iron, air, and plastic.[5]

So far as is known now, the primary cosmic rays are mainly protons that have been accelerated by electromagnetic fields within our own galaxy. Because they are deflected by the earth's magnetic field, they can come into the earth more easily near the poles than near the equator. However, over the comparatively small difference in latitude between Cambridge, Mass., and Mt. Washington, N. H., we are not troubled by this latitude effect. The numbers of incident protons that have energies sufficient to give rise to the mesons we counted are also independent of time; so that we can take data at the two different altitudes at different times as well as at different places.

When the cosmic-ray protons come in toward the earth, they hit the nuclei of nitrogen and oxygen atoms high in the atmosphere. Positive and negative π-mesons are made in these collisions, and they decay rapidly into positive or negative μ-mesons, respectively. Some neutral π-mesons are also formed, and they decay, even more rapidly, into gamma rays. The gamma rays from the neutral π-mesons in turn cause showers of electron–positron pairs. We do not have to worry about the electrons and positrons because they do not give rise to any pairs of pulses that

[5] B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1950), see particularly Sec. 4.9 and Chap. 8.

come separated in time by a few microseconds as do the decay electrons from the μ-mesons.

By the time cosmic-ray particles have penetrated down to the 6000-ft level, they have come down past about 80% of the matter in the atmosphere, and most of the primary protons have already been absorbed. This is important, because if there were appreciable numbers of primary protons left, they would produce π-mesons, which in turn would decay rapidly into μ-mesons, and we would be measuring the indirect production of μ-mesons as well as the decay of μ-mesons as a function of altitude. A partial compensation of the decay rate by such production would be interpreted as an additional time dilation, and the relativistic time dilation factor we calculate from our observations would be an overestimate.

In neglecting such possible creation of μ-mesons in proton and π-meson interactions between Mt. Washington and sea level, we rely on the results of other experiments. These have shown by direct search for μ-meson production that it is highly improbable that there is an appreciable number of interactions in the 0–6000 ft altitude range which create μ-mesons of the energies we would observe. So the μ-mesons we count at sea level are primarily survivors of μ-mesons which were created above the 6000-ft level of altitude.

B. Interactions of μ-Mesons in Flight

A related question is whether the μ-mesons themselves are absorbed by the molecules of air, or by the nuclei of air or of iron, thus living a shorter time than they would if they came down through a vacuum. A lifetime shortened by absorption in flight would be interpreted as an apparent time contraction rather than a time dilation. Thus, if μ-mesons were to interact strongly in flight, the relativistic time dilation factor we calculate from our observations would be an underestimate.

It is known from many experiments[6] that neither positive nor negative fast μ-mesons are appreciably absorbed by the nuclei of atoms; but it is known that there is the Rutherford scattering, caused by the electromagnetic repulsion or attraction between mesons and nuclei. This scattering changes the direction of a μ-meson which passes the nucleus, but usually by only a very small amount. The μ-mesons we detect have been slowed down in the air and the iron and have undergone many small-angle scatterings, but very few of them have been deviated more than a few degrees by the time they are close to the plastic or have already entered it. Thus, except for those lost by decay, we will treat all mesons as coming all the way down from Mt. Washington altitude to sea level in an approximately straight line through the air and iron.

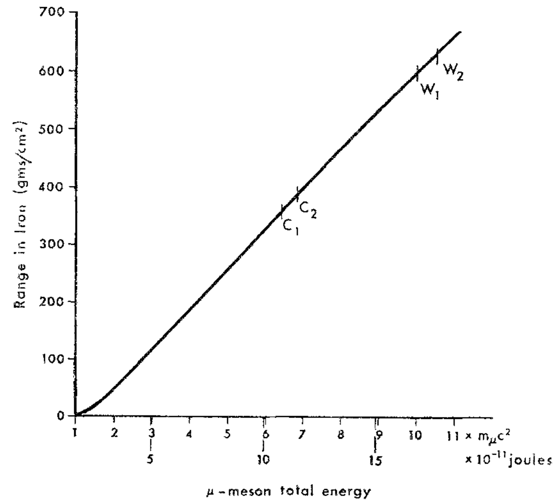


FIG. 7. The curve represents the approximate range-energy relationship for μ-mesons traversing iron. W_1 and C_1 represent the thicknesses of the iron on top of Mt. Washington and at Cambridge, respectively. The total thicknesses of the iron plus plastic are represented by W_2 (on Mt. Washington) and C_2 (at Cambridge). The difference between W_1 and C_1 represents the thickness of iron equivalent to 6255 ft of air, and the difference C_2-C_1 or W_2-W_1 is the equivalent thickness of the scintillator.

C. Measurement of the Speed of the μ-Mesons

A charged particle, such as a μ-meson, loses energy when it passes through matter by knocking electrons out of the atoms which it passes or by exciting these atoms. Some of the particle's energy is given to these electrons, and the charged particle slows down and eventually stops. The amount of material which a charged particle can penetrate gives a good measure of its energy.

Figure 7 shows a graph of the number of grams

[6] R. B. Leighton, *Principles of Modern Physics* (McGraw-Hill Book Company, Inc., New York, 1959), see p. 633 for the properties of μ-mesons.

per square centimeter of iron which μ-mesons of various energies can penetrate before stopping. The energy is given both in joules and in multiples of the rest energy ($m_\mu c^2$) of the particle. Notice that the total energy is used for the abscissa. At a total energy of one rest energy, the particle is at rest and hence, penetrates no iron. The graph of Fig. 7 is a theoretical one,[7] but the theory is one which has been extensively checked in various experiments.

On top of Mt. Washington the plastic scintillator was covered with $2\frac{1}{2}$ ft of iron. Iron has a density of about 7.9 g/cm^3, so $2\frac{1}{2}$ ft corresponds to 600 g/cm^2 of iron. Referring to point W_1 in Fig. 7, we see that a meson which could just penetrate the $2\frac{1}{2}$ ft of iron and stop in our scintillator must have arrived at the altitude of Mt. Washington with a total energy of 10.0 rest energies.

To a first approximation, the loss in energy of a charged particle passing through matter depends only on the number of electrons it passes, and not on the particular chemical composition of the material. The electron density is closely proportional to the mass density for most materials, so that we may use Fig. 7, approximately, for all materials. Our scintillator was about 30-g/cm^2 thick. μ-mesons with energies greater than 10.5 rest energies (point W_2) would penetrate not only the iron but also get through the plastic. On top of Mt. Washington, therefore, we stopped and observed the decays of mesons with energies between 10.0 and 10.5 rest energies when they arrived at the altitude of Mt. Washington.

When the apparatus was set up at Cambridge only $1\frac{1}{2}$ ft of iron, or 359 g/cm^2 (point C_1) were placed above the apparatus. The barometric pressure on Mt. Washington is approximately 60.0 cm of mercury, whereas it is 76.0 at sea level. Since mercury has a density 13.6 g/cm^3, this means that there are 13.6 (76.0–60.0) = 218 g/cm^2 of air between sea level and the altitude of Mt. Washington. To a first approximation, then, the apparatus was "covered" to the same altitude as before by 359+218 = 577 g/cm^2 of material, whereas on top of Mt. Washington 600 g/cm^2 of iron were used. Air is, however, a slightly more efficient slower of μ-mesons than iron. Not only

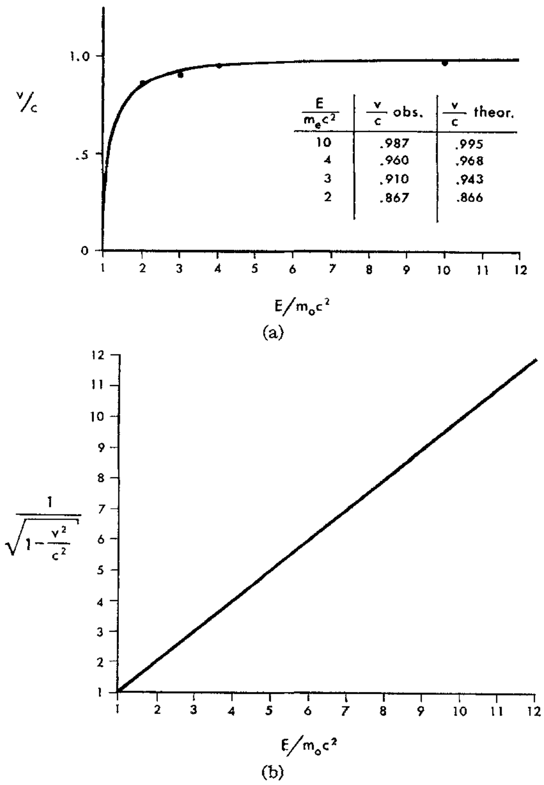


FIG. 8. (a) The solid curve gives the theoretical relationship between the speed of a body, as a fraction of the speed of light, versus the total energy of the body in units of its rest energy. The experimental points are for electrons (reference 9). (b) The theoretical relationship between the time dilation factor γ of a body and the total energy of that body in units of its rest energy.

are there somewhat more electrons per gram of air than per gram of iron, but the electrons in light atoms such as those of nitrogen and oxygen are on the average more effective than those in iron. All told, the 218 g/cm^2 of air are equivalent to 270 g/cm^2 of iron. Mesons arriving at the 6265-ft level above Cambridge had to penetrate the equivalent of 270+359 = 629 g/cm^2 of iron, in order to enter our scintillator. Thus, the band of meson energies acceptable at sea level was of the same width, as set by the stopping power of the plastic, but was centered on an incident energy some 5% higher than appropriate to the band selected on top of Mt. Washington. (We should have used one less layer of iron bars, making the average energy some 2% too low, but we misestimated.) Fortunately, the experiment is quite insensitive to the exact stopping power of the iron removed, because at these energies the number of incident mesons per energy interval

[7] B. Rossi, reference 5, Chap. 2.

does not change rapidly as their mean energy is changed.[8]

Point C_2 marks the maximum energy a meson could have and still stop in the scintillator at Cambridge.

To convert these energies to the speeds of the mesons, we use the expression given by the Special Theory of Relativity, $E=m_\mu c^2/(1-v^2/c^2)^{\frac{1}{2}}$ or, transposing, $v/c=[1-(m_\mu c^2/E)^2]^{\frac{1}{2}}$.

This relativistic prediction for the relation between the total energy and the speed has been checked in many experiments. For example, Fig. 8(a) gives the experimental results for v/c vs $E/m_e c^2$, obtained with electrons.[9] Figure 8(b) shows the quantity $1/(1-v^2/c^2)^{\frac{1}{2}}$ plotted vs $E/m_0 c^2$, where E is the total energy and m_0 the rest energy of any particle. We can see that v/c levels off near unity, at energies which are large compared with the rest energy of the particle, but that $1/(1-v^2c^2)^{\frac{1}{2}}$ keeps increasing. We note that the total energy of the meson is thus just its rest energy $m_\mu c^2$ times the same factor $1/(1-v^2/c^2)^{\frac{1}{2}}$ used in computing the time dilation.

At speeds very near that of light, where m_0c^2/E is very small, we can expand the radical in the expression for v/c and get, to good approximation, $v/c\approx 1-\frac{1}{2}(m_0c^2/E)^2$. For example, for $m_0c^2/E=10.0$, $v/c\approx 1-\frac{1}{2}\times 1/100$, which is $0.9950c$. Those mesons which are incident on the $2\frac{1}{2}$ ft of iron on Mt. Washington, and are slowed down and stop in the plastic, had incident speeds between $0.9950c$ and $0.9954c$. At sea level the band of speeds incident on the $1\frac{1}{2}$ ft of iron extends from $0.9881c$ to $0.9897c$. We discuss in Sec. V the effective average speed in flight for the mesons we count.

D. The Independence of this Experiment on the Shape of the Decay Distribution

We have avoided much discussion of the exponential nature of the radioactive decay of μ-mesons because it is not important for our purposes that they decay in any particular way, as long as they decay in the same way under all conditions.

[8] W. L. Kraushaar, Phys. Rev. 16, 1045 (1949).

[9] W. Bertozzi, *The Ultimate Speed, An Exploration With High Energy Electrons* (a filmed experiment produced by Educational Services Incorporated, Watertown, Massachusetts, 1962).

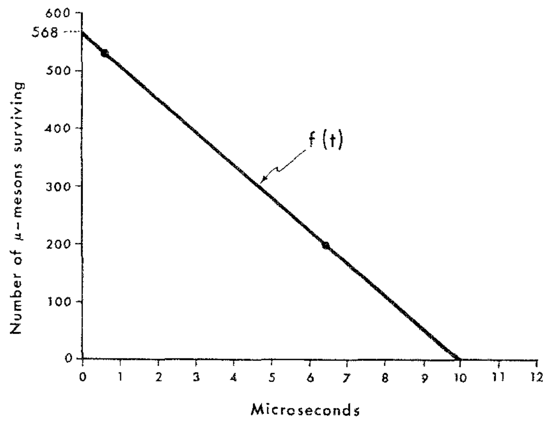


FIG. 9. Time dilation for a hypothetical straight-line decay distribution.

Let us first show that the functional form of the decay curve is unimportant. Let us call $f(t)$ the probability that a μ-meson that has entered the scintillator at the time $t=0$ survives until at least time t. If $f(t)$ decreases monotonically as time increases, there is a unique time that corresponds to every value of $f(t)$. If we observe a certain $f(t_1)$ after an elapsed time t_1 after the mesons come to rest, then we would expect to observe the same number of counts $f(t_1)$ after what we compute to be t_1 elapsed time while the mesons are in flight—that is, if we did not know about time dilation. Instead, we observe a different number, and, whatever value this number has, it is some $f(t_2)$, thus defining the time t_2 that elapses in the system of the mesons. For example, for the exponential form of decay, which we have observed, we expected a decrease from 568 counts to 27 counts in 6.4 μsec. That is, $f(6.4)/f(0)=f(6.4)/1=27/568=0.047$. We observed, instead, the ratio 412/568, i.e., $f(t_2)=412/568=0.725$. On the basis of an exponential decay with 2.21-μsec mean life, the t_2 which gives $f(t_2)=0.725$ must be 0.67 μsec.

Suppose, instead, that there were a different form of decay—for example, a straight line from 0–10 μsec, as illustrated in Fig. 9. Then $f(6.4)=(10.0-6.4)/10=0.36$, and without knowing about time dilation, we would predict $0.36\times 568=203$ decays after 6.4 μsec. The observed number would not have been 203, but would again have corresponded to a t_2 of 0.67 μsec because of the time dilation; that is, the number observed would have been $(10.0-0.67)568/10=530$. Of course, we have no way of checking this confident prediction with mesons or other elementary particles because they have only the exponential form of radioactive decay, but the emission of light of a given frequency by a moving atom can be viewed as a linear decay of stored beats, and it indeed shows the time dilation as we have predicted.

Next, we show that the time at which we begin to observe the decay of the mesons in the scintillator is unimportant. Suppose there is a delay of elapsed time τ before we begin counting meson decays, then the probability that a meson has lived at least $t+\tau$ is $f(t+\tau)$. In this experiment, τ was approximately 0.5 μsec, because we masked over the initial pulse so that the trace emerged half a μsec after the meson came to rest. This delay time was the same both on Mt. Washington and at sea level, because it was a property of the oscilloscope trace only, and the oscilloscope was always at rest relative to us. Thus, we defined $f(0.5)\equiv 1$, and observed on Mt. Washington that there were 568 decays after that time τ, with a distribution $f(t+0.5)$. Then we went down to sea level and observed 412 mesons. The ratio 412/568 was read from our graph as $f(0.7+0.5)$. The 0.7 μsec which described the time the mesons were in flight was dilated quite independent of the added undilated 0.5 μsec.

We now see that this experiment can be interpreted in either of two ways. The first method is the one used in Sec. III of this paper. The distribution of decay times of mesons at rest was measured, and from it the time the mesons spent in flight according to their own clocks was inferred. We have just seen that this method is independent of the form of the time distribution of decays. It requires, however, that the decay distribution be the same in the air as in the scintillator, except for the time dilation.

The second method, used in Sec. V, uses the information from other experiments that the decay distribution of μ-mesons is exponential, and that their mean life is $2.211\pm0.003\times10^{-6}$ sec, to combine with our total counts above and below to get the time dilation factor. This second method gives a much more accurate result than the first.

E. Interactions of μ-Mesons at Rest

When the μ-mesons are brought to rest, those mesons which have a negative charge are captured into orbits in the atoms of the scintillator (principally, carbon). Some of these mesons are captured by carbon nuclei before they decay, giving a slightly shorter mean life[4] for the μ^- than for the μ^+. Since the cosmic rays contain approximately equal numbers of μ^+'s and μ^-'s incident, we should get a decay distribution in our plastic which can be approximated by a single mean life of slightly less than the $2.211\pm0.003\times10^{-6}$ sec observed for free mesons—about 2.1×10^{-6} sec—and we do so within our experimental error.

Thus, in our calculation of the time dilation factor for mesons in flight from the observed distribution of decay times of mesons at rest in the scintillator, we overestimated the time dilation factor by using our observed mean life of mesons at rest in plastic, rather than the mean life of mesons at rest *in vacuo*. However, our distribution was not measured accurately enough to be sensitive to the difference.

F. The Loss of Counts after 8.5 μsec

Our observed integral distribution of decay pulses is not exactly exponential, for another reason. The end of the oscilloscope sweep was at 8.5 μsec, and we lost any decays that occurred after that time. Since there were 568 decays in 8.5 μsec, there should be, for an exponential decay with a 2.2×10^{-6} sec mean life, about 10 counts missed, i.e., only a 2% correction to the total number of decays.

G. Background from Accidental Near-Coincidences in Time from Straight-Through Pulses

Even though we could have an arbitrary distribution of decay times, it is of course important that the pulses we find are true decay pulses and not random pulses from straight-through cosmic rays. The following rough estimate of the random occurrence of delay pulses within the 8.5-μsec period shows that we do not have an intolerably large background of accidental apparent delays. The chance of any one straight-through pulse following another within 8.5 μsec is $8.5\times10^{-6}n$, where n is the average number of straight-throughs per second. The number of chance coincidences per sec is $n(8.5\times10^{-6}\,n)$.

On Mt. Washington, n was measured to be about 75 000 per h, divided by 3600 sec per h, about 21 per sec. Therefore, there is only a fraction $8.5\times10^{-6}\times21=1.8\times10^{-4}$ accidentals per trace, or an absolute rate of $21\times1.8\times10^{-4}=3.8\times10^{-3}$ accidentals per sec. The number of true decays per second is, on Mt. Washington, 568 per h/3600 seconds per h equals 0.15 per sec. Thus, the ratio of accidental decays from coincidences between two straight-throughs to true decay pulses is $3.8\times10^{-3}/0.16=2.4\times10^{-2}$, or about $2\frac{1}{2}\%$ of the true decay pulses.

This number of accidentals is, in fact, somewhat smaller at sea level because the rate of straight-throughs is down to approximately 19 per sec, and the number of accidentals is proportional to the rate squared. If the ratio of accidentals to true events were exactly independent of altitude, the correction for accidentals would make no difference in the ratio of counts at sea level to counts on Mt. Washington, and this is almost the case. To correct for accidentals requires a subtraction of about 13 counts from the average number on top of Mt. Washington, and of 11 counts from the number at Cambridge. The trues stood in the ratio $412/568\approx9.4/13$ so that instead of giving about a $2\frac{1}{2}\%$ effect on the ratio, the correction for accidentals gives an $\{(11/13)-(9.4/13)\}\times2\frac{1}{2}\%$ effect, i.e., not much more than a 0.3% effect. Thus, the correction for accidentals is negligible compared with the statistical errors.

H. Effects on the Time Dilation of the Deceleration of Mesons in Flight

The μ-mesons observed in this experiment are being decelerated greatly in coming down through the earth's atmosphere. It can be shown that the acceleration of a particle of rest mass m_0 in the frame of reference in which it is at rest is simply $a=(1/m_0)(dE/dx)=(c^2/m_0c^2)(dE/dx)$, where dE/dx is the rate at which it loses energy per unit distance. Figure 7 shows that high speed μ-mesons lose about one rest energy ($m_\mu c^2$), every 55 g/cm², or every 4.4×10^4 cm of air. Thus,

$$a=(9\times10^{20}/m_\mu c^2)(m_\mu c^2/4.4\times10^4)$$
$$=2\times10^{16}\text{ cm/sec}^2=2\times10^{13}\,g,$$

where g is the acceleration of gravity.

Although this seems to be a very large acceleration, it is minute compared with those experienced by μ-mesons in close encounters with atomic nuclei, where they undergo accelerations of magnitude $10^{29}\,g$. Even in such encounters μ-mesons are not torn apart, so we expect that their internal time-keeping mechanisms are not seriously affected by the much smaller accelerations in this experiment.

Note that in computing the time dilation we need apply only special relativistic—rather than general relativistic—transformations to these accelerated particles, as long as our calculation itself is made in an inertial frame of reference.

I. Effect of Nonvertical Mesons

We come at last to a consideration which is clearly of great importance. The μ-mesons we observe are not coming exactly straight down. The angular distribution about the vertical direction of the high energy mesons we are counting is roughly proportional to $\cos^3\theta$, where θ is the angle to the vertical direction.[8] This means that there are only $\cos^3 45°=(0.7)^3=0.35$ as many μ-mesons coming down per unit solid angle at 45° to the vertical as in a unit-solid-angle close to the vertical direction. At 60°, $\cos^3\theta=\frac{1}{8}$. However, there is more solid angle in a given interval of polar angle at larger angles to the vertical, so that there has been an appreciable contribution to our counting rates from angles greater than 60°.

Since the μ-mesons do not all come straight down, some go through a thickness of air and iron which is much greater than the vertical distances we have discussed. How can we interpret our experiment quantitatively without being able to correct for the time dilations of those mesons we stopped as a function of their angles?

As we saw in Sec. IV-C, the range of a particle is almost linearly proportional to its total energy in this region of ultrarelativistic energies; and since the total energy increases by the factor $1/(1-v^2/c^2)^{\frac{1}{2}}$ as the speed v increases, the range of the particle also increases by the same factor. Thus, a meson coming in at 45°, and stopping in our scintillator, has greater range than one coming in vertically by a factor $1/\cos 45° = 1.4$, and it has a greater energy by almost exactly the same factor.

The time dilation factor we are investigating is also[10] proportional to $1/(1-v^2/c^2)^{\frac{1}{2}}$. Particles which have a greater distance to travel by a factor of 1.4, and which therefore, because of our method of selection, have a greater energy by a factor of 1.4, should also have a lifetime dilated by the same factor, provided Special Relativity is correct. Hence, the larger fraction we expect to decay, because of the greater distance they go, is just compensated by the smaller interval of elapsed time we read on their clocks per unit distance they move.

We conclude that the apparent time dilation factor for high-energy particles which come down diagonally is the same as for those which come straight down, provided they both come down the same vertical distance and are selected out by passage through the same horizontal slab of air or iron.

Note that this happy accident occurs only if both the atmosphere and the iron are wide enough that a diagonal particle goes a distance through them actually proportional to $1/\cos\theta$. While the atmosphere was surely wide enough, the roof of our pile of iron was small enough, for practical reasons (it weighed 10 tons), that the mesons coming in at an angle greater than 45° to the vertical passed through the sides of the pile rather than through flat roof. Therefore, mesons at large angles to the vertical passed through less material and had less energy than they should have had to give this cancellation of effects.

Those mesons coming in at large angles to the vertical caused the greatest uncertainty in our experiment. We could have restricted the range of angles by requiring a coincidence with another scintillation counter placed some distance above our big scintillator, but we wanted to keep the experiment simple for filming.

[10] The fact that the dilation of energy from the "rest energy" and the dilation of time from the "proper time" (in this case the mean life for mesons at rest) have the same functional dependence on speed relative to the observer is described as follows: energy is the fourth component of the momentum-energy four-vector and time is the analogous fourth component of the space-time four-vector.

V. FINAL COMPARISON OF THEORY AND EXPERIMENT

A. Theory

The average incident meson energy on top of Mt. Washington is, for a vertical meson, approximately 10.2 $m_\mu c^2$. Similarly the average energy of those incident on top of the iron at sea level is about 6.8 $m_\mu c^2$. In their flight down through the atmosphere, the meson clocks ran slow with factors ranging, on the average, from 10.2 down to 6.8. For rough purposes we could take the median between these extremes, 8.5, as the approximate time dilation factor expected. For an exact calculation, we must take a weighted average which accounts for the following effects: (1) the range-energy curve is not exactly linear, and the atmospheric density changes as a function of height, so that the time spent at various energies is a complicated function of energy; (2) even if the mesons had spent equal times in each energy interval, the correct average energy would not be the median energy, because the decay rate is not linear in the time dilation factor, but is an exponential function of its reciprocal (see below).

The correct averaging gives an expected time dilation factor of 8.4, only slightly below the linear average. Because of the uncertainty in the numbers of incident mesons as a function of angle to the vertical, and consequent uncertainty in the amount of matter they go through, and because of the possibility of a small amount of μ-meson production between Mt. Washington and sea level altitudes, our estimate of the time dilation factor is good only to the order of two rest energies, i.e., the predicted factor is 8.4 ± 2. This rather large uncertainty is essentially only guessed at, because of the great amount of research it would take to make our knowledge of the sources of error more precise.

B. Experiment

The time dilation factor calculated from the data is quite sensitive to the quality of the experiment, because the time dilation is so great that the number of counts at sea level is not very much less than the number at Mt. Washington altitude. The same uncertainties in the observations would give a much smaller uncertainty in the time dilation factor if the path over which the decays were measured had been appreciably longer.

We made a rough estimate of a factor of 9 for the time dilation (Sec. III) by reading from the chart the time corresponding to a count of 412 mesons. We now make a more accurate calculation using all the data we collected. Since there is every reason to assume here that we have successfully identified the particles we are counting as those μ-mesons whose mean life at rest has been measured accurately to be $2.21\pm.003$ μsec, we use this information to make our more accurate calculation, rather than take the functional form of the decay distribution from our own data. We are on more certain ground, in any case, because the interactions of the μ-mesons stopped in the detector give a distribution of decay times slightly different from the distribution in the proper system of the mesons as they pass through the atmosphere (Sec. IV-E).

We also must use the speed which allows for the slowing down in the air between Mt. Washington and sea level, an average of about $0.992c$.

After calculating the statistical standard deviation from the mean of the numbers we have observed, we can write as the rate on Mt. Washington 563 ± 10, and at sea level 408 ± 9. If we call the time dilation factor γ, we have as our experimental statement:

$$\exp\left(-\frac{1907}{\gamma\times2.998\times10^8\times2.211\times10^{-6}}\right)=\frac{(408\pm9)\;-(11\pm2)}{(563\pm10)-(13\pm2)}=\frac{397\pm9}{550\pm10}=0.722\pm0.021.$$

Therefore, $\gamma_{\rm obs}=8.8\pm0.8$. (Note that the error in the μ-meson half-life is negligible by comparison with the greater error from our statistical fluctuations. Also, the errors in the backgrounds subtracted are estimated systematic errors rather than statistical ones, which are even more completely negligible.)

The statistical error quoted for the experimental result is probably an underestimate, particularly since we were troubled during part of the experiment with some drift of the amplification due to imperfectly stabilized electronics. Nevertheless, we do not know explicitly any other source of error that we can estimate quantitatively; but can only caution that the over-all error in the experimental value of γ is probably somewhat greater than the 0.8 quoted.

So our final comparison is between a predicted time dilation factor 8.4 ± 2 and an observed time dilation factor 8.8 ± 0.8 (statistical standard deviation only). This is eminently satisfactory agreement, easily compatible with the errors in both the theoretical and experimental numbers.

Thus, not only does our experiment give direct qualitative evidence of time dilation, but the observed numbers support the quantitative predictions of the Special Theory of Relativity.

ACKNOWLEDGMENTS

We wish to thank Richard Knowles for his energetic assistance in all phases of this experiment; Dr. George Clark for technical advice; Dr. James Strickland and John Friedman for great help in the preparation of both the film and this paper; the MIT Laboratory for Nuclear Science and the Cambridge Electron Accelerator for lending most of the equipment with which this experiment was done; and the members of the ESI Film group for their patience under adverse technical conditions.

Our film was one of a series planned by Professor Francis L. Friedman, and we were exceptionally fortunate in having his careful and vigorous guidance throughout its making, even toward the end of his fatal illness.