# Data and Error Analysis II

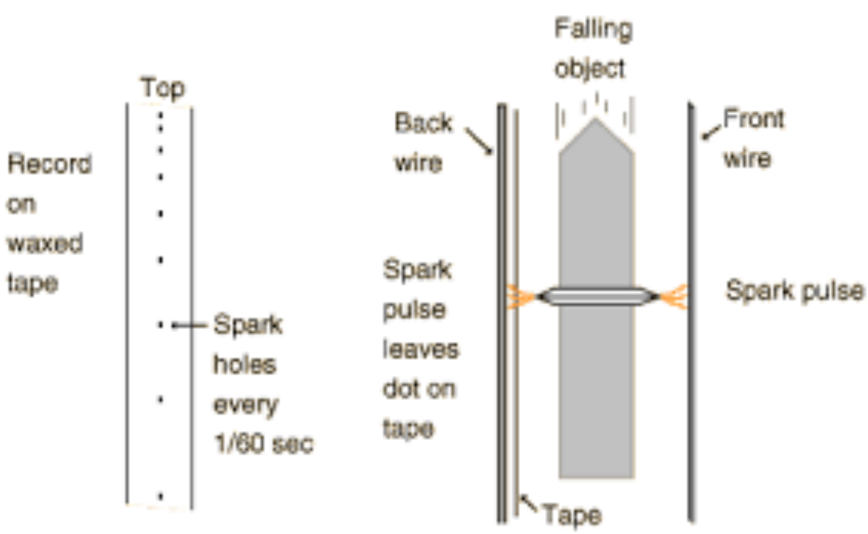
## Introduction

This lab consists of two parts. In the first part, you will practice data and error analysis by measuring the acceleration due to gravity using data from a free fall experiment. You will analyze the data three different ways, culminating in the data linearization technique that will be used often in later labs. In the second part, you will measure the area of one face of a rectangular block and calculate the uncertainty using error propagation.

You will need to use a spreadsheet program that can make plots and perform linear fits of plotted data. Microsoft Excel is the best choice because it is ubiquitous and has excellent plotting utilities. Google Sheets and Logger Pro are also good choices. Do not use the online version of Microsoft Office 365, which as far as I can tell cannot do linear fitting.

## The Experiment

In the apparatus shown below, a freely falling object causes electrical sparks to singe a long, vertical strip of paper at regular time intervals (60 times per second). By determining the position of each singe (with a meter stick) and knowing the time interval, the acceleration of the falling object can be measured.




Let's use a coordinate system in which positive $y$ is vertically downward. If the falling object is initially at rest at $y = 0$ when released, then its position $y(t)$ as a function of time $t$ is

$$y(t) = \frac{1}{2}gt^2 \qquad (1)$$

in which $g$ is the acceleration due to gravity.

## Part 1-1: One Data Point

Let's start with one data point. The position of the falling object at $t = $ (1/60) seconds is measured to be $y = $ 0.15 +/- 0.05 cm. Use equation (1) and this single data point to measure the acceleration due to gravity (first, use algebra to rearrange equation (1) to solve for $g$). Use the worst-case scenario error propagation (section 4.1 of the Error Analysis guide) to determine the uncertainty on $g$ due to the uncertainty on $y$ (assume that the error on $t$ is negligible).

**Informal Lab Report:** Show your work for the calculation of $g$ and its error and report your result in +/- notation with the correct number of figures. Compare your result to the accepted value of $g = $ 9.80 +/- 0.01 m/s$^2$ and comment on whether your measurement is consistent with the accepted value. Show your result to your lab instructor before starting Part 2.

## Part 1-2: Averaging the Data Points

Now let's use all of the data points. Download the .csv file called Data1 from Brightspace and open or import it into your preferred spreadsheet program. You should see two columns, one corresponding to time measurements and the other corresponding to vertical position. Make a new column that uses a formula to automatically calculate $g$ for each data point. In two other cells, use formulas to calculate the mean and standard error of $g$.

**Informal Lab Report:** Report your result for $g$, using the standard error as the uncertainty, and comment on whether your measurement is consistent with the accepted value. Show your result to your lab instructor before moving on.

Next, repeat Part 2 using a new set of data points from another run of the experiment. The second set of data is in the .csv file called Data2 on Brightspace.

**Informal Lab Report:** Report your result for $g$, with uncertainty, from Data2 and comment on whether your measurement is consistent with the accepted value.

## Part 1-3: Plotting the Data

You should have noticed that the result for $g$ using Data2 is quite bad. The reason for this is that there is a systematic error present and the data analysis procedure used in Part 2 (and also Part 1) is very sensitive to this particular error. Fortunately, we can improve our data analysis procedure to make it less sensitive to many kinds of systematic error, including the one that plagues Data2. As a bonus, this improved procedure will give us a nice visualization of the data that is much better than looking at a table of numbers.

First, we need to linearize the data so that it is expected to be linear when plotted, just as is explained in section 3.1 of the Error Analysis guide on Brightspace. In fact, the situation in this lab exactly matches the example used in the guide. By plotting $y$ on the y-axis vs. $t^2$ on the x-axis, we expect to get a linear trend with a slope equal to $\frac{1}{2}g$ and a y-intercept of zero.

In your spreadsheet that contains the problematic dataset Data2, create a new column that uses a formula to calculate $t^2$ automatically for each data point. Make a scatter plot (do not connect the data points with lines!) of $y$ vs. $t^2$ ($y$ on the vertical axis and $t^2$ on the horizontal axis) and fit it to a linear trendline. Use the LINEST formula in Excel or Google Sheets (or the equivalent formula if you're using different software) to find the standard error of the slope of your line of best fit. Be careful with units: the slope will have the units of the quantity plotted on the y-axis divided by the units of the quantity plotted on the x axis. If you want your slope to be reported in SI units, it is best to plot all quantities in SI units.

**Informal Lab Report:** Calculate the measured value of $g$ from the slope of your plot and the uncertainty on $g$ by propagating the uncertainty on the slope, and comment on whether your measurement is consistent with the accepted value. Include a printout of your plot with your lab report. The plot should have a title, axis labels with correct units, and the linear trendline.

**Question 1: What might have caused the systematic error in the Data2 data? Explain. Hint: look at the y-intercept of your fit and think about what this means given that your model for calculating $g$ is equation (1).**

**Question 2: Why is the process of linearizing, plotting, and fitting the data not sensitive to the systematic error in the Data2 data?**

## Part 2: Measuring the Surface Area of One Side of a Wooden Block

In this exercise, your goal is to measure the surface area of one side of a wooden block. Use the side that has the largest area – bounded by the two longest edges. You will model the side as a rectangle, so the area $A$ will be the product of the length $L$ and width $W$ of the edges:

$$A = LW \tag{1}$$

**Informal Lab Report:** Measure the length and the width five times each with a caliper or ruler and report all 10 measurements. Report a best measurement of $L$ with uncertainty using the mean and standard error of the data. Do the same for the best measurement of $W$. Next, calculate the area of the side using equation (1). Determine the uncertainty on the area by using the uncertainties on the length and width and the error propagation method taught by your instructor. Show all your work and report your measurement for the area with uncertainties using plus-minus notation.

## Informal Lab Report

In your informal lab report this week, include the following on a separate piece of paper in this order:

1) A heading with the title of the lab, your name, your partners' names, and the lab date.

2) The items specified under "**Informal Lab Report**" in Part 1-1 of this procedure.

3) The items specified under both "**Informal Lab Report**" paragraphs in Part 1-2 of this procedure.

4) A printout of your plot produced for Part 1-3 of this procedure.

5) The data analysis described under "**Informal Lab Report**" in Part 1-3 of this procedure.

6) The answers to questions 1 and 2 at the end of Part 1-3.

7) The items specified under "**Informal Lab Report**" in Part 2 of this procedure.