

# CONCEPTUAL QUESTIONS

- How many significant figures does each of the following numbers have?  
a. 53.2      b. 0.53      c. 5.320      d. 0.0532
- How many significant figures does each of the following numbers have?  
a. 310      b. 0.00310      c. 1.031      d.  $3.10 \times 10^5$
- Is the particle in **FIGURE Q1.3** speeding up? Slowing down? Or can you tell? Explain.

FIGURE Q1.3



- Does the object represented in **FIGURE Q1.4** have a positive or negative value of  $a_x$ ? Explain.
- Does the object represented in **FIGURE Q1.5** have a positive or negative value of  $a_y$ ? Explain.



FIGURE Q1.4

FIGURE Q1.5



- Determine the signs (positive or negative) of the position, velocity, and acceleration for the particle in **FIGURE Q1.6**.

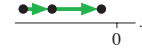


FIGURE Q1.6

- Determine the signs (positive or negative) of the position, velocity, and acceleration for the particle in **FIGURE Q1.7**.

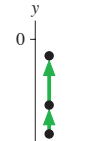


FIGURE Q1.7

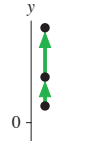


FIGURE Q1.8

- Determine the signs (positive or negative) of the position, velocity, and acceleration for the particle in **FIGURE Q1.8**.

# EXERCISES AND PROBLEMS

## Exercises

### Section 1.1 Motion Diagrams

- A car skids to a halt to avoid hitting an object in the road. Draw a basic motion diagram, using the images from the movie, from the time the skid begins until the car is stopped.
- A rocket is launched straight up. Draw a basic motion diagram, using the images from the movie, from the moment of liftoff until the rocket is at an altitude of 500 m.
- You're driving along the highway at 60 mph until you enter a town where the speed limit is 30 mph. You slow quickly, but not instantly, to 30 mph. Draw a basic motion diagram of your car, using images from the movie, from 30 s before reaching the city limit until 30 s afterward.

### Section 1.2 The Particle Model

- Write a paragraph describing the particle model. What is it, and why is it important?
  - Give two examples of situations, different from those described in the text, for which the particle model is appropriate.
  - Give an example of a situation, different from those described in the text, for which it would be inappropriate.

### Section 1.3 Position and Time

### Section 1.4 Velocity

- You drop a soccer ball from your third-story balcony. Use the particle model to draw a motion diagram showing the ball's position and average velocity vectors from the time you release the ball until the instant it touches the ground.

- A softball player hits the ball and starts running toward first base. Use the particle model to draw a motion diagram showing her position and her average velocity vectors during the first few seconds of her run.
- A softball player slides into second base. Use the particle model to draw a motion diagram showing his position and his average velocity vectors from the time he begins to slide until he reaches the base.

### Section 1.5 Linear Acceleration

- FIGURE EX1.8** shows the first three points of a motion diagram. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.
  - Use Tactics Box 1.3 to find the average acceleration vector at point 1. Draw the completed motion diagram, showing the velocity vectors and acceleration vector.



FIGURE EX1.8



FIGURE EX1.9

- FIGURE EX1.9** shows the first three points of a motion diagram. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.
  - Use Tactics Box 1.3 to find the average acceleration vector at point 1. Draw the completed motion diagram, showing the velocity vectors and acceleration vector.

10. **FIGURE EX1.10** shows two dots of a motion diagram and vector  $\vec{v}_1$ . Copy this figure and add vector  $\vec{v}_2$  and dot 3 if the acceleration vector  $\vec{a}$  at dot 2 (a) points up and (b) points down.

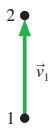


FIGURE EX1.10

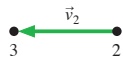


FIGURE EX1.11

11. **FIGURE EX1.11** shows two dots of a motion diagram and vector  $\vec{v}_2$ . Copy this figure and add vector  $\vec{v}_1$  and dot 1 if the acceleration vector  $\vec{a}$  at dot 2 (a) points to the right and (b) points to the left.
12. **I** A car travels to the left at a steady speed for a few seconds, then brakes for a stop sign. Draw a complete motion diagram of the car.
13. **I** A child is sledding on a smooth, level patch of snow. She encounters a rocky patch and slows to a stop. Draw a complete motion diagram of the child and her sled.
14. **I** You use a long rubber band to launch a paper wad straight up. Draw a complete motion diagram of the paper wad from the moment you release the stretched rubber band until the paper wad reaches its highest point.
15. **I** A roof tile falls straight down from a two-story building. It lands in a swimming pool and settles gently to the bottom. Draw a complete motion diagram of the tile.
16. **I** Your roommate drops a tennis ball from a third-story balcony. It hits the sidewalk and bounces as high as the second story. Draw a complete motion diagram of the tennis ball from the time it is released until it reaches the maximum height on its bounce. Be sure to determine and show the acceleration at the lowest point.
17. **I** A toy car rolls down a ramp, then across a smooth, horizontal floor. Draw a complete motion diagram of the toy car.

### Section 1.6 Motion in One Dimension

18. **FIGURE EX1.18** shows the motion diagram of a drag racer. The camera took one frame every 2 s.

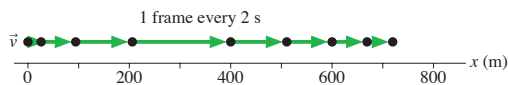


FIGURE EX1.18

- a. Measure the  $x$ -value of the racer at each dot. List your data in a table similar to Table 1.1, showing each position and the time at which it occurred.
- b. Make a position-versus-time graph for the drag racer. Because you have data only at certain instants, your graph should consist of dots that are not connected together.
19. **I** Write a short description of the motion of a real object for which **FIGURE EX1.19** would be a realistic position-versus-time graph.

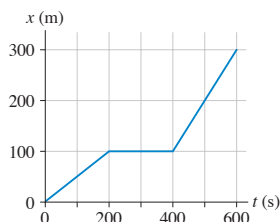


FIGURE EX1.19

20. **I** Write a short description of the motion of a real object for which **FIGURE EX1.20** would be a realistic position-versus-time graph.

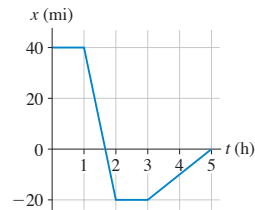


FIGURE EX1.20

### Section 1.7 Solving Problems in Physics

21. **I** Draw a pictorial representation for the following problem. Do *not* solve the problem. The light turns green, and a bicyclist starts forward with an acceleration of  $1.5 \text{ m/s}^2$ . How far must she travel to reach a speed of  $7.5 \text{ m/s}$ ?
22. **I** Draw a pictorial representation for the following problem. Do *not* solve the problem. What acceleration does a rocket need to reach a speed of  $200 \text{ m/s}$  at a height of  $1.0 \text{ km}$ ?

### Section 1.8 Units and Significant Figures

23. **I** Convert the following to SI units:  
 a.  $6.15 \text{ ms}$                       b.  $27.2 \text{ km}$   
 c.  $112 \text{ km/h}$                       d.  $72 \mu\text{m/ms}$
24. **I** Convert the following to SI units:  
 a.  $8.0 \text{ in}$                       b.  $66 \text{ ft/s}$   
 c.  $60 \text{ mph}$                       d.  $14 \text{ in}^2$
25. **I** Convert the following to SI units:  
 a.  $3 \text{ hours}$                       b.  $2 \text{ days}$   
 c.  $1 \text{ year}$                       d.  $215 \text{ ft/s}$
26. **I** Using the approximate conversion factors in Table 1.5, convert the following to SI units *without* using your calculator.  
 a.  $20 \text{ ft}$                       b.  $60 \text{ mi}$   
 c.  $60 \text{ mph}$                       d.  $8 \text{ in}$
27. **I** Using the approximate conversion factors in Table 1.5, convert the following SI units to English units *without* using your calculator.  
 a.  $30 \text{ cm}$                       b.  $25 \text{ m/s}$   
 c.  $5 \text{ km}$                       d.  $0.5 \text{ cm}$
28. **I** Compute the following numbers, applying the significant figure rule adopted in this textbook.  
 a.  $33.3 \times 25.4$                       b.  $33.3 - 25.4$   
 c.  $\sqrt{33.3}$                       d.  $333.3 \div 25.4$
29. **I** Compute the following numbers, applying the significant figure rule adopted in this textbook.  
 a.  $12.5^3$                       b.  $12.5 \times 5.21$   
 c.  $\sqrt{12.5} - 1.2$                       d.  $12.5^{-1}$
30. **I** Estimate (don't measure!) the length of a typical car. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.
31. **I** Estimate the height of a telephone pole. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.
32. **I** Estimate the average speed with which you go from home to campus via whatever mode of transportation you use most commonly. Give your answer in both mph and m/s. Briefly describe how you arrived at this estimate.

33. | Estimate the average speed with which the hair on your head grows. Give your answer in both m/s and  $\mu\text{m}/\text{hour}$ . Briefly describe how you arrived at this estimate.

### Problems

For Problems 34 through 43, draw a complete pictorial representation. Do *not* solve these problems or do any mathematics.

34. | A Porsche accelerates from a stoplight at  $5.0 \text{ m/s}^2$  for five seconds, then coasts for three more seconds. How far has it traveled?
35. | A jet plane is cruising at  $300 \text{ m/s}$  when suddenly the pilot turns the engines up to full throttle. After traveling  $4.0 \text{ km}$ , the jet is moving with a speed of  $400 \text{ m/s}$ . What is the jet's acceleration as it speeds up?
36. | Sam is recklessly driving  $60 \text{ mph}$  in a  $30 \text{ mph}$  speed zone when he suddenly sees the police. He steps on the brakes and slows to  $30 \text{ mph}$  in three seconds, looking nonchalant as he passes the officer. How far does he travel while braking?
37. | You would like to stick a wet spit wad on the ceiling, so you toss it straight up with a speed of  $10 \text{ m/s}$ . How long does it take to reach the ceiling,  $3.0 \text{ m}$  above?
38. | A speed skater moving across frictionless ice at  $8.0 \text{ m/s}$  hits a  $5.0\text{-m}$ -wide patch of rough ice. She slows steadily, then continues on at  $6.0 \text{ m/s}$ . What is her acceleration on the rough ice?
39. | Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of  $30^\circ$ . If Santa slides  $10 \text{ m}$  before reaching the edge, what is his speed as he leaves the roof?
40. | A motorist is traveling at  $20 \text{ m/s}$ . He is  $60 \text{ m}$  from a stoplight when he sees it turn yellow. His reaction time, before stepping on the brake, is  $0.50 \text{ s}$ . What steady deceleration while braking will bring him to a stop right at the light?
41. | A car traveling at  $30 \text{ m/s}$  runs out of gas while traveling up a  $10^\circ$  slope. How far up the hill will the car coast before starting to roll back down?
42. || Ice hockey star Bruce Blades is  $5.0 \text{ m}$  from the blue line and gliding toward it at a speed of  $4.0 \text{ m/s}$ . You are  $20 \text{ m}$  from the blue line, directly behind Bruce. You want to pass the puck to Bruce. With what speed should you shoot the puck down the ice so that it reaches Bruce exactly as he crosses the blue line?
43. || David is driving a steady  $30 \text{ m/s}$  when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady  $2.0 \text{ m/s}^2$  at the instant when David passes. How far does Tina drive before passing David?

Problems 44 through 48 show a motion diagram. For each of these problems, write a one or two sentence "story" about a *real object* that has this motion diagram. Your stories should talk about people or objects by name and say what they are doing. Problems 34 through 43 are examples of motion short stories.

44. |

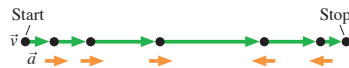


FIGURE P1.44

45. |

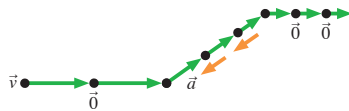


FIGURE P1.45

46. |

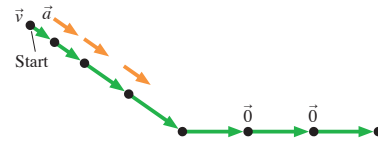


FIGURE P1.46

47. |

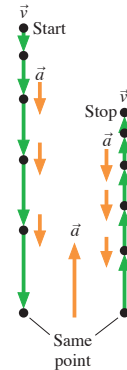


FIGURE P1.47

48. |

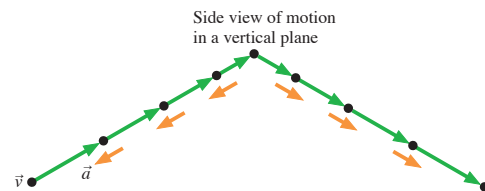


FIGURE P1.48

Problems 49 through 52 show a partial motion diagram. For each:

- Complete the motion diagram by adding acceleration vectors.
- Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.
- Draw a pictorial representation for your problem.

49. |



FIGURE P1.49

50. |



FIGURE P1.50

51. |

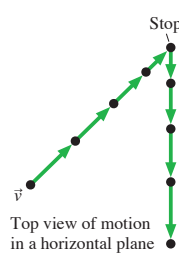


FIGURE P1.51

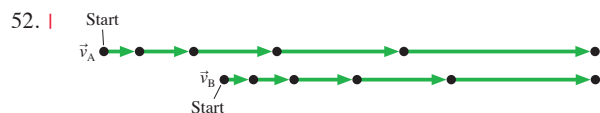


FIGURE P1.52

53. | A regulation soccer field for international play is a rectangle with a length between 100 m and 110 m and a width between 64 m and 75 m. What are the smallest and largest areas that the field could be?
54. | The quantity called *mass density* is the mass per unit volume of a substance. Express the following mass densities in SI units.
- Aluminum,  $2.7 \times 10^{-3} \text{ kg/cm}^3$
  - Alcohol,  $0.81 \text{ g/cm}^3$
55. | FIGURE P1.55 shows a motion diagram of a car traveling down a street. The camera took one frame every 10 s. A distance scale is provided.

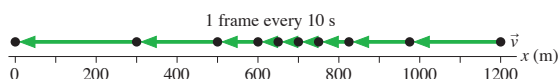


FIGURE P1.55

- Measure the  $x$ -value of the car at each dot. Place your data in a table, similar to Table 1.1, showing each position and the instant of time at which it occurred.
  - Make a position-versus-time graph for the car. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.
56. | Write a short description of a real object for which FIGURE P1.56 would be a realistic position-versus-time graph.

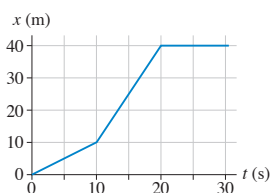


FIGURE P1.56

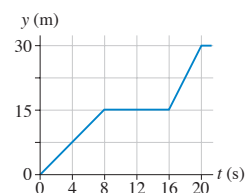


FIGURE P1.57

57. | Write a short description of a real object for which FIGURE P1.57 would be a realistic position-versus-time graph.

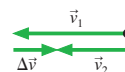
## STOP TO THINK ANSWERS

**Stop to Think 1.1: B.** The images of B are farther apart, so it travels a larger distance than does A during the same intervals of time.

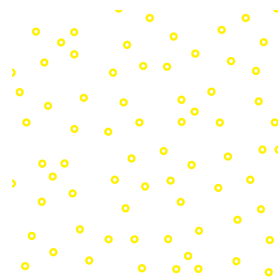
**Stop to Think 1.2: a.** Dropped ball. **b.** Dust particle. **c.** Descending rocket.

**Stop to Think 1.3: e.** The average velocity vector is found by connecting one dot in the motion diagram to the next.

**Stop to Think 1.4: b.**  $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$ , and  $\Delta\vec{v}$  points in the direction of  $\vec{a}$ .



**Stop to Think 1.5: d > c > b = a.**



# CONCEPTUAL QUESTIONS

For Questions 1 through 3, interpret the position graph given in each figure by writing a very short “story” of what is happening. Be creative! Have characters and situations! Simply saying that “a car moves 100 meters to the right” doesn’t qualify as a story. Your stories should make *specific reference* to information you obtain from the graph, such as distance moved or time elapsed.

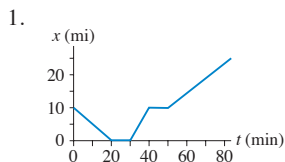


FIGURE Q2.1

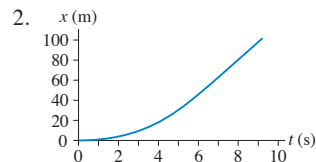


FIGURE Q2.2

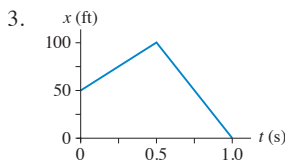


FIGURE Q2.3

4. FIGURE Q2.4 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
- At the instant  $t = 1$  s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
  - Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

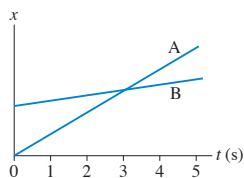


FIGURE Q2.4

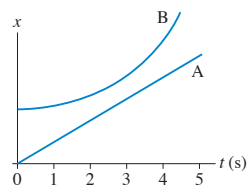


FIGURE Q2.5

5. FIGURE Q2.5 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
- At the instant  $t = 1$  s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
  - Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.
6. FIGURE Q2.6 shows the position-versus-time graph for a moving object. At which lettered point or points:
- Is the object *moving* the slowest?
  - Is the object moving the fastest?
  - Is the object at rest?
  - Is the object moving to the left?

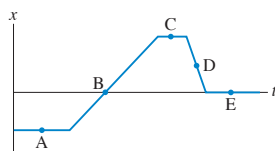


FIGURE Q2.6

7. FIGURE Q2.7 shows the position-versus-time graph for a moving object. At which lettered point or points:
- Is the object moving the fastest?
  - Is the object moving to the left?
  - Is the object speeding up?
  - Is the object turning around?

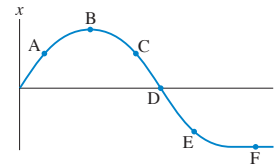


FIGURE Q2.7

8. FIGURE Q2.8 shows six frames from the motion diagrams of two moving cars, A and B.
- Do the two cars ever have the same position at one instant of time? If so, in which frame number (or numbers)?
  - Do the two cars ever have the same velocity at one instant of time? If so, between which two frames?

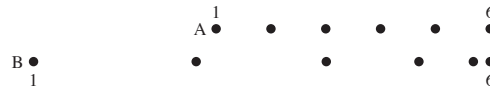


FIGURE Q2.8

9. You’re driving along the highway at a steady speed of 60 mph when another driver decides to pass you. At the moment when the front of his car is exactly even with the front of your car, and you turn your head to smile at him, do the two cars have equal velocities? Explain.
10. A bicycle is traveling east. Can its acceleration vector ever point west? Explain.
11. (a) Give an example of a vertical motion with a positive velocity and a negative acceleration. (b) Give an example of a vertical motion with a negative velocity and a negative acceleration.
12. A ball is thrown straight up into the air. At each of the following instants, is the magnitude of the ball’s acceleration greater than  $g$ , equal to  $g$ , less than  $g$ , or 0? Explain.
- Just after leaving your hand.
  - At the very top (maximum height).
  - Just before hitting the ground.
13. A rock is *thrown* (not dropped) straight down from a bridge into the river below. At each of the following instants, is the magnitude of the rock’s acceleration greater than  $g$ , equal to  $g$ , less than  $g$ , or 0? Explain.
- Immediately after being released.
  - Just before hitting the water.
14. A rubber ball dropped from a height of 2 m bounces back to a height of 1 m. Draw the ball’s position, velocity, and acceleration graphs, stacked vertically, from the instant you release it until it returns to its maximum bounce height. Pay close attention to the time the ball is in contact with the ground; this is a short interval of time, but it’s not zero.

# EXERCISES AND PROBLEMS

## Exercises

### Section 2.1 Uniform Motion

- Alan leaves Los Angeles at 8:00 a.m. to drive to San Francisco, 400 mi away. He travels at a steady 50 mph. Beth leaves Los Angeles at 9:00 a.m. and drives a steady 60 mph.
  - Who gets to San Francisco first?
  - How long does the first to arrive have to wait for the second?
- Larry leaves home at 9:05 and runs at constant speed to the lamppost seen in **FIGURE EX2.2**. He reaches the lamppost at 9:07, immediately turns, and runs to the tree at 9:10.
  - What is Larry's average velocity, in m/min, during each of these two intervals.
  - What is Larry's average velocity for the entire run?



FIGURE EX2.2

- Julie drives 100 mi to Grandmother's house. On the way to Grandmother's, Julie drives half the distance at 40 mph and half the distance at 60 mph. On her return trip, she drives half the time at 40 mph and half the time at 60 mph.
  - What is Julie's average speed on the way to Grandmother's house?
  - What is her average speed on the return trip?
- FIGURE EX2.4** is the position-versus-time graph of a jogger. What is the jogger's velocity at  $t = 10$  s, at  $t = 25$  s, and at  $t = 35$  s?

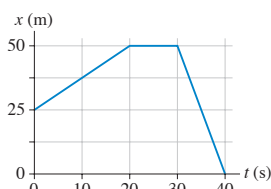


FIGURE EX2.4

### Section 2.2 Instantaneous Velocity

### Section 2.3 Finding Position from Velocity

- FIGURE EX2.5** shows the position graph of a particle.
  - Draw the particle's velocity graph for the interval  $0 \text{ s} \leq t \leq 4 \text{ s}$ .
  - Does this particle have a turning point or points? If so, at what time or times?

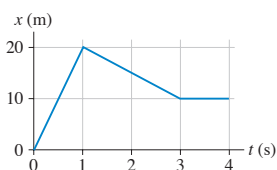


FIGURE EX2.5

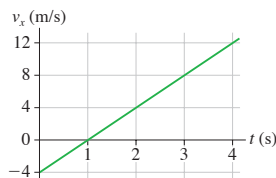


FIGURE EX2.6

- A particle starts from  $x_0 = 10 \text{ m}$  at  $t_0 = 0 \text{ s}$  and moves with the velocity graph shown in **FIGURE EX2.6**.
  - Does this particle have a turning point? If so, at what time?
  - What is the object's position at  $t = 2 \text{ s}$ ,  $3 \text{ s}$ , and  $4 \text{ s}$ ?
- FIGURE EX2.7** is a somewhat idealized graph of the velocity of blood in the ascending aorta during one beat of the heart.
 

BIO Approximately how far, in cm, does the blood move during one beat?

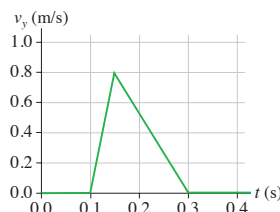


FIGURE EX2.7

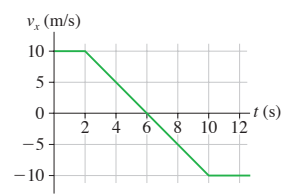


FIGURE EX2.8

- FIGURE EX2.8** shows the velocity graph for a particle having initial position  $x_0 = 0 \text{ m}$  at  $t_0 = 0 \text{ s}$ .
  - At what time or times is the particle found at  $x = 35 \text{ m}$ ? Work with the geometry of the graph, not with kinematic equations.
  - Draw a motion diagram for the particle.

### Section 2.4 Motion with Constant Acceleration

- FIGURE EX2.9** shows the velocity graph of a particle. Draw the particle's acceleration graph for the interval  $0 \text{ s} \leq t \leq 4 \text{ s}$ . Give both axes an appropriate numerical scale.

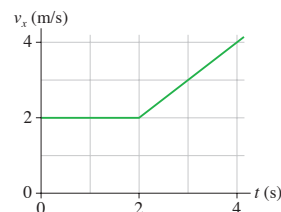


FIGURE EX2.9

- FIGURE EX2.7** showed the velocity graph of blood in the aorta.
 

BIO Estimate the blood's acceleration during each phase of the motion, speeding up and slowing down.
- FIGURE EX2.11** shows the velocity graph of a particle moving along the  $x$ -axis. Its initial position is  $x_0 = 2.0 \text{ m}$  at  $t_0 = 0 \text{ s}$ . At  $t = 2.0 \text{ s}$ , what are the particle's (a) position, (b) velocity, and (c) acceleration?

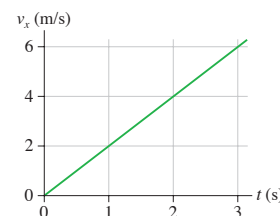


FIGURE EX2.11

12. **|** **FIGURE EX2.12** shows the velocity-versus-time graph for a particle moving along the  $x$ -axis. Its initial position is  $x_0 = 2.0$  m at  $t_0 = 0$  s.
- What are the particle's position, velocity, and acceleration at  $t = 1.0$  s?
  - What are the particle's position, velocity, and acceleration at  $t = 3.0$  s?



FIGURE EX2.12

13. **|** A jet plane is cruising at 300 m/s when suddenly the pilot turns the engines up to full throttle. After traveling 4.0 km, the jet is moving with a speed of 400 m/s. What is the jet's acceleration, assuming it to be a constant acceleration?
14. **|** When you sneeze, the air in your lungs accelerates from rest to 150 km/h in approximately 0.50 s. What is the acceleration of the air in  $\text{m/s}^2$ ?
15. **|** A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?
16. **|** A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of  $3.5 \text{ m/s}^2$  is larger than the Honda's  $3.0 \text{ m/s}^2$ , the Honda gets a 1.0 s head start. Who wins?

### Section 2.5 Free Fall

17. **|** Ball bearings are made by letting spherical drops of molten metal fall inside a tall tower—called a *shot tower*—and solidify as they fall.
- If a bearing needs 4.0 s to solidify enough for impact, how high must the tower be?
  - What is the bearing's impact velocity?
18. **|** A ball is thrown vertically upward with a speed of 19.6 m/s.
- What is the ball's velocity and its height after 1.0, 2.0, 3.0, and 4.0 s?
  - Draw the ball's velocity-versus-time graph. Give both axes an appropriate numerical scale.
19. **|** A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)
20. **|** A rock is tossed straight up with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.
- What is the rock's velocity as it hits the bottom of the hole?
  - How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?

### Section 2.6 Motion on an Inclined Plane

21. **|** A skier is gliding along at 3.0 m/s on horizontal, frictionless snow. He suddenly starts down a  $10^\circ$  incline. His speed at the bottom is 15 m/s.
- What is the length of the incline?
  - How long does it take him to reach the bottom?
22. **|** A car traveling at 30 m/s runs out of gas while traveling up a  $10^\circ$  slope. How far up the hill will it coast before starting to roll back down?

### Section 2.7 Instantaneous Acceleration

23. **|** A particle moving along the  $x$ -axis has its position described by the function  $x = (2t^2 - t + 1)$  m, where  $t$  is in s. At  $t = 2$  s what are the particle's (a) position, (b) velocity, and (c) acceleration?
24. **|** A particle moving along the  $x$ -axis has its velocity described by the function  $v_x = 2t^2$  m/s, where  $t$  is in s. Its initial position is  $x_0 = 1$  m at  $t_0 = 0$  s. At  $t = 1$  s what are the particle's (a) position, (b) velocity, and (c) acceleration?
25. **|** **FIGURE EX2.25** shows the acceleration-versus-time graph of a particle moving along the  $x$ -axis. Its initial velocity is  $v_{0x} = 8.0$  m/s at  $t_0 = 0$  s. What is the particle's velocity at  $t = 4.0$  s?

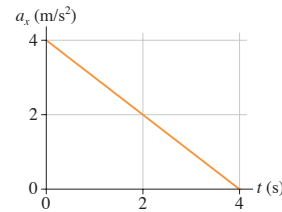


FIGURE EX2.25

### Problems

26. **|** A particle's position on the  $x$ -axis is given by the function  $x = (t^2 - 4t + 2)$  m, where  $t$  is in s.
- Make a position-versus-time graph for the interval  $0 \text{ s} \leq t \leq 5 \text{ s}$ . Do this by calculating and plotting  $x$  every 0.5 s from 0 s to 5 s, then drawing a smooth curve through the points.
  - Determine the particle's velocity at  $t = 1.0$  s by drawing the tangent line on your graph and measuring its slope.
  - Determine the particle's velocity at  $t = 1.0$  s by evaluating the derivative at that instant. Compare this to your result from part b.
  - Are there any turning points in the particle's motion? If so, at what position or positions?
  - Where is the particle when  $v_x = 4.0$  m/s?
  - Draw a motion diagram for the particle.
27. **|** Three particles move along the  $x$ -axis, each starting with  $v_{0x} = 10$  m/s at  $t_0 = 0$  s. In **FIGURE P2.27**, the graph for A is a position-versus-time graph; the graph for B is a velocity-versus-time graph; the graph for C is an acceleration-versus-time graph. Find each particle's velocity at  $t = 7.0$  s. Work with the geometry of the graphs, not with kinematic equations.

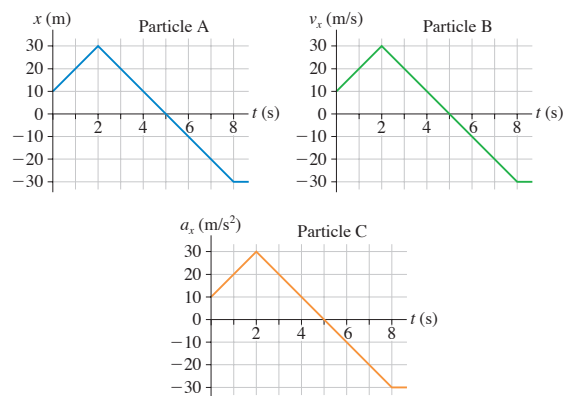


FIGURE P2.27



28. **FIGURE P2.28** shows the acceleration graph for a particle that starts from rest at  $t = 0$  s. Determine the object's velocity at times  $t = 0$  s, 2 s, 4 s, 6 s, and 8 s.

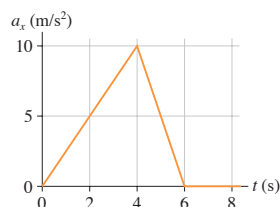


FIGURE P2.28

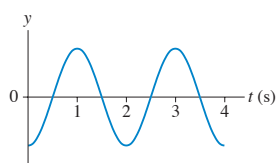


FIGURE P2.29

29. **I** A block is suspended from a spring, pulled down, and released. The block's position-versus-time graph is shown in **FIGURE P2.29**.
- At what times is the velocity zero? At what times is the velocity most positive? Most negative?
  - Draw a reasonable velocity-versus-time graph.
30. **II** A particle's velocity is described by the function  $v_x = t^2 - 7t + 10$  m/s, where  $t$  is in s.
- At what times does the particle reach its turning points?
  - What is the particle's acceleration at each of the turning points?
31. **I** The position of a particle is given by the function  $x = (2t^3 - 9t^2 + 12)$  m, where  $t$  is in s.
- At what time or times is  $v_x = 0$  m/s?
  - What are the particle's position and its acceleration at this time(s)?
32. **I** An object starts from rest at  $x = 0$  m at time  $t = 0$  s. Five seconds later, at  $t = 5.0$  s, the object is observed to be at  $x = 40$  m and to have velocity  $v_x = 11$  m/s.
- Was the object's acceleration uniform or nonuniform? Explain your reasoning.
  - Sketch the velocity-versus-time graph implied by these data. Is the graph a straight line or curved? If curved, is it concave upward or downward?
33. **III** A particle's velocity is described by the function  $v_x = kt^2$  m/s, where  $k$  is a constant and  $t$  is in s. The particle's position at  $t_0 = 0$  s is  $x_0 = -9.0$  m. At  $t_1 = 3.0$  s, the particle is at  $x_1 = 9.0$  m. Determine the value of the constant  $k$ . Be sure to include the proper units.
34. **II** A particle's acceleration is described by the function  $a_x = (10 - t)$  m/s<sup>2</sup>, where  $t$  is in s. Its initial conditions are  $x_0 = 0$  m and  $v_{0x} = 0$  m/s at  $t = 0$  s.
- At what time is the velocity again zero?
  - What is the particle's position at that time?

35. **II** A ball rolls along the frictionless track shown in **FIGURE P2.35**. Each segment of the track is straight, and the ball passes smoothly from one segment to the next without changing speed or leaving the track. Draw three vertically stacked graphs showing position, velocity, and acceleration versus time. Each graph should have the same time axis, and the proportions of the graph should be qualitatively correct. Assume that the ball has enough speed to reach the top.

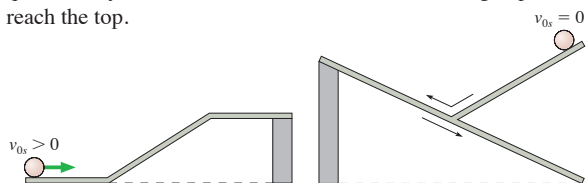


FIGURE P2.35

FIGURE P2.36

36. **II** Draw position, velocity, and acceleration graphs for the ball shown in **FIGURE P2.36**. See Problem 35 for more information.
37. **II** Draw position, velocity, and acceleration graphs for the ball shown in **FIGURE P2.37**. See Problem 35 for more information. The ball changes direction but not speed as it bounces from the reflecting wall.

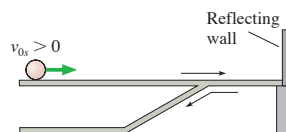


FIGURE P2.37

38. **II** **FIGURE P2.38** shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

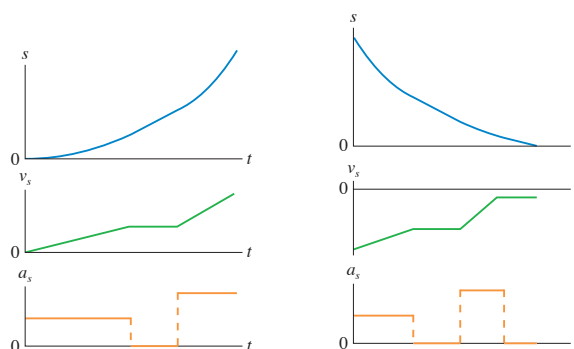


FIGURE P2.38

FIGURE P2.39

39. **II** **FIGURE P2.39** shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.
40. **I** The takeoff speed for an Airbus A320 jetliner is 80 m/s. Velocity data measured during takeoff are as shown.
- | $t$ (s) | $v_x$ (m/s) |
|---------|-------------|
| 0       | 0           |
| 10      | 23          |
| 20      | 46          |
| 30      | 69          |
- What is the takeoff speed in miles per hour?
  - Is the jetliner's acceleration constant during takeoff? Explain.
  - At what time do the wheels leave the ground?
  - For safety reasons, in case of an aborted takeoff, the runway must be three times the takeoff distance. Can an A320 take off safely on a 2.5-mi-long runway?
41. **I**
- What constant acceleration, in SI units, must a car have to go from zero to 60 mph in 10 s?
  - What fraction of  $g$  is this?
  - How far has the car traveled when it reaches 60 mph? Give your answer both in SI units and in feet.
42. **II**
- How many days will it take a spaceship to accelerate to the speed of light ( $3.0 \times 10^8$  m/s) with the acceleration  $g$ ?
  - How far will it travel during this interval?
  - What fraction of a light year is your answer to part b? A light year is the distance light travels in one year.

**NOTE**  $\blacktriangleright$  We know, from Einstein's theory of relativity, that no object can travel at the speed of light. So this problem, while interesting and instructive, is not realistic.  $\blacktriangleleft$



43. I You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.50 s and that your car brakes with constant acceleration.
- How far are you from the intersection when you begin to apply the brakes?
  - What acceleration will bring you to rest right at the intersection?
  - How long does it take you to stop after the light turns red?
44. II a. Suppose you are driving at speed  $v_0$  when a sudden obstacle in the road forces you to make a quick stop. If your reaction time before applying the brakes is  $t_R$ , what constant deceleration (absolute value of  $a_x$ ) do you need to stop in distance  $d$ ? Assume that  $d$  is larger than the car travels during your reaction time.
- b. Suppose you are driving at 21 m/s when you suddenly see an obstacle 50 m ahead. If your reaction time is 0.50 s and if your car's maximum deceleration is  $6.0 \text{ m/s}^2$ , can you stop in time to avoid a collision?
45. II You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is  $10 \text{ m/s}^2$ .
- How much distance is between you and the deer when you come to a stop?
  - What is the maximum speed you could have and still not hit the deer?
46. III The minimum stopping distance for a car traveling at a speed of 30 m/s is 60 m, including the distance traveled during the driver's reaction time of 0.50 s.
- What is the minimum stopping distance for the same car traveling at a speed of 40 m/s?
  - Draw a position-versus-time graph for the motion of the car in part a. Assume the car is at  $x_0 = 0 \text{ m}$  when the driver first sees the emergency situation ahead that calls for a rapid halt.
47. II When jumping, a flea accelerates at an astounding  $1000 \text{ m/s}^2$ , but over only the very short distance of 0.50 mm. If a flea jumps straight up, and if air resistance is neglected (a rather poor approximation in this situation), how high does the flea go?
48. II BIO A cheetah spots a Thomson's gazelle, its preferred prey, and leaps into action, quickly accelerating to its top speed of 30 m/s, the highest of any land animal. However, a cheetah can maintain this extreme speed for only 15 s before having to let up. The cheetah is 170 m from the gazelle as it reaches top speed, and the gazelle sees the cheetah at just this instant. With negligible reaction time, the gazelle heads directly away from the cheetah, accelerating at  $4.6 \text{ m/s}^2$  for 5.0 s, then running at constant speed. Does the gazelle escape?
49. III A 200 kg weather rocket is loaded with 100 kg of fuel and fired straight up. It accelerates upward at  $30 \text{ m/s}^2$  for 30 s, then runs out of fuel. Ignore any air resistance effects.
- What is the rocket's maximum altitude?
  - How long is the rocket in the air before hitting the ground?
  - Draw a velocity-versus-time graph for the rocket from liftoff until it hits the ground.
50. II A 1000 kg weather rocket is launched straight up. The rocket motor provides a constant acceleration for 16 s, then the motor stops. The rocket altitude 20 s after launch is 5100 m. You can ignore any effects of air resistance.
- What was the rocket's acceleration during the first 16 s?
  - What is the rocket's speed as it passes through a cloud 5100 m above the ground?
51. III A lead ball is dropped into a lake from a diving board 5.0 m above the water. After entering the water, it sinks to the bottom with a constant velocity equal to the velocity with which it hit the water. The ball reaches the bottom 3.0 s after it is released. How deep is the lake?
52. II A hotel elevator ascends 200 m with a maximum speed of 5.0 m/s. Its acceleration and deceleration both have a magnitude of  $1.0 \text{ m/s}^2$ .
- How far does the elevator move while accelerating to full speed from rest?
  - How long does it take to make the complete trip from bottom to top?
53. II A car starts from rest at a stop sign. It accelerates at  $4.0 \text{ m/s}^2$  for 6.0 s, coasts for 2.0 s, and then slows down at a rate of  $3.0 \text{ m/s}^2$  for the next stop sign. How far apart are the stop signs?
54. II A car accelerates at  $2.0 \text{ m/s}^2$  along a straight road. It passes two marks that are 30 m apart at times  $t = 4.0 \text{ s}$  and  $t = 5.0 \text{ s}$ . What was the car's velocity at  $t = 0 \text{ s}$ ?
55. II Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of  $30^\circ$ . If Santa slides 10 m before reaching the edge, what is his speed as he leaves the roof?
56. II Ann and Carol are driving their cars along the same straight road. Carol is located at  $x = 2.4 \text{ mi}$  at  $t = 0 \text{ h}$  and drives at a steady 36 mph. Ann, who is traveling in the same direction, is located at  $x = 0.0 \text{ mi}$  at  $t = 0.50 \text{ h}$  and drives at a steady 50 mph.
- At what time does Ann overtake Carol?
  - What is their position at this instant?
  - Draw a position-versus-time graph showing the motion of both Ann and Carol.
57. II a. A very slippery block of ice slides down a smooth ramp tilted at angle  $\theta$ . The ice is released from rest at vertical height  $h$  above the bottom of the ramp. Find an expression for the speed of the ice at the bottom.
- b. Evaluate your answer to part a for ice released at a height of 30 cm on ramps tilted at  $20^\circ$  and  $40^\circ$ .
58. II A toy train is pushed forward and released at  $x_0 = 2.0 \text{ m}$  with a speed of 2.0 m/s. It rolls at a steady speed for 2.0 s, then one wheel begins to stick. The train comes to a stop 6.0 m from the point at which it was released. What is the magnitude of the train's acceleration after its wheel begins to stick?
59. II Bob is driving the getaway car after the big bank robbery. He's going 50 m/s when his headlights suddenly reveal a nail strip that the cops have placed across the road 150 m in front of him. If Bob can stop in time, he can throw the car into reverse and escape. But if he crosses the nail strip, all his tires will go flat and he will be caught. Bob's reaction time before he can hit the brakes is 0.60 s, and his car's maximum deceleration is  $10 \text{ m/s}^2$ . Is Bob in jail?
60. II One game at the amusement park has you push a puck up a long, frictionless ramp. You win a stuffed animal if the puck, at its highest point, comes to within 10 cm of the end of the ramp without going off. You give the puck a push, releasing it with a speed of 5.0 m/s when it is 8.5 m from the end of the ramp. The puck's speed after traveling 3.0 m is 4.0 m/s. Are you a winner?
61. II a. Your goal in laboratory is to launch a ball of mass  $m$  straight up so that it reaches exactly height  $h$  above the top of the launching tube. You and your lab partners will earn fewer points if the ball goes too high or too low. The launch tube uses compressed air to accelerate the ball over a distance  $d$ , and you have a table of data telling you how to set the

air compressor to achieve a desired acceleration. Find an expression for the acceleration that will earn you maximum points.

- b. Evaluate your answer to part a to achieve a height of 3.2 m using a 45-cm-long launch tube.
62. || Nicole throws a ball straight up. Chad watches the ball from a window 5.0 m above the point where Nicole released it. The ball passes Chad on the way up, and it has a speed of 10 m/s as it passes him on the way back down. How fast did Nicole throw the ball?
63. || A motorist is driving at 20 m/s when she sees that a traffic light 200 m ahead has just turned red. She knows that this light stays red for 15 s, and she wants to reach the light just as it turns green again. It takes her 1.0 s to step on the brakes and begin slowing. What is her speed as she reaches the light at the instant it turns green?
64. || When a 1984 Alfa Romeo Spider sports car accelerates at the maximum possible rate, its motion during the first 20 s is extremely well modeled by the simple equation

$$v_x^2 = \frac{2P}{m}t$$

where  $P = 3.6 \times 10^4$  watts is the car's power output,  $m = 1200$  kg is its mass, and  $v_x$  is in m/s. That is, the square of the car's velocity increases linearly with time.

- a. What is the car's speed at  $t = 10$  s and at  $t = 20$  s?
- b. Find an algebraic expression in terms of  $P$ ,  $m$ , and  $t$ , for the car's acceleration at time  $t$ .
- c. Evaluate the acceleration at  $t = 1$  s and  $t = 10$  s.
- d. This simple model fails for  $t$  less than about 0.5 s. Explain how you can recognize the failure.
65. || David is driving a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady  $2.0 \text{ m/s}^2$  at the instant when David passes.
- a. How far does Tina drive before passing David?
- b. What is her speed as she passes him?
66. || A cat is sleeping on the floor in the middle of a 3.0-m-wide room when a barking dog enters with a speed of 1.50 m/s. As the dog enters, the cat (as only cats can do) immediately accelerates at  $0.85 \text{ m/s}^2$  toward an open window on the opposite side of the room. The dog (all bark and no bite) is a bit startled by the cat and begins to slow down at  $0.10 \text{ m/s}^2$  as soon as it enters the room. Does the dog catch the cat before the cat is able to leap through the window?
67. || Jill has just gotten out of her car in the grocery store parking lot. The parking lot is on a hill and is tilted  $3^\circ$ . Twenty meters downhill from Jill, a little old lady lets go of a fully loaded shopping cart. The cart, with frictionless wheels, starts to roll straight downhill. Jill immediately starts to sprint after the cart with her top acceleration of  $2.0 \text{ m/s}^2$ . How far has the cart rolled before Jill catches it?
68. || As a science project, you drop a watermelon off the top of the Empire State Building, 320 m above the sidewalk. It so happens that Superman flies by at the instant you release the watermelon. Superman is headed straight down with a speed of 35 m/s. How fast is the watermelon going when it passes Superman?
69. || I was driving along at 20 m/s, trying to change a CD and not watching where I was going. When I looked up, I found myself 45 m from a railroad crossing. And wouldn't you know it, a train moving at 30 m/s was only 60 m from the crossing. In

a split second, I realized that the train was going to beat me to the crossing and that I didn't have enough distance to stop. My only hope was to accelerate enough to cross the tracks before the train arrived. If my reaction time before starting to accelerate was 0.50 s, what minimum acceleration did my car need for me to be here today writing these words?

70. || As an astronaut visiting Planet X, you're assigned to measure the free-fall acceleration. Getting out your meter stick and stop watch, you time the fall of a heavy ball from several heights. Your data are as follows:

Height (m)	Fall time (s)
0.0	0.00
1.0	0.54
2.0	0.72
3.0	0.91
4.0	1.01
5.0	1.17

Analyze these data to determine the free-fall acceleration on Planet X. Your analysis method should involve fitting a straight line to an appropriate graph, similar to the analysis in Example 2.15.

71. || Your engineering firm has been asked to determine the deceleration of a car during hard braking. To do so, you decide to measure the lengths of the skid marks when stopping from various initial speeds. Your data are as follows:

Speed (m/s)	Skid length (m)
10	7
15	14
20	27
25	37
30	58

- a. Do the data support an assertion that the deceleration is constant, independent of speed? Explain.
- b. Determine an experimental value for the car's deceleration—that is, the absolute value of the acceleration. Your analysis method should involve fitting a straight line to an appropriate graph, similar to the analysis in Example 2.15.

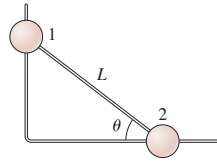
In Problems 72 through 75, you are given the kinematic equation or equations that are used to solve a problem. For each of these, you are to:

- a. Write a *realistic* problem for which this is the correct equation(s). Be sure that the answer your problem requests is consistent with the equation(s) given.
- b. Draw the pictorial representation for your problem.
- c. Finish the solution of the problem.
72.  $64 \text{ m} = 0 \text{ m} + (32 \text{ m/s})(4 \text{ s} - 0 \text{ s}) + \frac{1}{2}a_x(4 \text{ s} - 0 \text{ s})^2$
73.  $(10 \text{ m/s})^2 = v_{0y}^2 - 2(9.8 \text{ m/s}^2)(10 \text{ m} - 0 \text{ m})$
74.  $(0 \text{ m/s})^2 = (5 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(\sin 10^\circ)(x_1 - 0 \text{ m})$
75.  $v_{1x} = 0 \text{ m/s} + (20 \text{ m/s}^2)(5 \text{ s} - 0 \text{ s})$   
 $x_1 = 0 \text{ m} + (0 \text{ m/s})(5 \text{ s} - 0 \text{ s}) + \frac{1}{2}(20 \text{ m/s}^2)(5 \text{ s} - 0 \text{ s})^2$   
 $x_2 = x_1 + v_{1x}(10 \text{ s} - 5 \text{ s})$

## Challenge Problems

76. The two masses in **FIGURE CP2.76** slide on frictionless wires. They are connected by a pivoting rigid rod of length  $L$ . Prove that  $v_{2x} = -v_{1y} \tan \theta$ .

FIGURE CP2.76



77. A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration?
78. Your school science club has devised a special event for homecoming. You've attached a rocket to the rear of a small car that has been decorated in the blue-and-gold school colors. The rocket provides a constant acceleration for 9.0 s. As the rocket shuts off, a parachute opens and slows the car at a rate of  $5.0 \text{ m/s}^2$ . The car passes the judges' box in the center of the grandstand, 990 m from the starting line, exactly 12 s after you fire the rocket. What is the car's speed as it passes the judges?
79. Careful measurements have been made of Olympic sprinters in the 100-meter dash. A simple but reasonably accurate model is that a sprinter accelerates at  $3.6 \text{ m/s}^2$  for  $3\frac{1}{3}$  s, then runs at constant velocity to the finish line.
- What is the race time for a sprinter who follows this model?
  - A sprinter could run a faster race by accelerating faster at the beginning, thus reaching top speed sooner. If a sprinter's top speed is the same as in part a, what acceleration would he need to run the 100-meter dash in 9.9 s?
  - By what percent did the sprinter need to increase his acceleration in order to decrease his time by 1%?
80. Careful measurements have been made of Olympic sprinters in the 100-meter dash. A quite realistic model is that the sprinter's velocity is given by

$$v_x = a(1 - e^{-bt})$$

where  $t$  is in s,  $v_x$  is in m/s, and the constants  $a$  and  $b$  are characteristic of the sprinter. Sprinter Carl Lewis's run at the 1987 World Championships is modeled with  $a = 11.81 \text{ m/s}$  and  $b = 0.6887 \text{ s}^{-1}$ .

- What was Lewis's acceleration at  $t = 0 \text{ s}$ ,  $2.00 \text{ s}$ , and  $4.00 \text{ s}$ ?
  - Find an expression for the distance traveled at time  $t$ .
  - Your expression from part b is a transcendental equation, meaning that you can't solve it for  $t$ . However, it's not hard to use trial and error to find the time needed to travel a specific distance. To the nearest 0.01 s, find the time Lewis needed to sprint 100.0 m. His official time was 0.01 s more than your answer, showing that this model is very good, but not perfect.
81. A sprinter can accelerate with constant acceleration for 4.0 s before reaching top speed. He can run the 100-meter dash in 10.0 s. What is his speed as he crosses the finish line?
82. A rubber ball is shot straight up from the ground with speed  $v_0$ . Simultaneously, a second rubber ball at height  $h$  directly above the first ball is dropped from rest.
- At what height above the ground do the balls collide? Your answer will be an *algebraic expression* in terms of  $h$ ,  $v_0$ , and  $g$ .
  - What is the maximum value of  $h$  for which a collision occurs before the first ball falls back to the ground?
  - For what value of  $h$  does the collision occur at the instant when the first ball is at its highest point?
83. The Starship Enterprise returns from warp drive to ordinary space with a forward speed of 50 km/s. To the crew's great surprise, a Klingon ship is 100 km directly ahead, traveling in the same direction at a mere 20 km/s. Without evasive action, the Enterprise will overtake and collide with the Klingons in just slightly over 3.0 s. The Enterprise's computers react instantly to brake the ship. What magnitude acceleration does the Enterprise need to just barely avoid a collision with the Klingon ship? Assume the acceleration is constant.

**Hint:** Draw a position-versus-time graph showing the motions of both the Enterprise and the Klingon ship. Let  $x_0 = 0 \text{ km}$  be the location of the Enterprise as it returns from warp drive. How do you show graphically the situation in which the collision is "barely avoided"? Once you decide what it looks like graphically, express that situation mathematically.

## STOP TO THINK ANSWERS

**Stop to Think 2.1: d.** The particle starts with positive  $x$  and moves to negative  $x$ .

**Stop to Think 2.2: c.** The velocity is the slope of the position graph. The slope is positive and constant until the position graph crosses the axis, then positive but decreasing, and finally zero when the position graph is horizontal.

**Stop to Think 2.3: b.** A constant positive  $v_x$  corresponds to a linearly increasing  $x$ , starting from  $x_i = -10 \text{ m}$ . The constant negative  $v_x$  then corresponds to a linearly decreasing  $x$ .

**Stop to Think 2.4: a and b.** The velocity is constant while  $a = 0$ , it decreases linearly while  $a$  is negative. Graphs a, b, and c all have the same acceleration, but only graphs a and b have a positive initial velocity that represents a particle moving to the right.

**Stop to Think 2.5: d.** The acceleration vector points downhill (negative  $s$ -direction) and has the constant value  $-g \sin \theta$  throughout the motion.

**Stop to Think 2.6: c.** Acceleration is the slope of the graph. The slope is zero at B. Although the graph is steepest at A, the slope at that point is negative, and so  $a_A < a_B$ . Only C has a positive slope, so  $a_C > a_B$ .

# CONCEPTUAL QUESTIONS

- Can the magnitude of the displacement vector be more than the distance traveled? Less than the distance traveled? Explain.
- If  $\vec{C} = \vec{A} + \vec{B}$ , can  $C = A + B$ ? Can  $C > A + B$ ? For each, show how or explain why not.
- If  $\vec{C} = \vec{A} + \vec{B}$ , can  $C = 0$ ? Can  $C < 0$ ? For each, show how or explain why not.
- Is it possible to add a scalar to a vector? If so, demonstrate. If not, explain why not.
- How would you define the *zero vector*  $\vec{0}$ ?
- Can a vector have a component equal to zero and still have non-zero magnitude? Explain.
- Can a vector have zero magnitude if one of its components is nonzero? Explain.
- Suppose two vectors have unequal magnitudes. Can their sum be zero? Explain.
- Are the following statements true or false? Explain your answer.
  - The magnitude of a vector can be different in different coordinate systems.
  - The direction of a vector can be different in different coordinate systems.
  - The components of a vector can be different in different coordinate systems.

# EXERCISES AND PROBLEMS

## Exercises

### Section 3.1 Vectors

#### Section 3.2 Properties of Vectors

- Trace the vectors in **FIGURE EX3.1** onto your paper. Then find (a)  $\vec{A} + \vec{B}$  and (b)  $\vec{A} - \vec{B}$ .

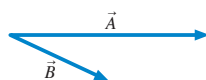


FIGURE EX3.1



FIGURE EX3.2

- Trace the vectors in **FIGURE EX3.2** onto your paper. Then find (a)  $\vec{A} + \vec{B}$  and (b)  $\vec{A} - \vec{B}$ .

#### Section 3.3 Coordinate Systems and Vector Components

- What are the  $x$ - and  $y$ -components of vector  $\vec{E}$  shown in **FIGURE EX3.3** in terms of the angle  $\theta$  and the magnitude  $E$ ?
  - For the same vector, what are the  $x$ - and  $y$ -components in terms of the angle  $\phi$  and the magnitude  $E$ ?

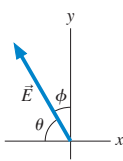


FIGURE EX3.3

- A velocity vector  $40^\circ$  below the positive  $x$ -axis has a  $y$ -component of  $-10$  m/s. What is the value of its  $x$ -component?
- A position vector in the first quadrant has an  $x$ -component of  $8$  m and a magnitude of  $10$  m. What is the value of its  $y$ -component?
- Draw each of the following vectors, then find its  $x$ - and  $y$ -components.
  - $\vec{r} = (100 \text{ m}, 45^\circ \text{ below positive } x\text{-axis})$
  - $\vec{v} = (300 \text{ m/s}, 20^\circ \text{ above positive } x\text{-axis})$
  - $\vec{a} = (5.0 \text{ m/s}^2, \text{negative } y\text{-direction})$
- Draw each of the following vectors, then find its  $x$ - and  $y$ -components.
  - $\vec{v} = (10 \text{ m/s}, \text{negative } y\text{-direction})$
  - $\vec{a} = (20 \text{ m/s}^2, 30^\circ \text{ below positive } x\text{-axis})$
  - $\vec{F} = (100 \text{ N}, 36.9^\circ \text{ counterclockwise from positive } y\text{-axis})$

- Let  $\vec{C} = (3.15 \text{ m}, 15^\circ \text{ above the negative } x\text{-axis})$  and  $\vec{D} = (25.6 \text{ m}, 30^\circ \text{ to the right of the negative } y\text{-axis})$ . Find the magnitude, the  $x$ -component, and the  $y$ -component of each vector.
- The *magnetic field* inside an instrument is  $\vec{B} = (2.0\hat{i} - 1.0\hat{j}) \text{ T}$  where  $\vec{B}$  represents the magnetic field vector and T stands for *tesla*, the unit of the magnetic field. What are the magnitude and direction of the magnetic field?

#### Section 3.4 Vector Algebra

- Draw each of the following vectors, label an angle that specifies the vector's direction, then find its magnitude and direction.
  - $\vec{B} = -4\hat{i} + 4\hat{j}$
  - $\vec{r} = (-2.0\hat{i} - 1.0\hat{j}) \text{ cm}$
  - $\vec{v} = (-10\hat{i} - 100\hat{j}) \text{ m/s}$
  - $\vec{a} = (20\hat{i} + 10\hat{j}) \text{ m/s}^2$
- Draw each of the following vectors, label an angle that specifies the vector's direction, then find the vector's magnitude and direction.
  - $\vec{A} = 4\hat{i} - 6\hat{j}$
  - $\vec{r} = (50\hat{i} + 80\hat{j}) \text{ m}$
  - $\vec{v} = (-20\hat{i} + 40\hat{j}) \text{ m/s}$
  - $\vec{a} = (2.0\hat{i} - 6.0\hat{j}) \text{ m/s}^2$
- Let  $\vec{A} = 2\hat{i} + 3\hat{j}$  and  $\vec{B} = 4\hat{i} - 2\hat{j}$ .
  - Draw a coordinate system and on it show vectors  $\vec{A}$  and  $\vec{B}$ .
  - Use graphical vector subtraction to find  $\vec{C} = \vec{A} - \vec{B}$ .
- Let  $\vec{A} = 4\hat{i} - 2\hat{j}$ ,  $\vec{B} = -3\hat{i} + 5\hat{j}$ , and  $\vec{C} = \vec{A} + \vec{B}$ .
  - Write vector  $\vec{C}$  in component form.
  - Draw a coordinate system and on it show vectors  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$ .
  - What are the magnitude and direction of vector  $\vec{C}$ ?
- Let  $\vec{A} = 4\hat{i} - 2\hat{j}$ ,  $\vec{B} = -3\hat{i} + 5\hat{j}$ , and  $\vec{D} = \vec{A} - \vec{B}$ .
  - Write vector  $\vec{D}$  in component form.
  - Draw a coordinate system and on it show vectors  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{D}$ .
  - What are the magnitude and direction of vector  $\vec{D}$ ?
- Let  $\vec{A} = 4\hat{i} - 2\hat{j}$ ,  $\vec{B} = -3\hat{i} + 5\hat{j}$ , and  $\vec{E} = 4\vec{A} + 2\vec{B}$ .
  - Write vector  $\vec{E}$  in component form.
  - Draw a coordinate system and on it show vectors  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{E}$ .
  - What are the magnitude and direction of vector  $\vec{E}$ ?
- Let  $\vec{A} = 4\hat{i} - 2\hat{j}$ ,  $\vec{B} = -3\hat{i} + 5\hat{j}$ , and  $\vec{F} = \vec{A} - 4\vec{B}$ .
  - Write vector  $\vec{F}$  in component form.
  - Draw a coordinate system and on it show vectors  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{F}$ .
  - What are the magnitude and direction of vector  $\vec{F}$ ?

17. || Let  $\vec{B} = (5.0 \text{ m}, 60^\circ \text{ counterclockwise from vertical})$ . Find the  $x$ - and  $y$ -components of  $\vec{B}$  in each of the two coordinate systems shown in **FIGURE EX3.17**.

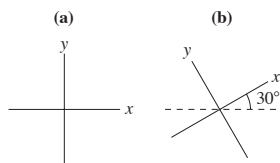


FIGURE EX3.17

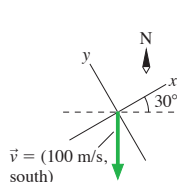


FIGURE EX3.18

18. | What are the  $x$ - and  $y$ -components of the velocity vector shown in **FIGURE EX3.18**?

### Problems

19. || Let  $\vec{A} = (3.0 \text{ m}, 20^\circ \text{ south of east})$ ,  $\vec{B} = (2.0 \text{ m}, \text{north})$ , and  $\vec{C} = (5.0 \text{ m}, 70^\circ \text{ south of west})$ .
- Draw and label  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  with their tails at the origin. Use a coordinate system with the  $x$ -axis to the east.
  - Write  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  in component form, using unit vectors.
  - Find the magnitude and the direction of  $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ .
20. | Let  $\vec{E} = 2\hat{i} + 3\hat{j}$  and  $\vec{F} = 2\hat{i} - 2\hat{j}$ . Find the magnitude of
- $\vec{E}$  and  $\vec{F}$
  - $\vec{E} + \vec{F}$
  - $-\vec{E} - 2\vec{F}$

21. | The position of a particle as a function of time is given by  $\vec{r} = (5.0\hat{i} + 4.0\hat{j})t^2 \text{ m}$ , where  $t$  is in seconds.
- What is the particle's distance from the origin at  $t = 0$ , 2, and 5 s?
  - Find an expression for the particle's velocity  $\vec{v}$  as a function of time.
  - What is the particle's speed at  $t = 0$ , 2, and 5 s?

22. || **FIGURE P3.22** shows vectors  $\vec{A}$  and  $\vec{B}$ . Let  $\vec{C} = \vec{A} + \vec{B}$ .

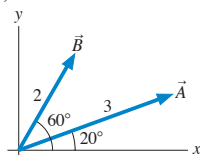


FIGURE P3.22

- Reproduce the figure on your page as accurately as possible, using a ruler and protractor. Draw vector  $\vec{C}$  on your figure, using the graphical addition of  $\vec{A}$  and  $\vec{B}$ . Then determine the magnitude and direction of  $\vec{C}$  by measuring it with a ruler and protractor.
  - Based on your figure of part a, use geometry and trigonometry to calculate the magnitude and direction of  $\vec{C}$ .
  - Decompose vectors  $\vec{A}$  and  $\vec{B}$  into components, then use these to calculate algebraically the magnitude and direction of  $\vec{C}$ .
23. || For the three vectors shown in **FIGURE P3.23**,  $\vec{A} + \vec{B} + \vec{C} = 1\hat{j}$ . What is vector  $\vec{B}$ ?
- Write  $\vec{B}$  in component form.
  - Write  $\vec{B}$  as a magnitude and a direction.

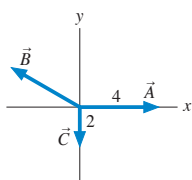


FIGURE P3.23

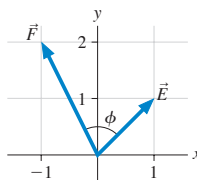


FIGURE P3.24

24. | a. What is the angle  $\phi$  between vectors  $\vec{E}$  and  $\vec{F}$  in **FIGURE P3.24**?
- Use geometry and trigonometry to determine the magnitude and direction of  $\vec{G} = \vec{E} + \vec{F}$ .
  - Use components to determine the magnitude and direction of  $\vec{G} = \vec{E} + \vec{F}$ .

25. || **FIGURE P3.25** shows vectors  $\vec{A}$  and  $\vec{B}$ . Find vector  $\vec{C}$  such that  $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ . Write your answer in component form.

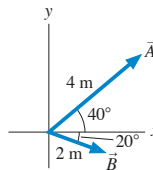


FIGURE P3.25

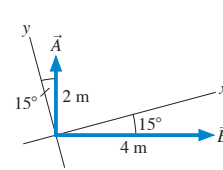


FIGURE P3.26

26. || **FIGURE P3.26** shows vectors  $\vec{A}$  and  $\vec{B}$ . Find  $\vec{D} = 2\vec{A} + \vec{B}$ . Write your answer in component form.
27. || Find a vector that points in the same direction as the vector  $(\hat{i} + \hat{j})$  and whose magnitude is 1.
28. || Carlos runs with velocity  $\vec{v} = (5.0 \text{ m/s}, 25^\circ \text{ north of east})$  for 10 minutes. How far to the north of his starting position does Carlos end up?
29. || While vacationing in the mountains you do some hiking. In the morning, your displacement is  $\vec{S}_{\text{morning}} = (2000 \text{ m}, \text{east}) + (3000 \text{ m}, \text{north}) + (200 \text{ m}, \text{vertical})$ . After lunch, your displacement is  $\vec{S}_{\text{afternoon}} = (1500 \text{ m}, \text{west}) + (2000 \text{ m}, \text{north}) - (300 \text{ m}, \text{vertical})$ .
- At the end of the hike, how much higher or lower are you compared to your starting point?
  - What is the magnitude of your net displacement for the day?
30. || The minute hand on a watch is 2.0 cm in length. What is the displacement vector of the tip of the minute hand
- From 8:00 to 8:20 A.M.?
  - From 8:00 to 9:00 A.M.?
31. || Bob walks 200 m south, then jogs 400 m southwest, then walks 200 m in a direction  $30^\circ$  east of north.
- Draw an accurate graphical representation of Bob's motion. Use a ruler and a protractor!
  - Use either trigonometry or components to find the displacement that will return Bob to his starting point by the most direct route. Give your answer as a distance and a direction.
  - Does your answer to part b agree with what you can measure on your diagram of part a?
32. || Jim's dog Sparky runs 50 m northeast to a tree, then 70 m west to a second tree, and finally 20 m south to a third tree.
- Draw a picture and establish a coordinate system.
  - Calculate Sparky's net displacement in component form.
  - Calculate Sparky's net displacement as a magnitude and an angle.
33. || A field mouse trying to escape a hawk runs east for 5.0 m, darts southeast for 3.0 m, then drops 1.0 m straight down a hole into its burrow. What is the magnitude of its net displacement?
34. | A cannon tilted upward at  $30^\circ$  fires a cannonball with a speed of 100 m/s. What is the component of the cannonball's velocity parallel to the ground?
35. | Jack and Jill ran up the hill at 3.0 m/s. The horizontal component of Jill's velocity vector was 2.5 m/s.
- What was the angle of the hill?
  - What was the vertical component of Jill's velocity?
36. | A pine cone falls straight down from a pine tree growing on a  $20^\circ$  slope. The pine cone hits the ground with a speed of 10 m/s. What is the component of the pine cone's impact velocity (a) parallel to the ground and (b) perpendicular to the ground?



37. || Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 1.0 m/s. Mary can row the boat with a speed of 2.0 m/s relative to the water.
- If Mary rows straight north, how far downstream will she land?
  - Draw a picture showing Mary's displacement due to rowing, her displacement due to the river's motion, and her net displacement.

38. || The treasure map in FIGURE P3.38 gives the following directions to the buried treasure: "Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig." But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle  $60^\circ$  east of north. After walking 300 paces you see an opening through the woods. Which direction should you go, and how far, to reach the treasure?

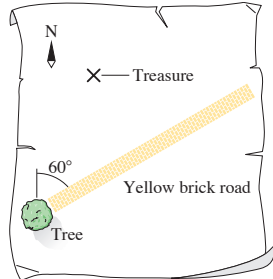


FIGURE P3.38

39. || A jet plane is flying horizontally with a speed of 500 m/s over a hill that slopes upward with a 3% grade (i.e., the "rise" is 3% of the "run"). What is the component of the plane's velocity perpendicular to the ground?

40. || The bacterium *E. coli* is a single-cell organism that lives in the gut of healthy animals, including humans. When grown in a uniform medium in the laboratory, these bacteria swim along zigzag paths at a constant speed of  $20 \mu\text{m/s}$ .

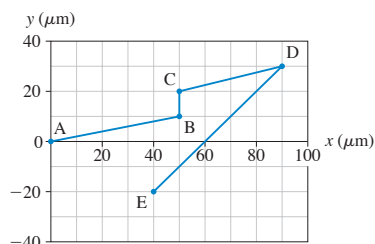


FIGURE P3.40

FIGURE P3.40 shows the trajectory of an *E. coli* as it moves from point A to point E. What are the magnitude and direction of the bacterium's average velocity for the entire trip?

41. || A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. A wise elder duck finally realizes that the solution is to fly at an angle to the wind. If the ducks can fly at 8.0 m/s relative to the air, what direction should they head in order to move directly south?

42. || FIGURE P3.42 shows three ropes tied together in a knot. One of your friends pulls on a rope with 3.0 units of force and another pulls on a second rope with 5.0 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving?

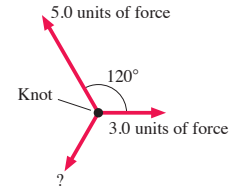


FIGURE P3.42

43. || Three forces are exerted on an object placed on a tilted floor in FIGURE P3.43. The forces are measured in newtons (N). Assuming that forces are vectors,

- What is the component of the net force  $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$  parallel to the floor?
- What is the component of  $\vec{F}_{\text{net}}$  perpendicular to the floor?
- What are the magnitude and direction of  $\vec{F}_{\text{net}}$ ?

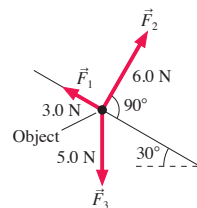


FIGURE P3.43

44. || FIGURE P3.44 shows four electric charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly away from B) on charge A of 3.0 N. Charge C exerts an attractive force (directly toward C) on charge A of 6.0 N. Finally, charge D exerts an attractive force of 2.0 N on charge A. Assuming that forces are vectors, what are the magnitude and direction of the net force  $\vec{F}_{\text{net}}$  exerted on charge A?

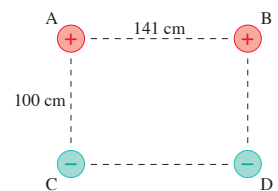


FIGURE P3.44

## STOP TO THINK ANSWERS

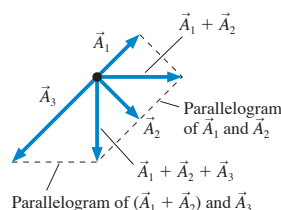
**Stop to Think 3.1:** C. The graphical construction of  $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$  is shown at right.

**Stop to Think 3.2:** a. The graphical construction of  $2\vec{A} - \vec{B}$  is shown at right.

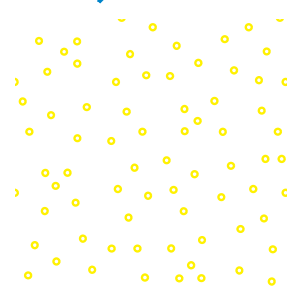
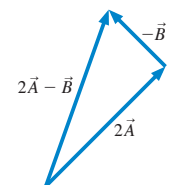
**Stop to Think 3.3:**  $C_x = -4 \text{ cm}$ ,  $C_y = 2 \text{ cm}$ .

**Stop to Think 3.4:** c. Vector  $\vec{C}$  points to the left and down, so both  $C_x$  and  $C_y$  are negative.  $C_x$  is in the numerator because it is the side opposite  $\phi$ .

## STOP TO THINK 3.1



## STOP TO THINK 3.2





## Terms and Notation

projectile  
launch angle,  $\theta$   
reference frame  
Galilean transformation of  
velocity

uniform circular motion  
period,  $T$   
angular position,  $\theta$   
arc length,  $s$   
radians

angular displacement,  $\Delta\theta$   
angular velocity,  $\omega$   
centripetal acceleration,  $a$   
nonuniform circular motion  
angular acceleration,  $\alpha$

radial acceleration,  $a_r$   
tangential acceleration,  $a_t$

## CONCEPTUAL QUESTIONS

- At this instant, is the particle in **FIGURE Q4.1** speeding up, slowing down, or traveling at constant speed?
- Is this particle curving to the right, curving to the left, or traveling straight?

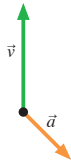


FIGURE Q4.1

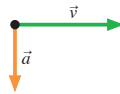


FIGURE Q4.2

- At this instant, is the particle in **FIGURE Q4.2** speeding up, slowing down, or traveling at constant speed?
- Is this particle curving upward, curving downward, or traveling straight?
- Tarzan swings through the jungle by hanging from a vine.
  - Immediately after stepping off a branch to swing over to another tree, is Tarzan's acceleration  $\vec{a}$  zero or not zero? If not zero, which way does it point? Explain.
  - Answer the same question at the lowest point in Tarzan's swing.
- A projectile is launched at an angle of  $30^\circ$ .
  - Is there any point on the trajectory where  $\vec{v}$  and  $\vec{a}$  are parallel to each other? If so, where?
  - Is there any point where  $\vec{v}$  and  $\vec{a}$  are perpendicular to each other? If so, where?
- For a projectile, which of the following quantities are constant during the flight:  $x$ ,  $y$ ,  $r$ ,  $v_x$ ,  $v_y$ ,  $v$ ,  $a_x$ ,  $a_y$ ? Which of these quantities are zero throughout the flight?
- A cart that is rolling at constant velocity on a level table fires a ball straight up.
  - When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in the tube? Explain.
  - Will your answer change if the cart is accelerating in the forward direction? If so, how?
- A rock is thrown from a bridge at an angle  $30^\circ$  below horizontal. Immediately after the rock is released, is the magnitude of its acceleration greater than, less than, or equal to  $g$ ? Explain.
- Anita is running to the right at 5 m/s in **FIGURE Q4.8**. Balls 1 and 2 are thrown toward her by friends standing on the ground. According to Anita, both balls are approaching her at 10 m/s.

Which ball was thrown at a faster speed? Or were they thrown with the same speed? Explain.

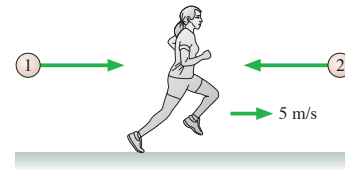


FIGURE Q4.8

- An electromagnet on the ceiling of an airplane holds a steel ball. When a button is pushed, the magnet releases the ball. The experiment is first done while the plane is parked on the ground, and the point where the ball hits the floor is marked with an X. Then the experiment is repeated while the plane is flying level at a steady 500 mph. Does the ball land slightly in front of the X (toward the nose of the plane), on the X, or slightly behind the X (toward the tail of the plane)? Explain.
- Zack is driving past his house in **FIGURE Q4.10**. He wants to toss his physics book out the window and have it land in his driveway. If he lets go of the book exactly as he passes the end of the driveway, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain.

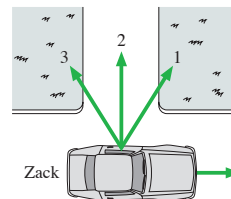


FIGURE Q4.10

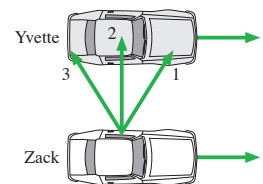


FIGURE Q4.11

- In **FIGURE Q4.11**, Yvette and Zack are driving down the freeway side by side with their windows down. Zack wants to toss his physics book out the window and have it land in Yvette's front seat. Ignoring air resistance, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain.
- In uniform circular motion, which of the following quantities are constant: speed, instantaneous velocity, tangential velocity, radial acceleration, tangential acceleration? Which of these quantities are zero throughout the motion?

13. FIGURE Q4.13 shows three points on a steadily rotating wheel.

- Rank in order, from largest to smallest, the angular velocities  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$  of these points. Explain.
- Rank in order, from largest to smallest, the speeds  $v_1$ ,  $v_2$ , and  $v_3$  of these points. Explain.

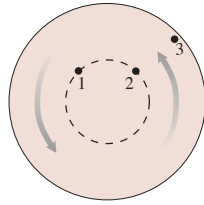


FIGURE Q4.13

14. FIGURE Q4.14 shows four rotating wheels. For each, determine the signs (+ or -) of  $\omega$  and  $\alpha$ .

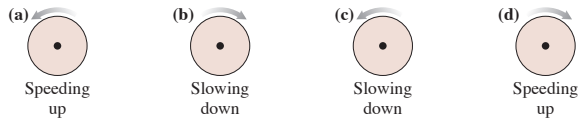


FIGURE Q4.14

15. FIGURE Q4.15 shows a pendulum at one end point of its arc.

- At this point, is  $\omega$  positive, negative, or zero? Explain.
- At this point, is  $\alpha$  positive, negative, or zero? Explain.

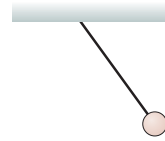


FIGURE Q4.15

## EXERCISES AND PROBLEMS

### Exercises

#### Section 4.1 Acceleration

Problems 1 and 2 show a partial motion diagram. For each:

- Complete the motion diagram by adding acceleration vectors.
- Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.

1. I

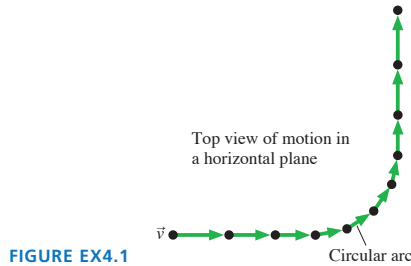


FIGURE EX4.1

2. I

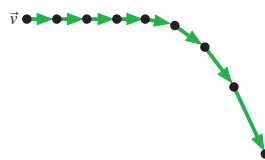


FIGURE EX4.2

Answer Problems 3 through 5 by choosing one of the eight labeled acceleration vectors or selecting option I:  $\vec{a} = \vec{0}$ .

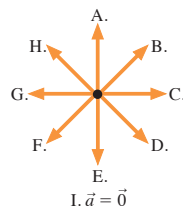


FIGURE EX4.3

- At this instant, the particle is slowing and curving upward. What is the direction of its acceleration?
- At this instant, the particle has steady speed and is curving to the right. What is the direction of its acceleration?

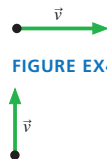


FIGURE EX4.4

5. I At this instant, the particle is speeding up and curving downward. What is the direction of its acceleration?



FIGURE EX4.5

#### Section 4.2 Two-Dimensional Kinematics

- A sailboat is traveling east at 5.0 m/s. A sudden gust of wind gives the boat an acceleration  $\vec{a} = (0.80 \text{ m/s}^2, 40^\circ \text{ north of east})$ . What are the boat's speed and direction 6.0 s later when the gust subsides?
- A model rocket is launched from rest with an upward acceleration of  $6.00 \text{ m/s}^2$  and, due to a strong wind, a horizontal acceleration of  $1.50 \text{ m/s}^2$ . How far is the rocket from the launch pad 6.00 s later when the rocket engine runs out of fuel?
- A particle's trajectory is described by  $x = (\frac{1}{2}t^3 - 2t^2) \text{ m}$  and  $y = (\frac{1}{2}t^2 - 2t) \text{ m}$ , where  $t$  is in s.
  - What are the particle's position and speed at  $t = 0 \text{ s}$  and  $t = 4 \text{ s}$ ?
  - What is the particle's direction of motion, measured as an angle from the  $x$ -axis, at  $t = 0 \text{ s}$  and  $t = 4 \text{ s}$ ?
- A rocket-powered hockey puck moves on a horizontal frictionless table. FIGURE EX4.9 shows graphs of  $v_x$  and  $v_y$ , the  $x$ - and  $y$ -components of the puck's velocity. The puck starts at the origin.
  - In which direction is the puck moving at  $t = 2 \text{ s}$ ? Give your answer as an angle from the  $x$ -axis.
  - How far from the origin is the puck at  $t = 5 \text{ s}$ ?

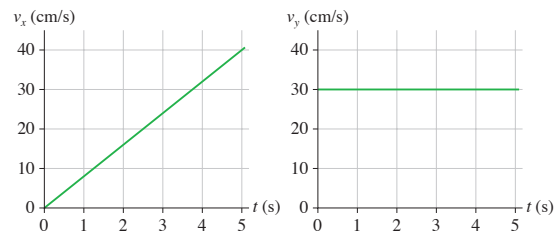


FIGURE EX4.9

10. || A rocket-powered hockey puck moves on a horizontal frictionless table. **FIGURE EX4.10** shows graphs of  $v_x$  and  $v_y$ , the  $x$ - and  $y$ -components of the puck's velocity. The puck starts at the origin. What is the magnitude of the puck's acceleration at  $t = 5$  s?

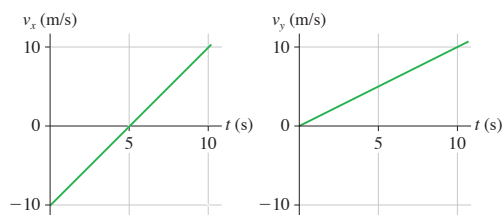


FIGURE EX4.10

### Section 4.3 Projectile Motion

11. | A physics student on Planet Exidor throws a ball, and it follows the parabolic trajectory shown in **FIGURE EX4.11**. The ball's position is shown at 1 s intervals until  $t = 3$  s. At  $t = 1$  s, the ball's velocity is  $\vec{v} = (2.0\hat{i} + 2.0\hat{j})$  m/s.
- Determine the ball's velocity at  $t = 0$  s, 2 s, and 3 s.
  - What is the value of  $g$  on Planet Exidor?
  - What was the ball's launch angle?

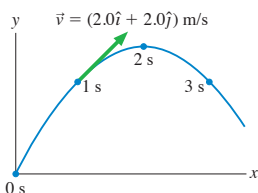


FIGURE EX4.11

12. | A ball thrown horizontally at 25 m/s travels a horizontal distance of 50 m before hitting the ground. From what height was the ball thrown?
13. || A rifle is aimed horizontally at a target 50 m away. The bullet hits the target 2.0 cm below the aim point.
- What was the bullet's flight time?
  - What was the bullet's speed as it left the barrel?
14. || A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 100 m above the glacier at a speed of 150 m/s. How far short of the target should it drop the package?

### Section 4.4 Relative Motion

15. || A boat takes 3.0 hours to travel 30 km down a river, then 5.0 hours to return. How fast is the river flowing?
16. || When the moving sidewalk at the airport is broken, as it often seems to be, it takes you 50 s to walk from your gate to baggage claim. When it is working and you stand on the moving sidewalk the entire way, without walking, it takes 75 s to travel the same distance. How long will it take you to travel from the gate to baggage claim if you walk while riding on the moving sidewalk?
17. || Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 1.0 m/s. Mary can row with a speed of 2.0 m/s.
- If Mary points her boat due north, how far from her intended landing spot will she be when she reaches the opposite shore?
  - What is her speed with respect to the shore?

18. || Susan, driving north at 60 mph, and Trent, driving east at 45 mph, are approaching an intersection. What is Trent's speed relative to Susan's reference frame?

### Section 4.5 Uniform Circular Motion

19. | **FIGURE EX4.19** shows the angular-position-versus-time graph for a particle moving in a circle. What is the particle's angular velocity at (a)  $t = 1$  s, (b)  $t = 4$  s, and (c)  $t = 7$  s?

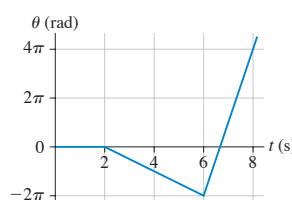


FIGURE EX4.19

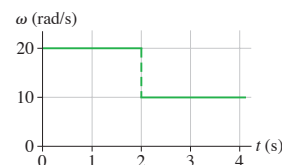


FIGURE EX4.20

20. || **FIGURE EX4.20** shows the angular-velocity-versus-time graph for a particle moving in a circle. How many revolutions does the object make during the first 4 s?
21. | **FIGURE EX4.21** shows the angular-velocity-versus-time graph for a particle moving in a circle, starting from  $\theta_0 = 0$  rad at  $t = 0$  s. Draw the angular-position-versus-time graph. Include an appropriate scale on both axes.

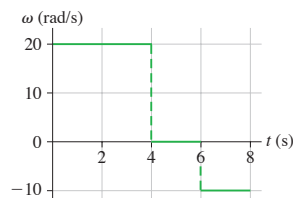


FIGURE EX4.21

22. | An old-fashioned single-play vinyl record rotates on a turntable at 45 rpm. What are (a) the angular velocity in rad/s and (b) the period of the motion?
23. || The earth's radius is about 4000 miles. Kampala, the capital of Uganda, and Singapore are both nearly on the equator. The distance between them is 5000 miles. The flight from Kampala to Singapore takes 9.0 hours. What is the plane's angular velocity with respect to the earth's surface? Give your answer in  $^\circ/\text{h}$ .

### Section 4.6 Velocity and Acceleration in Uniform Circular Motion

24. || A 3000-m-high mountain is located on the equator. How much faster does a climber on top of the mountain move than a surfer at a nearby beach? The earth's radius is 6400 km.
25. | How fast must a plane fly along the earth's equator so that the sun stands still relative to the passengers? In which direction must the plane fly, east to west or west to east? Give your answer in both km/h and mph. The earth's radius is 6400 km.
26. | **BIO** To withstand "g-forces" of up to  $10 g$ 's, caused by suddenly pulling out of a steep dive, fighter jet pilots train on a "human centrifuge."  $10 g$ 's is an acceleration of  $98 \text{ m/s}^2$ . If the length of the centrifuge arm is 12 m, at what speed is the rider moving when she experiences  $10 g$ 's?

27. I The radius of the earth's very nearly circular orbit around the sun is  $1.5 \times 10^{11}$  m. Find the magnitude of the earth's (a) velocity, (b) angular velocity, and (c) centripetal acceleration as it travels around the sun. Assume a year of 365 days.
28. II Your roommate is working on his bicycle and has the bike upside down. He spins the 60-cm-diameter wheel, and you notice that a pebble stuck in the tread goes by three times every second. What are the pebble's speed and acceleration?

### Section 4.7 Nonuniform Circular Motion and Angular Acceleration

29. I FIGURE EX4.29 shows the angular velocity graph of the crankshaft in a car. What is the crankshaft's angular acceleration at (a)  $t = 1$  s, (b)  $t = 3$  s, and (c)  $t = 5$  s?

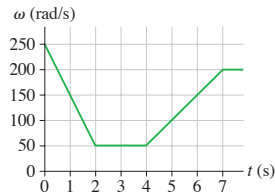


FIGURE EX4.29

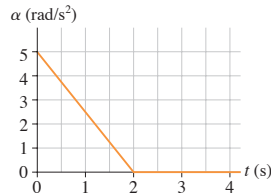


FIGURE EX4.30

30. II FIGURE EX4.30 shows the angular acceleration graph of a turntable that starts from rest. What is the turntable's angular velocity at (a)  $t = 1$  s, (b)  $t = 2$  s, and (c)  $t = 3$  s?
31. II FIGURE EX4.31 shows the angular-velocity-versus-time graph for a particle moving in a circle. How many revolutions does the object make during the first 4 s?

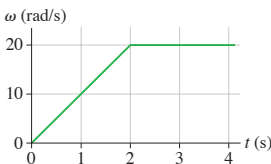


FIGURE EX4.31

32. II A 5.0-m-diameter merry-go-round is initially turning with a 4.0 s period. It slows down and stops in 20 s.
- Before slowing, what is the speed of a child on the rim?
  - How many revolutions does the merry-go-round make as it stops?
33. II An electric fan goes from rest to 1800 rpm in 4.0 s. What is its angular acceleration?
34. III A bicycle wheel is rotating at 50 rpm when the cyclist begins to pedal harder, giving the wheel a constant angular acceleration of  $0.50 \text{ rad/s}^2$ .
- What is the wheel's angular velocity, in rpm, 10 s later?
  - How many revolutions does the wheel make during this time?
35. III A 3.0-cm-diameter crankshaft that is rotating at 2500 rpm comes to a halt in 1.5 s.
- What is the tangential acceleration of a point on the surface?
  - How many revolutions does the crankshaft make as it stops?

### Problems

36. II A particle starts from rest at  $\vec{r}_0 = 9.0\hat{j}$  m and moves in the xy-plane with the velocity shown in FIGURE P4.36. The particle passes through a wire hoop located at  $\vec{r}_1 = 20\hat{i}$  m, then continues onward.

- At what time does the particle pass through the hoop?
- What is the value of  $v_{4y}$ , the y-component of the particle's velocity at  $t = 4$  s?

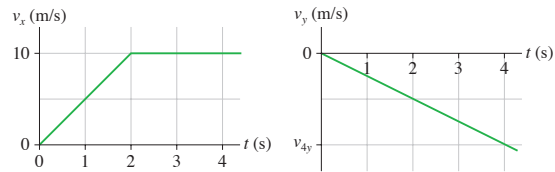


FIGURE EX4.36

37. II A spaceship maneuvering near Planet Zeta is located at  $\vec{r} = (600\hat{i} - 400\hat{j} + 200\hat{k}) \times 10^3$  km, relative to the planet, and traveling at  $\vec{v} = 9500\hat{i}$  m/s. It turns on its thruster engine and accelerates with  $\vec{a} = (40\hat{i} - 20\hat{k}) \text{ m/s}^2$  for 35 min. Where is the spaceship located when the engine shuts off? Give your answer as a vector measured in km.
38. II A projectile's horizontal range on level ground is  $R = v_0^2 \sin 2\theta / g$ . At what launch angle or angles will the projectile land at half of its maximum possible range?
39. II
- A projectile is launched with speed  $v_0$  and angle  $\theta$ . Derive an expression for the projectile's maximum height  $h$ .
  - A baseball is hit with a speed of 33.6 m/s. Calculate its height and the distance traveled if it is hit at angles of  $30.0^\circ$ ,  $45.0^\circ$ , and  $60.0^\circ$ .
40. II A gray kangaroo can bound across level ground with each jump carrying it 10 m from the takeoff point. Typically the kangaroo leaves the ground at a  $20^\circ$  angle. If this is so:
- What is its takeoff speed?
  - What is its maximum height above the ground?
41. II A projectile is fired with an initial speed of 30 m/s at an angle of  $60^\circ$  above the horizontal. The object hits the ground 7.5 s later.
- How much higher or lower is the launch point relative to the point where the projectile hits the ground?
  - To what maximum height above the launch point does the projectile rise?
42. II In the Olympic shotput event, an athlete throws the shot with an initial speed of 12.0 m/s at a  $40.0^\circ$  angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground.
- How far does the shot travel?
  - Repeat the calculation of part (a) for angles  $42.5^\circ$ ,  $45.0^\circ$ , and  $47.5^\circ$ . Put all your results, including  $40.0^\circ$ , in a table. At what angle of release does she throw the farthest?
43. II On the Apollo 14 mission to the moon, astronaut Alan Shepard hit a golf ball with a 6 iron. The free-fall acceleration on the moon is  $1/6$  of its value on earth. Suppose he hit the ball with a speed of 25 m/s at an angle  $30^\circ$  above the horizontal.
- How much farther did the ball travel on the moon than it would have on earth?
  - For how much more time was the ball in flight?
44. II A ball is thrown toward a cliff of height  $h$  with a speed of 30 m/s and an angle of  $60^\circ$  above horizontal. It lands on the edge of the cliff 4.0 s later.
- How high is the cliff?
  - What was the maximum height of the ball?
  - What is the ball's impact speed?

45. || A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20.0 m/s at an angle  $5.0^\circ$  above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high. Does the ball clear the net? If so, by how much? If not, by how much does it miss?
46. || A baseball player friend of yours wants to determine his pitching speed. You have him stand on a ledge and throw the ball horizontally from an elevation 4.0 m above the ground. The ball lands 25 m away.
- What is his pitching speed?
  - As you think about it, you're not sure he threw the ball exactly horizontally. As you watch him throw, the pitches seem to vary from  $5^\circ$  below horizontal to  $5^\circ$  above horizontal. What are the lowest and highest speeds with which the ball might have left his hand?
47. || You are playing right field for the baseball team. Your team is up by one run in the bottom of the last inning of the game when a ground ball slips through the infield and comes straight toward you. As you pick up the ball 65 m from home plate, you see a runner rounding third base and heading for home with the tying run. You throw the ball at an angle of  $30^\circ$  above the horizontal with just the right speed so that the ball is caught by the catcher, standing on home plate, at the same height as you threw it. As you release the ball, the runner is 20 m from home plate and running full speed at 8.0 m/s. Will the ball arrive in time for your team's catcher to make the tag and win the game?
48. || You're 6.0 m from one wall of the house seen in **FIGURE P4.48**. You want to toss a ball to your friend who is 6.0 m from the opposite wall. The throw and catch each occur 1.0 m above the ground.
- What minimum speed will allow the ball to clear the roof?
  - At what angle should you toss the ball?

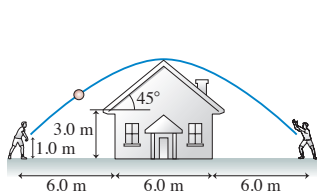


FIGURE P4.48

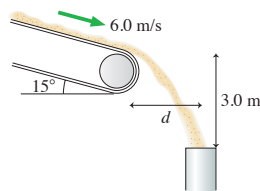


FIGURE P4.49

49. || Sand moves without slipping at 6.0 m/s down a conveyor that is tilted at  $15^\circ$ . The sand enters a pipe 3.0 m below the end of the conveyor belt, as shown in **FIGURE P4.49**. What is the horizontal distance  $d$  between the conveyor belt and the pipe?
50. || A stunt man drives a car at a speed of 20 m/s off a 30-m-high cliff. The road leading to the cliff is inclined upward at an angle of  $20^\circ$ .
- How far from the base of the cliff does the car land?
  - What is the car's impact speed?
51. || A javelin thrower standing at rest holds the center of the javelin behind her head, then accelerates it through a distance of 70 cm as she throws. She releases the javelin 2.0 m above the ground traveling at an angle of  $30^\circ$  above the horizontal. Top-rated javelin throwers do throw at about a  $30^\circ$  angle, not the  $45^\circ$  you might have expected, because the biomechanics of the arm allow them to throw the javelin much faster at  $30^\circ$  than they would be able to at  $45^\circ$ . In this throw, the javelin hits the ground 62 m away. What was the acceleration of the javelin during the throw? Assume that it has a constant acceleration.
52. || Ships A and B leave port together. For the next two hours, ship A travels at 20 mph in a direction  $30^\circ$  west of north while the ship B travels  $20^\circ$  east of north at 25 mph.
- What is the distance between the two ships two hours after they depart?
  - What is the speed of ship A as seen by ship B?
53. || A kayaker needs to paddle north across a 100-m-wide harbor. The tide is going out, creating a tidal current that flows to the east at 2.0 m/s. The kayaker can paddle with a speed of 3.0 m/s.
- In which direction should he paddle in order to travel straight across the harbor?
  - How long will it take him to cross?
54. || Mike throws a ball upward and toward the east at a  $63^\circ$  angle with a speed of 22 m/s. Nancy drives east past Mike at 30 m/s at the instant he releases the ball.
- What is the ball's initial angle in Nancy's reference frame?
  - Find and graph the ball's trajectory as seen by Nancy.
55. || While driving north at 25 m/s during a rainstorm you notice that the rain makes an angle of  $38^\circ$  with the vertical. While driving back home moments later at the same speed but in the opposite direction, you see that the rain is falling straight down. From these observations, determine the speed and angle of the raindrops relative to the ground.
56. || You've been assigned the task of using a shaft encoder—a device that measures the angle of a shaft or axle and provides a signal to a computer—to analyze the rotation of an engine crankshaft under certain conditions. The table lists the crankshaft's angles over a 0.6 s interval.

Time (s)	Angle (rad)
0.0	0.0
0.1	2.0
0.2	3.2
0.3	4.3
0.4	5.3
0.5	6.1
0.6	7.0

Is the crankshaft rotating with uniform circular motion? If so, what is its angular velocity in rpm? If not, is the angular acceleration positive or negative?

57. || A speck of dust on a spinning DVD has a centripetal acceleration of  $20 \text{ m/s}^2$ .
- What is the acceleration of a different speck of dust that is twice as far from the center of the disk?
  - What would be the acceleration of the first speck of dust if the disk's angular velocity was doubled?
58. || A typical laboratory centrifuge rotates at 4000 rpm. Test tubes have to be placed into a centrifuge very carefully because of the very large accelerations.
- What is the acceleration at the end of a test tube that is 10 cm from the axis of rotation?
  - For comparison, what is the magnitude of the acceleration a test tube would experience if dropped from a height of 1.0 m and stopped in a 1.0-ms-long encounter with a hard floor?
59. || Astronauts use a centrifuge to simulate the acceleration of a rocket launch. The centrifuge takes 30 s to speed up from rest to its top speed of 1 rotation every 1.3 s. The astronaut is strapped into a seat 6.0 m from the axis.
- What is the astronaut's tangential acceleration during the first 30 s?
  - How many g's of acceleration does the astronaut experience when the device is rotating at top speed? Each  $9.8 \text{ m/s}^2$  of acceleration is 1 g.



60. **BIO** Peregrine falcons are known for their maneuvering ability. In a tight circular turn, a falcon can attain a centripetal acceleration 1.5 times the free-fall acceleration. What is the radius of the turn if the falcon is flying at 25 m/s?
61. **||** As the earth rotates, what is the speed of (a) a physics student in Miami, Florida, at latitude  $26^\circ$ , and (b) a physics student in Fairbanks, Alaska, at latitude  $65^\circ$ ? Ignore the revolution of the earth around the sun. The radius of the earth is 6400 km.
62. **||** Communications satellites are placed in a circular orbit where they stay directly over a fixed point on the equator as the earth rotates. These are called *geosynchronous orbits*. The radius of the earth is  $6.37 \times 10^6$  m, and the altitude of a geosynchronous orbit is  $3.58 \times 10^7$  m ( $\approx 22,000$  miles). What are (a) the speed and (b) the magnitude of the acceleration of a satellite in a geosynchronous orbit?
63. **||** A computer hard disk 8.0 cm in diameter is initially at rest. A small dot is painted on the edge of the disk. The disk accelerates at  $600 \text{ rad/s}^2$  for  $\frac{1}{2}$  s, then coasts at a steady angular velocity for another  $\frac{1}{2}$  s.
- a. What is the speed of the dot at  $t = 1.0$  s?
- b. Through how many revolutions has the disk turned?
64. **||** a. A turbine spinning with angular velocity  $\omega_0$  rad/s comes to a halt in  $T$  seconds. Find an expression for the angle  $\Delta\theta$  through which the turbine turns while stopping.
- b. A turbine is spinning at 3800 rpm. Friction in the bearings is so low that it takes 10 min to coast to a stop. How many revolutions does the turbine make while stopping?
65. **||** A high-speed drill rotating ccw at 2400 rpm comes to a halt in 2.5 s.
- a. What is the drill's angular acceleration?
- b. How many revolutions does it make as it stops?
66. **||** A wheel initially rotating at 60 rpm experiences the angular acceleration shown in **FIGURE P4.66**. What is the wheel's angular velocity, in rpm, at  $t = 3.0$  s?

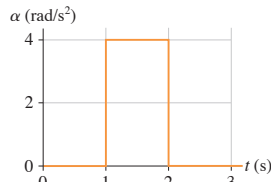


FIGURE P4.66

67. **||** Your car tire is rotating at 3.5 rev/s when suddenly you press down hard on the accelerator. After traveling 200 m, the tire's rotation has increased to 6.0 rev/s. What was the tire's angular acceleration? Give your answer in  $\text{rad/s}^2$ .
68. **||** The angular velocity of a process control motor is  $\omega = (20 - \frac{1}{2}t^2) \text{ rad/s}$ , where  $t$  is in seconds.
- a. At what time does the motor reverse direction?
- b. Through what angle does the motor turn between  $t = 0$  s and the instant at which it reverses direction?
69. **||** A Ferris wheel of radius  $R$  speeds up with angular acceleration  $\alpha$  starting from rest. Find an expression for the (a) velocity and (b) centripetal acceleration of a rider after the Ferris wheel has rotated through angle  $\Delta\theta$ .
70. **||** A 6.0-cm-diameter gear rotates with angular velocity  $\omega = (2.0 + \frac{1}{2}t^2) \text{ rad/s}$ , where  $t$  is in seconds. At  $t = 4.0$  s, what are:
- a. The gear's angular acceleration?
- b. The tangential acceleration of a tooth on the gear?

71. **||** On a lonely highway, with no other cars in sight, you decide to measure the angular acceleration of your engine's crankshaft while braking gently. Having excellent memory, you are able to read the tachometer every 1.0 s and remember seven values long enough to later write them down. The table shows your data:

Time (s)	rpm
0.0	3010
1.0	2810
2.0	2450
3.0	2250
4.0	1940
5.0	1810
6.0	1510

What is the *magnitude* of the crankshaft's angular acceleration? Give your result in  $\text{rad/s}^2$ .

72. **||** A car starts from rest on a curve with a radius of 120 m and accelerates at  $1.0 \text{ m/s}^2$ . Through what angle will the car have traveled when the magnitude of its total acceleration is  $2.0 \text{ m/s}^2$ ?
73. **||** A long string is wrapped around a 6.0-cm-diameter cylinder, initially at rest, that is free to rotate on an axle. The string is then pulled with a constant acceleration of  $1.5 \text{ m/s}^2$  until 1.0 m of string has been unwound. If the string unwinds without slipping, what is the cylinder's angular speed, in rpm, at this time?

In Problems 74 through 76 you are given the equations that are used to solve a problem. For each of these, you are to

- a. Write a realistic problem for which these are the correct equations. Be sure that the answer your problem requests is consistent with the equations given.
- b. Finish the solution of the problem, including a pictorial representation.
74.  $100 \text{ m} = 0 \text{ m} + (50 \cos \theta \text{ m/s})t_1$   
 $0 \text{ m} = 0 \text{ m} + (50 \sin \theta \text{ m/s})t_1 - \frac{1}{2}(9.80 \text{ m/s}^2)t_1^2$
75.  $v_x = -(6.0 \cos 45^\circ) \text{ m/s} + 3.0 \text{ m/s}$   
 $v_y = (6.0 \sin 45^\circ) \text{ m/s} + 0 \text{ m/s}$   
 $100 \text{ m} = v_y t_1, x_1 = v_x t_1$
76.  $2.5 \text{ rad} = 0 \text{ rad} + \omega_i(10 \text{ s}) + \left( (1.5 \text{ m/s}^2)/2(50 \text{ m}) \right)(10 \text{ s})^2$   
 $\omega_f = \omega_i + \left( (1.5 \text{ m/s}^2)/(50 \text{ m}) \right)(10 \text{ s})$

### Challenge Problems

77. **BIO** You are asked to consult for the city's research hospital, where a group of doctors is investigating the bombardment of cancer tumors with high-energy ions. The ions are fired directly toward the center of the tumor at speeds of  $5.0 \times 10^6 \text{ m/s}$ . To cover the entire tumor area, the ions are deflected sideways by passing them between two charged metal plates that accelerate the ions perpendicular to the direction of their initial motion. The acceleration region is 5.0 cm long, and the ends of the acceleration plates are 1.5 m from the patient. What sideways acceleration is required to deflect an ion 2.0 cm to one side?

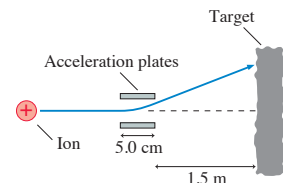


FIGURE CP4.77



78. In one contest at the county fair, seen in **FIGURE CP4.78**, a spring-loaded plunger launches a ball at a speed of 3.0 m/s from one corner of a smooth, flat board that is tilted up at a  $20^\circ$  angle. To win, you must make the ball hit a small target at the adjacent corner, 2.50 m away. At what angle  $\theta$  should you tilt the ball launcher?

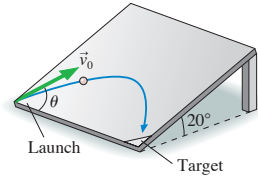


FIGURE CP4.78

79. You are watching an archery tournament when you start wondering how fast an arrow is shot from the bow. Remembering your physics, you ask one of the archers to shoot an arrow parallel to the ground. You find the arrow stuck in the ground 60 m away, making a  $3.0^\circ$  angle with the ground. How fast was the arrow shot?
80. An archer standing on a  $15^\circ$  slope shoots an arrow  $20^\circ$  above the horizontal, as shown in **FIGURE CP4.80**. How far down the slope does the arrow hit if it is shot with a speed of 50 m/s from 1.75 m above the ground?

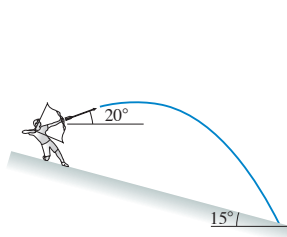


FIGURE CP4.80

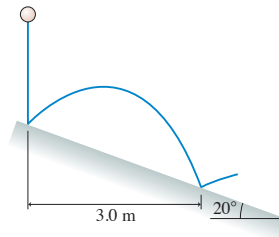


FIGURE CP4.81

81. A rubber ball is dropped onto a ramp that is tilted at  $20^\circ$ , as shown in **FIGURE CP4.81**. A bouncing ball obeys the “law of reflection,” which says that the ball leaves the surface at the same angle it approached the surface. The ball’s next bounce is 3.0 m to the right of its first bounce. What is the ball’s rebound speed on its first bounce?
82. A skateboarder starts up a 1.0-m-high,  $30^\circ$  ramp at a speed of 7.0 m/s. The skateboard wheels roll without friction. How far from the end of the ramp does the skateboarder touch down?
83. A motorcycle daredevil wants to set a record for jumping over burning school buses. He has hired you to help with the design.

He intends to ride off a horizontal platform at 40 m/s, cross the burning buses in a pit below him, then land on a ramp sloping down at  $20^\circ$ . It’s very important that he not bounce when he hits the landing ramp because that could cause him to lose control and crash. You immediately recognize that he won’t bounce if his velocity is parallel to the ramp as he touches down. This can be accomplished if the ramp is tangent to his trajectory *and* if he lands right on the front edge of the ramp. There’s no room for error! Your task is to determine where to place the landing ramp. That is, how far from the edge of the launching platform should the front edge of the landing ramp be horizontally and how far below it? There’s a clause in your contract that requires you to test your design before the hero goes on national television to set the record.

84. A cannon on a train car fires a projectile to the right with speed  $v_0$ , relative to the train, from a barrel elevated at angle  $\theta$ . The cannon fires just as the train, which had been cruising to the right along a level track with speed  $v_{\text{train}}$ , begins to accelerate with acceleration  $a$ . Find an expression for the angle at which the projectile should be fired so that it lands as far as possible from the cannon. You can ignore the small height of the cannon above the track.
85. A child in danger of drowning in a river is being carried downstream by a current that flows uniformly with a speed of 2.0 m/s. The child is 200 m from the shore and 1500 m upstream of the boat dock from which the rescue team sets out. If their boat speed is 8.0 m/s with respect to the water, at what angle from the shore should the pilot leave the shore to go directly to the child?
86. An amusement park game, shown in **FIGURE CP4.86**, launches a marble toward a small cup. The marble is placed directly on top of a spring-loaded wheel and held with a clamp. When released, the wheel spins around clockwise at constant angular acceleration, opening the clamp and releasing the marble after making  $\frac{11}{12}$  revolution. What angular acceleration is needed for the ball to land in the cup? The top of the cup is level with the center of the wheel.

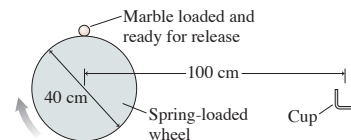


FIGURE CP4.86

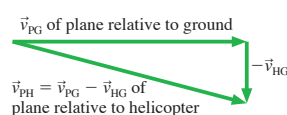
## STOP TO THINK ANSWERS

**Stop to Think 4.1: d.** The parallel component of  $\vec{a}$  is opposite  $\vec{v}$  and will cause the particle to slow down. The perpendicular component of  $\vec{a}$  will cause the particle to change direction downward.

**Stop to Think 4.2: c.**  $v = 0$  requires both  $v_x = 0$  and  $v_y = 0$ . Neither  $x$  nor  $y$  can be changing.

**Stop to Think 4.3: d.** A projectile’s acceleration  $\vec{a} = -g\hat{j}$  does not depend on its mass. The second marble has the same initial velocity and the same acceleration, so it follows the same trajectory and lands at the same position.

**Stop to Think 4.4: f.** The plane’s velocity relative to the helicopter is  $\vec{v}_{\text{PH}} = \vec{v}_{\text{PG}} + \vec{v}_{\text{GH}} = \vec{v}_{\text{PG}} - \vec{v}_{\text{HG}}$ , where G is the ground. The vector addition shows that  $\vec{v}_{\text{PH}}$  is to the right and down with a magnitude greater than the 100 m/s of  $\vec{v}_{\text{PG}}$ .



**Stop to Think 4.5: b.** An initial cw rotation causes the particle’s angular position to become increasingly negative. The speed drops to half after reversing direction, so the slope becomes positive and is half as steep as the initial slope. Turning through the same angle returns the particle to  $\theta = 0^\circ$ .

**Stop to Think 4.6:**  $a_b > a_c > a_a = a_c > a_d$ . Centripetal acceleration is  $v^2/r$ . Doubling  $r$  decreases  $a_r$  by a factor of 2. Doubling  $v$  increases  $a_r$  by a factor of 4. Reversing direction doesn’t change  $a_r$ .

**Stop to Think 4.7: c.**  $\omega$  is negative because the rotation is cw. Because  $\omega$  is negative but becoming *less* negative, the change  $\Delta\omega$  is *positive*. So  $\alpha$  is positive.

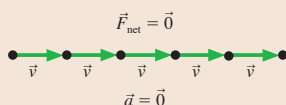
# SUMMARY

The goal of Chapter 5 has been to establish a connection between force and motion.

## General Principles

### Newton's First Law

An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.



The first law tells us that no “cause” is needed for motion. Uniform motion is the “natural state” of an object.

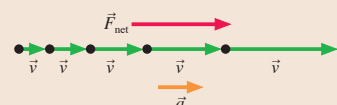
Newton's laws are valid only in inertial reference frames.

### Newton's Second Law

An object with mass  $m$  will undergo acceleration

$$\vec{a} = \frac{1}{m} \vec{F}_{\text{net}}$$

where  $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$  is the vector sum of all the individual forces acting on the object.



The second law tells us that a net force causes an object to accelerate. This is the connection between force and motion that we are seeking.

## Important Concepts

**Acceleration** is the link to kinematics.

From  $\vec{F}_{\text{net}}$ , find  $\vec{a}$ .

From  $a$ , find  $v$  and  $x$ .

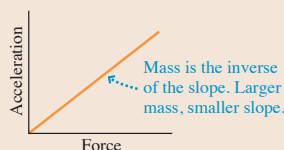
$\vec{a} = \vec{0}$  is the condition for **equilibrium**.

**Static equilibrium** if  $\vec{v} = \vec{0}$ .

**Dynamic equilibrium** if  $\vec{v} = \text{constant}$ .

Equilibrium occurs if and only if  $\vec{F}_{\text{net}} = \vec{0}$ .

**Mass** is the resistance of an object to acceleration. It is an intrinsic property of an object.



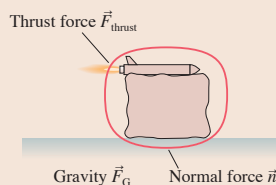
**Force** is a push or a pull on an object.

- Force is a vector, with a magnitude and a direction.
- Force requires an agent.
- Force is either a contact force or a long-range force.

## Key Skills

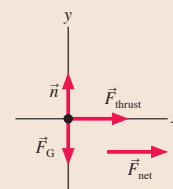
### Identifying Forces

Forces are identified by locating the points where other objects touch the object of interest. These are points where contact forces are exerted. In addition, objects with mass feel a long-range gravitational force.



### Free-Body Diagrams

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.



## Terms and Notation

dynamics  
mechanics  
force,  $\vec{F}$   
agent  
contact force  
long-range force  
net force,  $\vec{F}_{\text{net}}$   
superposition of forces

gravitational force,  $\vec{F}_G$   
spring force,  $\vec{F}_{\text{sp}}$   
tension force,  $\vec{T}$   
atomic model  
normal force,  $\vec{n}$   
friction,  $\vec{f}_k$  or  $\vec{f}_s$   
drag,  $\vec{D}$   
thrust,  $\vec{F}_{\text{thrust}}$

proportionality  
proportionality constant  
proportional reasoning  
newton, N  
inertia  
inertial mass,  $m$   
Newton's second law  
Newton's zeroth law

Newton's first law  
mechanical equilibrium  
static equilibrium  
dynamic equilibrium  
inertial reference frame  
free-body diagram

## CONCEPTUAL QUESTIONS

1. An elevator suspended by a cable is descending at constant velocity. How many force vectors would be shown on a free-body diagram? Name them.
2. A compressed spring is pushing a block across a rough horizontal table. How many force vectors would be shown on a free-body diagram? Name them.
3. A brick is falling from the roof of a three-story building. How many force vectors would be shown on a free-body diagram? Name them.

4. In **FIGURE Q5.4**, block B is falling and dragging block A across a table. How many force vectors would be shown on a free-body diagram of block A? Name them.
5. You toss a ball straight up in the air. Immediately after you let go of it, what forces are acting on the ball? For each force you name, (a) state whether it is a contact force or a long-range force and (b) identify the agent of the force.

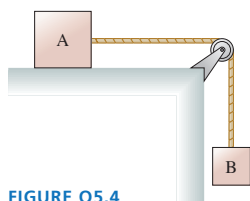


FIGURE Q5.4

6. A constant force applied to A causes A to accelerate at  $5 \text{ m/s}^2$ . The same force applied to B causes an acceleration of  $3 \text{ m/s}^2$ .
  - a. Which object has the largest mass? Explain.
  - b. Which object has the smallest mass?
  - c. What is the ratio  $m_A/m_B$  of the mass of A to the mass of B?
7. An object experiencing a constant force accelerates at  $10 \text{ m/s}^2$ . What will the acceleration of this object be if
  - a. The force is doubled? Explain.
  - b. The mass is doubled?
  - c. The force is doubled *and* the mass is doubled?
8. An object experiencing a constant force accelerates at  $8 \text{ m/s}^2$ . What will the acceleration of this object be if
  - a. The force is halved? Explain.
  - b. The mass is halved?
  - c. The force is halved *and* the mass is halved?

9. If an object is at rest, can you conclude that there are no forces acting on it? Explain.
10. If a force is exerted on an object, is it possible for that object to be moving with constant velocity? Explain.
11. Is the statement “An object always moves in the direction of the net force acting on it” true or false? Explain.
12. Newton’s second law says  $\vec{F}_{\text{net}} = m\vec{a}$ . So is  $m\vec{a}$  a force? Explain.
13. Is it possible for the friction force on an object to be in the direction of motion? If so, give an example. If not, why not?
14. Suppose you press your physics book against a wall hard enough to keep it from moving. Does the friction force on the book point (a) into the wall, (b) out of the wall, (c) up, (d) down, or (e) is there no friction force? Explain.
15. **FIGURE Q5.15** shows a hollow tube forming three-quarters of a circle. It is lying flat on a table. A ball is shot through the tube at high speed. As the ball emerges from the other end, does it follow path A, path B, or path C? Explain.

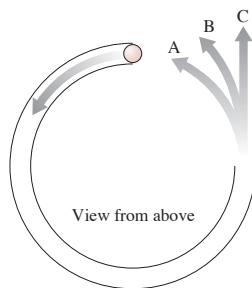


FIGURE Q5.15

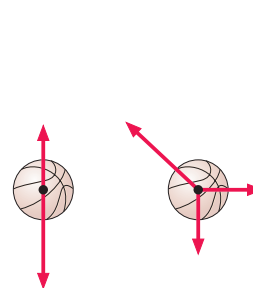


FIGURE Q5.16

16. Which, if either, of the basketballs in **FIGURE Q5.16** are in equilibrium? Explain.
17. Which of the following are inertial reference frames? Explain.
  - a. A car driving at steady speed on a straight and level road.
  - b. A car driving at steady speed up a  $10^\circ$  incline.
  - c. A car speeding up after leaving a stop sign.
  - d. A car driving at steady speed around a curve.

## EXERCISES AND PROBLEMS

### Exercises

#### Section 5.3 Identifying Forces

1. I A chandelier hangs from a chain in the middle of a dining room. Identify the forces on the chandelier.
2. I A car is parked on a steep hill. Identify the forces on the car.
3. II A jet plane is speeding down the runway during takeoff. Air resistance is not negligible. Identify the forces on the jet.
4. I A baseball player is sliding into second base. Identify the forces on the baseball player.
5. II A bullet has just been shot from a gun and is now traveling horizontally. Air resistance is not negligible. Identify the forces on the bullet.

#### Section 5.4 What Do Forces Do? A Virtual Experiment

6. I Two rubber bands cause an object to accelerate with acceleration  $a$ . How many rubber bands are needed to cause an object with half the mass to accelerate three times as quickly?
7. I Two rubber bands pulling on an object cause it to accelerate at  $1.2 \text{ m/s}^2$ .
  - a. What will be the object’s acceleration if it is pulled by four rubber bands?
  - b. What will be the acceleration of two of these objects glued together if they are pulled by two rubber bands?

8. **FIGURE EX5.8** shows an acceleration-versus-force graph for three objects pulled by rubber bands. The mass of object 2 is 0.20 kg. What are the masses of objects 1 and 3? Explain your reasoning.

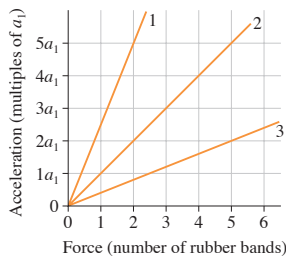


FIGURE EX5.8

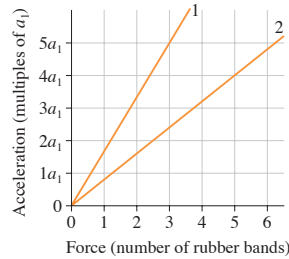


FIGURE EX5.9

9. **FIGURE EX5.9** shows acceleration-versus-force graphs for two objects pulled by rubber bands. What is the mass ratio  $m_1/m_2$ ?
10. For an object starting from rest and accelerating with constant acceleration, distance traveled is proportional to the square of the time. If an object travels 2.0 furlongs in the first 2.0 s, how far will it travel in the first 4.0 s?
11. The period of a pendulum is proportional to the square root of its length. A 2.0-m-long pendulum has a period of 3.0 s. What is the period of a 3.0-m-long pendulum?

### Section 5.5 Newton's Second Law

12. **FIGURE EX5.12** shows an acceleration-versus-force graph for a 500 g object. What acceleration values go in the blanks on the vertical scale?

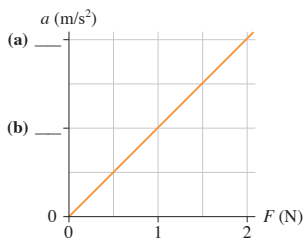


FIGURE EX5.12

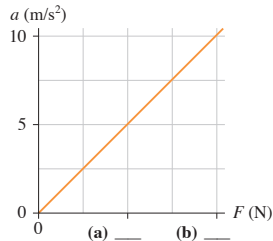


FIGURE EX5.13

13. **FIGURE EX5.13** shows an acceleration-versus-force graph for a 200 g object. What force values go in the blanks on the horizontal scale?
14. **FIGURE EX5.14** shows an object's acceleration-versus-force graph. What is the object's mass?

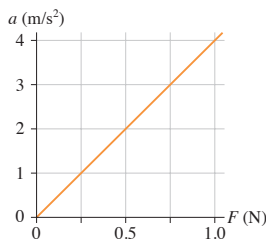


FIGURE EX5.14

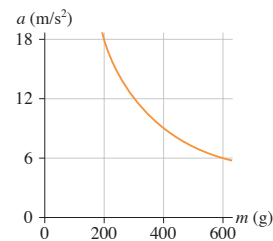


FIGURE EX5.15

15. **FIGURE EX5.15** shows the acceleration of objects of different mass that experience the same force. What is the magnitude of the force?

16. Based on the information in Table 5.1, *estimate*
- The weight of a laptop computer.
  - The propulsion force of a bicycle.
17. Based on the information in Table 5.1, *estimate*
- The weight of a pencil.
  - The propulsion force of a sprinter.

### Section 5.6 Newton's First Law

Exercises 18 through 20 show two of the three forces acting on an object in equilibrium. Redraw the diagram, showing all three forces. Label the third force  $\vec{F}_3$ .

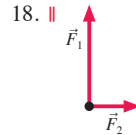


FIGURE EX5.18

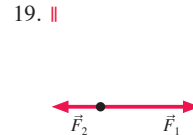


FIGURE EX5.19

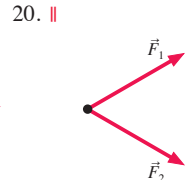


FIGURE EX5.20

### Section 5.7 Free-Body Diagrams

Exercises 21 through 23 show a free-body diagram. For each:

- Redraw the free-body diagram.
- Write a short description of a real object for which this is the correct free-body diagram. Use Examples 5.4, 5.5, and 5.6 as models of what a description should be like.

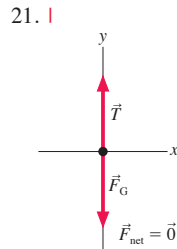


FIGURE EX5.21

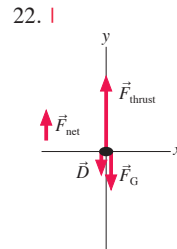


FIGURE EX5.22

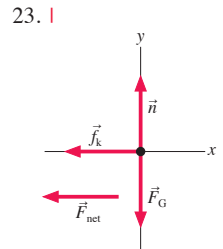


FIGURE EX5.23

Exercises 24 through 27 describe a situation. For each, identify all forces acting on the object and draw a free-body diagram of the object.

- A cat is sitting on a window sill.
- An ice hockey puck glides across frictionless ice.
- Your physics textbook is sliding across the table.
- A steel beam is being lowered at steady speed by a crane.

### Problems

28. Redraw the two motion diagrams shown in **FIGURE P5.28**, then draw a vector beside each one to show the direction of the net force acting on the object. Explain your reasoning.

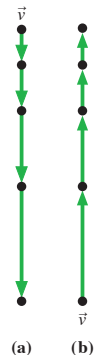


FIGURE P5.28

29. I Redraw the two motion diagrams shown in FIGURE P5.29, then draw a vector beside each one to show the direction of the net force acting on the object. Explain your reasoning.

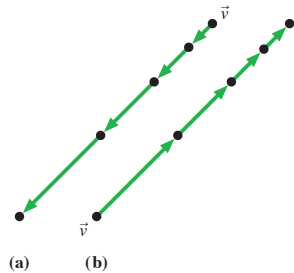


FIGURE P5.29

30. I A single force with  $x$ -component  $F_x$  acts on a 2.0 kg object as it moves along the  $x$ -axis. The object's acceleration graph ( $a_x$  versus  $t$ ) is shown in FIGURE P5.30. Draw a graph of  $F_x$  versus  $t$ .

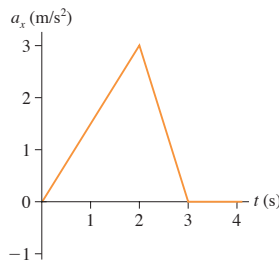


FIGURE P5.30

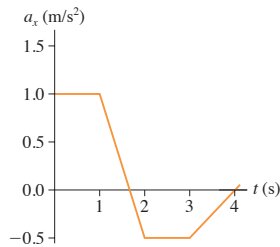


FIGURE P5.31

31. I A single force with  $x$ -component  $F_x$  acts on a 500 g object as it moves along the  $x$ -axis. The object's acceleration graph ( $a_x$  versus  $t$ ) is shown in FIGURE P5.31. Draw a graph of  $F_x$  versus  $t$ .
32. I A single force with  $x$ -component  $F_x$  acts on a 2.0 kg object as it moves along the  $x$ -axis. A graph of  $F_x$  versus  $t$  is shown in FIGURE P5.32. Draw an acceleration graph ( $a_x$  versus  $t$ ) for this object.

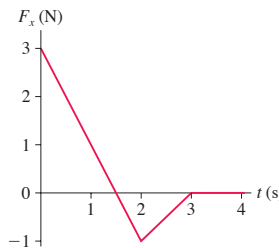


FIGURE P5.32

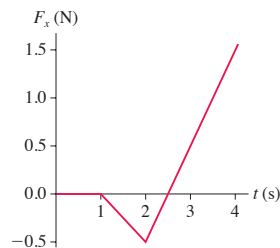


FIGURE P5.33

33. I A single force with  $x$ -component  $F_x$  acts on a 500 g object as it moves along the  $x$ -axis. A graph of  $F_x$  versus  $t$  is shown in FIGURE P5.33. Draw an acceleration graph ( $a_x$  versus  $t$ ) for this object.
34. I A constant force is applied to an object, causing the object to accelerate at  $8.0 \text{ m/s}^2$ . What will the acceleration be if
- The force is doubled?
  - The object's mass is doubled?
  - The force and the object's mass are both doubled?
  - The force is doubled and the object's mass is halved?
35. I A constant force is applied to an object, causing the object to accelerate at  $10 \text{ m/s}^2$ . What will the acceleration be if
- The force is halved?
  - The object's mass is halved?
  - The force and the object's mass are both halved?
  - The force is halved and the object's mass is doubled?

Problems 36 through 41 show a free-body diagram. For each:

- Redraw the diagram.
- Identify the direction of the acceleration vector  $\vec{a}$  and show it as a vector next to your diagram. Or, if appropriate, write  $\vec{a} = \vec{0}$ .
- If possible, identify the direction of the velocity vector  $\vec{v}$  and show it as a labeled vector.
- Write a short description of a real object for which this is the correct free-body diagram. Use Examples 5.4, 5.5, and 5.6 as models of what a description should be like.

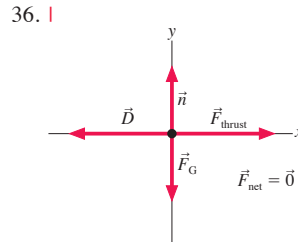


FIGURE P5.36

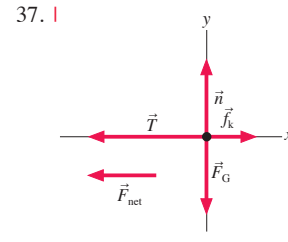


FIGURE P5.37

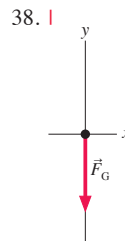


FIGURE P5.38

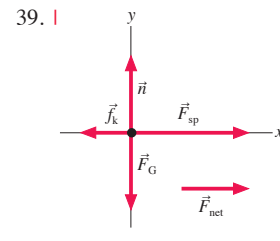


FIGURE P5.39

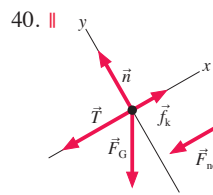


FIGURE P5.40

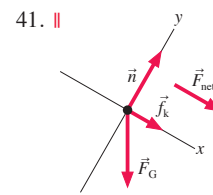


FIGURE P5.41

42. II In lab, you propel a cart with four known forces while using an ultrasonic motion detector to measure the cart's acceleration. Your data are as follows:

Force (N)	Acceleration ( $\text{m/s}^2$ )
0.25	0.5
0.50	0.8
0.75	1.3
1.00	1.8

- How should you graph these data so as to determine the mass of the cart from the slope of the line? That is, what values should you graph on the horizontal axis and what on the vertical axis?
- Is there another data point that would be reasonable to add, even though you made no measurements? If so, what is it?
- What is your best determination of the cart's mass?

Problems 43 through 52 describe a situation. For each, draw a motion diagram, a force-identification diagram, and a free-body diagram.

43. I An elevator, suspended by a single cable, has just left the tenth floor and is speeding up as it descends toward the ground floor.
44. II A rocket is being launched straight up. Air resistance is not negligible.
45. I A jet plane is speeding down the runway during takeoff. Air resistance is not negligible.
46. I You've slammed on the brakes and your car is skidding to a stop while going down a  $20^\circ$  hill.
47. II A skier is going down a  $20^\circ$  slope. A *horizontal* headwind is blowing in the skier's face. Friction is small, but not zero.
48. II You've just kicked a rock on the sidewalk and it is now sliding along the concrete.
49. I A Styrofoam ball has just been shot straight up. Air resistance is not negligible.
50. I A spring-loaded gun shoots a plastic ball. The trigger has just been pulled and the ball is starting to move down the barrel. The barrel is horizontal.
51. II A person on a bridge throws a rock straight down toward the water. The rock has just been released.
52. I A gymnast has just landed on a trampoline. She's still moving downward as the trampoline stretches.
53. BIO II The leaf hopper, champion jumper of the insect world, can jump straight up at  $4 \text{ m/s}^2$ . The jump itself lasts a mere 1 ms before the insect is clear of the ground.
  - a. Draw a free-body diagram of this mighty leaper while the jump is taking place.
  - b. While the jump is taking place, is the force of the ground on the leaf hopper greater than, less than, or equal to the force of gravity on the leaf hopper? Explain.

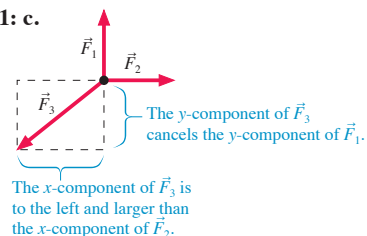
### Challenge Problems

54. A heavy box is in the back of a truck. The truck is accelerating to the right. Draw a motion diagram, a force-identification diagram, and a free-body diagram for the box.

55. A bag of groceries is on the seat of your car as you stop for a stop light. The bag does not slide. Draw a motion diagram, a force-identification diagram, and a free-body diagram for the bag.
56. A rubber ball bounces. We'd like to understand *how* the ball bounces.
  - a. A rubber ball has been dropped and is bouncing off the floor. Draw a motion diagram of the ball during the brief time interval that it is in contact with the floor. Show 4 or 5 frames as the ball compresses, then another 4 or 5 frames as it expands. What is the direction of  $\vec{a}$  during each of these parts of the motion?
  - b. Draw a picture of the ball in contact with the floor and identify all forces acting on the ball.
  - c. Draw a free-body diagram of the ball during its contact with the ground. Is there a net force acting on the ball? If so, in which direction?
  - d. Write a paragraph in which you describe what you learned from parts a to c and in which you answer the question: How does a ball bounce?
57. If a car stops suddenly, you feel "thrown forward." We'd like to understand what happens to the passengers as a car stops. Imagine yourself sitting on a *very* slippery bench inside a car. This bench has no friction, no seat back, and there's nothing for you to hold onto.
  - a. Draw a picture and identify all of the forces acting on you as the car travels at a perfectly steady speed on level ground.
  - b. Draw your free-body diagram. Is there a net force on you? If so, in which direction?
  - c. Repeat parts a and b with the car slowing down.
  - d. Describe what happens to you as the car slows down.
  - e. Use Newton's laws to explain why you seem to be "thrown forward" as the car stops. Is there really a force pushing you forward?
  - f. Suppose now that the bench is not slippery. As the car slows down, you stay on the bench and don't slide off. What force is responsible for your deceleration? In which direction does this force point? Include a free-body diagram as part of your answer.

### STOP TO THINK ANSWERS

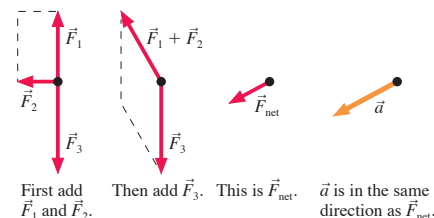
**Stop to Think 5.1: c.**



**Stop to Think 5.2: a, b, and d.** Friction and the normal force are the only contact forces. Nothing is touching the rock to provide a "force of the kick."

**Stop to Think 5.3: b.** Acceleration is proportional to force, so doubling the number of rubber bands doubles the acceleration of the original object from  $2 \text{ m/s}^2$  to  $4 \text{ m/s}^2$ . But acceleration is also inversely proportional to mass. Doubling the mass cuts the acceleration in half, back to  $2 \text{ m/s}^2$ .

**Stop to Think 5.4: d.**



**Stop to Think 5.5: c.** The acceleration vector points downward as the elevator slows.  $\vec{F}_{\text{net}}$  points in the same direction as  $\vec{a}$ , so  $\vec{F}_{\text{net}}$  also points down. This will be true if the tension is less than the gravitational force:  $T < F_G$ .



# SUMMARY

The goal of Chapter 6 has been to learn how to solve linear force-and-motion problems.

## General Strategy

All examples in this chapter follow a four-part strategy. You'll become a better problem solver if you adhere to it as you do the homework problems. The *Dynamics Worksheets* in the *Student Workbook* will help you structure your work in this way.

### Equilibrium Problems

Object at rest or moving with constant velocity.

**MODEL** Make simplifying assumptions.

#### VISUALIZE

- Translate words into symbols.
- Identify forces.
- Draw a free-body diagram.

**SOLVE** Use Newton's first law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{0}$$

“Read” the vectors from the free-body diagram.

**ASSESS** Is the result reasonable?

### Dynamics Problems

Object accelerating.

**MODEL** Make simplifying assumptions.

#### VISUALIZE

- Translate words into symbols.
- Draw a sketch to define the situation.
- Draw a motion diagram.
- Identify forces.
- Draw a free-body diagram.

**SOLVE** Use Newton's second law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

“Read” the vectors from the free-body diagram. Use kinematics to find velocities and positions.

**ASSESS** Is the result reasonable?

Go back and forth  
between these  
steps as needed.

## Important Concepts

Specific information about three important forces:

**Gravity**  $\vec{F}_G = (mg, \text{downward})$

**Friction**  $\vec{f}_s = (0 \text{ to } \mu_s n, \text{direction as necessary to prevent motion})$

$\vec{f}_k = (\mu_k n, \text{direction opposite the motion})$

$\vec{f}_r = (\mu_r n, \text{direction opposite the motion})$

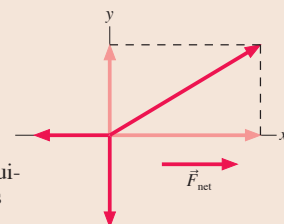
**Drag**  $\vec{D} = (\frac{1}{2} C \rho A v^2, \text{direction opposite the motion})$

Newton's laws are vector expressions. You must write them out by **components**:

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$(F_{\text{net}})_y = \sum F_y = ma_y$$

The acceleration is zero in equilibrium and also along an axis perpendicular to the motion.



## Applications

**Mass** is an intrinsic property of an object that describes the object's inertia and, loosely speaking, its quantity of matter.

The **weight** of an object is the reading of a calibrated spring scale on which the object is stationary. Weight is the result of weighing. An object's weight depends on its mass, its acceleration, and the strength of gravity. An object in free fall is weightless.

A falling object reaches **terminal speed**

$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}}$$



Terminal speed is reached when the drag force exactly balances the gravitational force:  $\vec{a} = 0$ .

## Terms and Notation

flat-earth approximation

weight

coefficient of static friction,  $\mu_s$

coefficient of kinetic friction,  $\mu_k$

rolling friction

coefficient of rolling friction,  $\mu_r$

drag coefficient,  $C$

terminal speed,  $v_{\text{term}}$

# CONCEPTUAL QUESTIONS

- Are the objects described here in static equilibrium, dynamic equilibrium, or not in equilibrium at all? Explain.
  - A 200 pound barbell is held over your head.
  - A girder is lifted at constant speed by a crane.
  - A girder is being lowered into place. It is slowing down.
  - A jet plane has reached its cruising speed and altitude.
  - A box in the back of a truck doesn't slide as the truck stops.
- A ball tossed straight up has  $v = 0$  at its highest point. Is it in equilibrium? Explain.
- Kat, Matt, and Nat are arguing about why a physics book on a table doesn't fall. According to Kat, "Gravity pulls down on it, but the table is in the way so it can't fall." "Nonsense," says Matt. "There are all kinds of forces acting on the book, but the upward forces overcome the downward forces to prevent it from falling." "But what about Newton's first law?" counters Nat. "It's not moving, so there can't be any forces acting on it." None of the statements is exactly correct. Who comes closest, and how would you change his or her statement to make it correct?
- If you know all of the forces acting on a moving object, can you tell the direction the object is moving? If yes, explain how. If no, give an example.
- An elevator, hanging from a single cable, moves upward at constant speed. Friction and air resistance are negligible. Is the tension in the cable greater than, less than, or equal to the gravitational force on the elevator? Explain. Include a free-body diagram as part of your explanation.
- An elevator, hanging from a single cable, moves downward and is slowing. Friction and air resistance are negligible. Is the tension in the cable greater than, less than, or equal to the gravitational force on the elevator? Explain. Include a free-body diagram as part of your explanation.
- Are the following statements true or false? Explain.
  - The mass of an object depends on its location.
  - The weight of an object depends on its location.
  - Mass and weight describe the same thing in different units.
- An astronaut takes his bathroom scale to the moon and then stands on it. Is the reading of the scale his weight? Explain.
- The four balls in **FIGURE Q6.9** have been thrown straight up. They have the same size, but different masses. Air resistance is negligible. Rank in order, from largest to smallest, the magnitude of the net force acting on each ball. Some may be equal. Give your answer in the form  $a > b = c > d$  and explain your ranking.

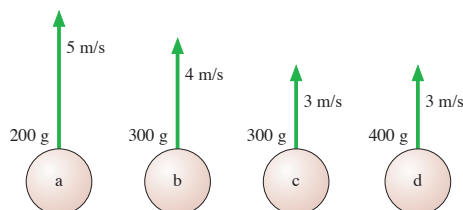


FIGURE Q6.9

- Suppose you attempt to pour out 100 g of salt, using a pan balance for measurements, while in a rocket accelerating upward. Will the quantity of salt be too much, too little, or the correct amount? Explain.

- A box with a passenger inside is launched straight up into the air by a giant rubber band. Before launch, the passenger stood on a scale and weighed 750 N. Once the box has left the rubber band but is still moving upward, is the passenger's weight more than 750 N, 750 N, less than 750 N but not zero, or zero? Explain.
- An astronaut orbiting the earth is handed two balls that have identical outward appearances. However, one is hollow while the other is filled with lead. How can the astronaut determine which is which? Cutting or altering the balls is not allowed.
- A hand presses down on the book in **FIGURE Q6.13**. Is the normal force of the table on the book larger than, smaller than, or equal to  $mg$ ?

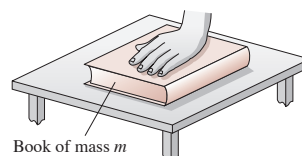


FIGURE Q6.13

- Suppose you push a hockey puck of mass  $m$  across frictionless ice for a time  $\Delta t$ , starting from rest, giving the puck speed  $v$  after traveling distance  $d$ . If you repeat the experiment with a puck of mass  $2m$ ,
  - How long will you have to push for the puck to reach the same speed  $v$ ?
  - How long will you have to push for the puck to travel the same distance  $d$ ?
- A block pushed along the floor with velocity  $v_{0x}$  slides a distance  $d$  after the pushing force is removed.
  - If the mass of the block is doubled but its initial velocity is not changed, what distance does the block slide before stopping?
  - If the initial velocity is doubled to  $2v_{0x}$  but the mass is not changed, what distance does the block slide before stopping?
- Can the friction force on an object ever point in the direction of the object's motion? If yes, give an example. If no, why not?
- A crate of fragile dishes is in the back of a pickup truck. The truck accelerates north from a stop sign, and the crate moves without slipping. Does the friction force on the crate point north or south? Or is the friction force zero? Explain.
- Five balls move through the air as shown in **FIGURE Q6.18**. All five have the same size and shape. Air resistance is not negligible. Rank in order, from largest to smallest, the magnitudes of the accelerations  $a_a$  to  $a_e$ . Some may be equal. Give your answer in the form  $a > b = c > d > e$  and explain your ranking.

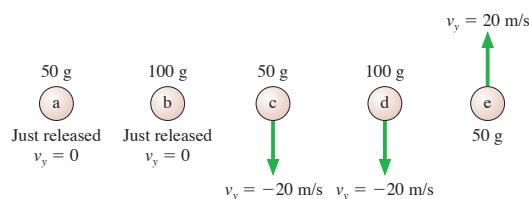


FIGURE Q6.18

# EXERCISES AND PROBLEMS

## Exercises

### Section 6.1 Equilibrium

1. I The three ropes in **FIGURE EX6.1** are tied to a small, very light ring. Two of the ropes are anchored to walls at right angles, and the third rope pulls as shown. What are  $T_1$  and  $T_2$ , the magnitudes of the tension forces in the first two ropes?

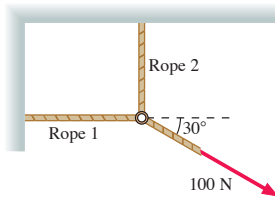


FIGURE EX6.1

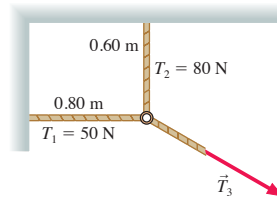


FIGURE EX6.2

2. I The three ropes in **FIGURE EX6.2** are tied to a small, very light ring. Two of these ropes are anchored to walls at right angles with the tensions shown in the figure. What are the magnitude and direction of the tension  $\vec{T}_3$  in the third rope?
3. II A 20 kg loudspeaker is suspended 2.0 m below the ceiling by two 3.0-m-long cables that angle outward at equal angles. What is the tension in the cables?
4. II A football coach sits on a sled while two of his players build their strength by dragging the sled across the field with ropes. The friction force on the sled is 1000 N and the angle between the two ropes is  $20^\circ$ . How hard must each player pull to drag the coach at a steady 2.0 m/s?
5. I A construction worker with a weight of 850 N stands on a roof that is sloped at  $20^\circ$ . What is the magnitude of the normal force of the roof on the worker?

### Section 6.2 Using Newton's Second Law

6. I In each of the two free-body diagrams in **FIGURE EX6.6**, the forces are acting on a 2.0 kg object. For each diagram, find the values of  $a_x$  and  $a_y$ , the  $x$ - and  $y$ -components of the acceleration.

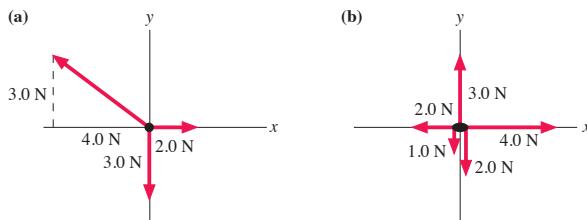


FIGURE EX6.6

7. II In each of the two free-body diagrams in **FIGURE EX6.7**, the forces are acting on a 2.0 kg object. For each diagram, find the values of  $a_x$  and  $a_y$ , the  $x$ - and  $y$ -components of the acceleration.

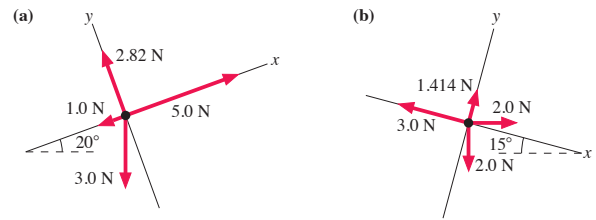


FIGURE EX6.7

8. I **FIGURE EX6.8** shows the velocity graph of a 2.0 kg object as it moves along the  $x$ -axis. What is the net force acting on this object at  $t = 1$  s? At 4 s? At 7 s?

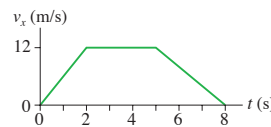


FIGURE EX6.8

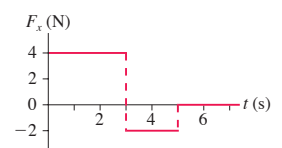


FIGURE EX6.9

9. I **FIGURE EX6.9** shows the force acting on a 2.0 kg object as it moves along the  $x$ -axis. The object is at rest at the origin at  $t = 0$  s. What are its acceleration and velocity at  $t = 6$  s?
10. I A horizontal rope is tied to a 50 kg box on frictionless ice. What is the tension in the rope if:
- The box is at rest?
  - The box moves at a steady 5.0 m/s?
  - The box has  $v_x = 5.0$  m/s and  $a_x = 5.0$  m/s<sup>2</sup>?
11. I A 50 kg box hangs from a rope. What is the tension in the rope if:
- The box is at rest?
  - The box moves up at a steady 5.0 m/s?
  - The box has  $v_y = 5.0$  m/s and is speeding up at 5.0 m/s<sup>2</sup>?
  - The box has  $v_y = 5.0$  m/s and is slowing down at 5.0 m/s<sup>2</sup>?
12. II a. The block in **FIGURE EX6.12** floats on a cushion of air. It is pushed to the right with a force that remains constant as the block moves from 0 to 1. The block
- Speeds up from 0 to 1.
  - Speeds up at first, then has constant speed.
  - Moves with constant speed from 0 to 1.
- b. From 1 to 2, the size of the force steadily decreases until it reaches half of its initial value. The block
- Continues to speed up from 1 to 2.
  - Moves with constant speed from 1 to 2.
  - Slows down.

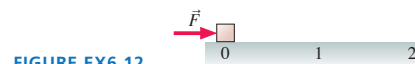


FIGURE EX6.12

### Section 6.3 Mass, Weight, and Gravity

13. I A woman has a mass of 55 kg.
- What is her weight while standing on earth?
  - What are her mass and her weight on Mars, where  $g = 3.76$  m/s<sup>2</sup>?

14. | It takes the elevator in a skyscraper 4.0 s to reach its cruising speed of 10 m/s. A 60 kg passenger gets aboard on the ground floor. What is the passenger's weight
- Before the elevator starts moving?
  - While the elevator is speeding up?
  - After the elevator reaches its cruising speed?
15. || FIGURE EX6.15 shows the velocity graph of a 75 kg passenger in an elevator. What is the passenger's weight at  $t = 1$  s? At 5 s? At 9 s?

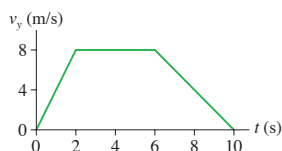


FIGURE EX6.15

16. || What thrust does a 200 g model rocket need in order to have a vertical acceleration of  $10 \text{ m/s}^2$
- On earth?
  - On the moon, where  $g = 1.62 \text{ m/s}^2$ ?

### Section 6.4 Friction

17. || Bonnie and Clyde are sliding a 300 kg bank safe across the floor to their getaway car. The safe slides with a constant speed if Clyde pushes from behind with 385 N of force while Bonnie pulls forward on a rope with 350 N of force. What is the safe's coefficient of kinetic friction on the bank floor?
18. | A stubborn, 120 kg mule sits down and refuses to move. To drag the mule to the barn, the exasperated farmer ties a rope around the mule and pulls with his maximum force of 800 N. The coefficients of friction between the mule and the ground are  $\mu_s = 0.8$  and  $\mu_k = 0.5$ . Is the farmer able to move the mule?
19. || A 10 kg crate is placed on a horizontal conveyor belt. The materials are such that  $\mu_s = 0.5$  and  $\mu_k = 0.3$ .
- Draw a free-body diagram showing all the forces on the crate if the conveyor belt runs at constant speed.
  - Draw a free-body diagram showing all the forces on the crate if the conveyor belt is speeding up.
  - What is the maximum acceleration the belt can have without the crate slipping?
20. | Bob is pulling a 30 kg filing cabinet with a force of 200 N, but the filing cabinet refuses to move. The coefficient of static friction between the filing cabinet and the floor is 0.80. What is the magnitude of the friction force on the filing cabinet?
21. || A 4000 kg truck is parked on a  $15^\circ$  slope. How big is the friction force on the truck? The coefficient of static friction between the tires and the road is 0.90.
22. | A 1500 kg car skids to a halt on a wet road where  $\mu_k = 0.50$ . How fast was the car traveling if it leaves 65-m-long skid marks?
23. || A 50,000 kg locomotive is traveling at 10 m/s when its engine and brakes both fail. How far will the locomotive roll before it comes to a stop? Assume the track is level.

### Section 6.5 Drag

24. || A 75 kg skydiver can be modeled as a rectangular "box" with dimensions  $20 \text{ cm} \times 40 \text{ cm} \times 180 \text{ cm}$ . What is his terminal speed if he falls feet first? Use 0.8 for the drag coefficient.
25. || A 6.5-cm-diameter tennis ball has a terminal speed of 26 m/s. What is the ball's mass?

### Problems

26. || A 5.0 kg object initially at rest at the origin is subjected to the time-varying force shown in FIGURE P6.26. What is the object's velocity at  $t = 6$  s?



FIGURE P6.26

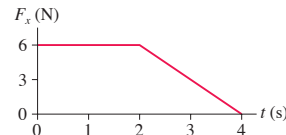


FIGURE P6.27

27. || A 2.0 kg object initially at rest at the origin is subjected to the time-varying force shown in FIGURE P6.27. What is the object's velocity at  $t = 4$  s?
28. || The 1000 kg steel beam in FIGURE P6.28 is supported by two ropes. What is the tension in each?

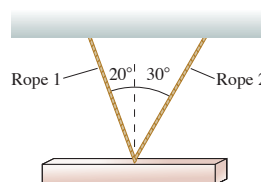


FIGURE P6.28

29. || In an electricity experiment, a 1.0 g plastic ball is suspended on a 60-cm-long string and given an electric charge. A charged rod brought near the ball exerts a horizontal electrical force  $\vec{F}_{\text{elec}}$  on it, causing the ball to swing out to a  $20^\circ$  angle and remain there.
- What is the magnitude of  $\vec{F}_{\text{elec}}$ ?
  - What is the tension in the string?
30. | A 500 kg piano is being lowered into position by a crane while two people steady it with ropes pulling to the sides. Bob's rope pulls to the left,  $15^\circ$  below horizontal, with 500 N of tension. Ellen's rope pulls toward the right,  $25^\circ$  below horizontal.
- What tension must Ellen maintain in her rope to keep the piano descending at a steady speed?
  - What is the tension in the main cable supporting the piano?
31. || Henry gets into an elevator on the 50th floor of a building and it begins moving at  $t = 0$  s. FIGURE P6.31 shows his weight over the next 12 s.
- Is the elevator's initial direction up or down? Explain how you can tell.
  - What is Henry's mass?
  - How far has Henry traveled at  $t = 12$  s?

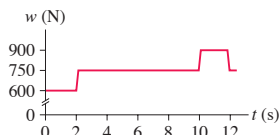


FIGURE P6.31

32. | Zach, whose mass is 80 kg, is in an elevator descending at 10 m/s. The elevator takes 3.0 s to brake to a stop at the first floor.
- What is Zach's weight before the elevator starts braking?
  - What is Zach's weight while the elevator is braking?
33. || BIO An accident victim with a broken leg is being placed in traction. The patient wears a special boot with a pulley attached to the sole. The foot and boot together have a mass of 4.0 kg, and the

doctor has decided to hang a 6.0 kg mass from the rope. The boot is held suspended by the ropes, as shown in FIGURE P6.33, and does not touch the bed.

- Determine the amount of tension in the rope by using Newton's laws to analyze the hanging mass.
- The net traction force needs to pull straight out on the leg. What is the proper angle  $\theta$  for the upper rope?
- What is the net traction force pulling on the leg?

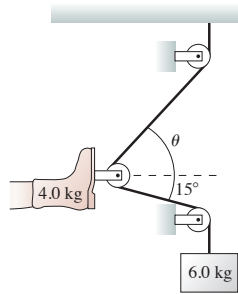


FIGURE P6.33

**Hint:** If the pulleys are frictionless, which we will assume, the tension in the rope is constant from one end to the other.

34. **BIO** **|** Seat belts and air bags save lives by reducing the forces exerted on the driver and passengers in an automobile collision. Cars are designed with a “crumple zone” in the front of the car. In the event of an impact, the passenger compartment decelerates over a distance of about 1 m as the front of the car crumples. An occupant restrained by seat belts and air bags decelerates with the car. By contrast, an unrestrained occupant keeps moving forward with no loss of speed (Newton's first law!) until hitting the dashboard or windshield. These are unyielding surfaces, and the unfortunate occupant then decelerates over a distance of only about 5 mm.
- A 60 kg person is in a head-on collision. The car's speed at impact is 15 m/s. Estimate the net force on the person if he or she is wearing a seat belt and if the air bag deploys.
  - Estimate the net force that ultimately stops the person if he or she is not restrained by a seat belt or air bag.
  - How do these two forces compare to the person's weight?
35. **|** The position of a 2.0 kg mass is given by  $x = (2t^3 - 3t^2)$  m, where  $t$  is in seconds. What is the net horizontal force on the mass at (a)  $t = 0$  s and (b)  $t = 1$  s?
36. **|** The piston of a machine exerts a constant force on a ball as it moves horizontally through a distance of 15 cm. You use a motion detector to measure the speed of five different balls as they come off the piston; the data are shown below. Use theory to find two quantities that, when graphed, should give a straight line. Then use the graph to find the size of the piston's force.

Mass (g)	Speed (m/s)
200	9.4
400	6.3
600	5.2
800	4.9
1000	4.0

37. **|** Compressed air is used to fire a 50 g ball vertically upward from a 1.0-m-tall tube. The air exerts an upward force of 2.0 N on the ball as long as it is in the tube. How high does the ball go above the top of the tube?
38. **|** a. A rocket of mass  $m$  is launched straight up with thrust  $\vec{F}_{\text{thrust}}$ . Find an expression for the rocket's speed at height  $h$  if air resistance is neglected.
- The motor of a 350 g model rocket generates 9.5 N thrust. If air resistance can be neglected, what will be the rocket's speed as it reaches a height of 85 m?
39. **|** A rifle with a barrel length of 60 cm fires a 10 g bullet with a horizontal speed of 400 m/s. The bullet strikes a block of wood and penetrates to a depth of 12 cm.
- What resistive force (assumed to be constant) does the wood exert on the bullet?
  - How long does it take the bullet to come to rest?
  - Draw a velocity-versus-time graph for the bullet in the wood.
40. **|** A 20,000 kg rocket has a rocket motor that generates  $3.0 \times 10^5$  N of thrust.
- What is the rocket's initial upward acceleration?
  - At an altitude of 5000 m the rocket's acceleration has increased to  $6.0 \text{ m/s}^2$ . What mass of fuel has it burned?
41. **|** a. An object of mass  $m$  is at rest at the top of a smooth slope of height  $h$  and length  $L$ . The coefficient of kinetic friction between the object and the surface,  $\mu_k$ , is small enough that the object will slide down the slope if given a very small push to get it started. Find an expression for the object's speed at the bottom of the slope.
- Sam, whose mass is 75 kg, stands at the top of a 12-m-high, 100-m-long snow-covered slope. His skis have a coefficient of kinetic friction on snow of 0.07. If he uses his poles to get started, then glides down, what is his speed at the bottom?
42. **|** Sam, whose mass is 75 kg, takes off across level snow on his jet-powered skis. The skis have a thrust of 200 N and a coefficient of kinetic friction on snow of 0.10. Unfortunately, the skis run out of fuel after only 10 s.
- What is Sam's top speed?
  - How far has Sam traveled when he finally coasts to a stop?
43. **|** Sam, whose mass is 75 kg, takes off down a 50-m-high,  $10^\circ$  slope on his jet-powered skis. The skis have a thrust of 200 N. Sam's speed at the bottom is 40 m/s. What is the coefficient of kinetic friction of his skis on snow?
44. **|** A baggage handler drops your 10 kg suitcase onto a conveyor belt running at 2.0 m/s. The materials are such that  $\mu_s = 0.50$  and  $\mu_k = 0.30$ . How far is your suitcase dragged before it is riding smoothly on the belt?
45. **|** You and your friend Peter are putting new shingles on a roof pitched at  $25^\circ$ . You're sitting on the very top of the roof when Peter, who is at the edge of the roof directly below you, 5.0 m away, asks you for the box of nails. Rather than carry the 2.5 kg box of nails down to Peter, you decide to give the box a push and have it slide down to him. If the coefficient of kinetic friction between the box and the roof is 0.55, with what speed should you push the box to have it gently come to rest right at the edge of the roof?
46. **|** It's moving day, and you need to push a 100 kg box up a  $20^\circ$  ramp into the truck. The coefficients of friction for the box on the ramp are  $\mu_s = 0.90$  and  $\mu_k = 0.60$ . Your largest pushing force is 1000 N. Can you get the box into the truck without assistance if you get a running start at the ramp? If you stop on the ramp, will you be able to get the box moving again?
47. **|** An Airbus A320 jetliner has a takeoff mass of 75,000 kg. It reaches its takeoff speed of 82 m/s (180 mph) in 35 s. What is the thrust of the engines? You can neglect air resistance but not rolling friction.
48. **|** A 2.0 kg wood block is launched up a wooden ramp that is inclined at a  $30^\circ$  angle. The block's initial speed is 10 m/s.
- What vertical height does the block reach above its starting point?
  - What speed does it have when it slides back down to its starting point?



49. || It's a snowy day and you're pulling a friend along a level road on a sled. You've both been taking physics, so she asks what you think the coefficient of friction between the sled and the snow is. You've been walking at a steady 1.5 m/s, and the rope pulls up on the sled at a  $30^\circ$  angle. You estimate that the mass of the sled, with your friend on it, is 60 kg and that you're pulling with a force of 75 N. What answer will you give?
50. || a. A large box of mass  $M$  is pulled across a horizontal, frictionless surface by a horizontal rope with tension  $T$ . A small box of mass  $m$  sits on top of the large box. The coefficients of static and kinetic friction between the two boxes are  $\mu_s$  and  $\mu_k$ , respectively. Find an expression for the maximum tension  $T_{\max}$  for which the small box rides on top of the large box without slipping.
- b. A horizontal rope pulls a 10 kg wood sled across frictionless snow. A 5.0 kg wood box rides on the sled. What is the largest tension force for which the box doesn't slip?
51. || a. A large box of mass  $M$  is moving on a horizontal surface at speed  $v_0$ . A small box of mass  $m$  sits on top of the large box. The coefficients of static and kinetic friction between the two boxes are  $\mu_s$  and  $\mu_k$ , respectively. Find an expression for the shortest distance  $d_{\min}$  in which the large box can stop without the small box slipping.
- b. A pickup truck with a steel bed is carrying a steel file cabinet. If the truck's speed is 15 m/s, what is the shortest distance in which it can stop without the file cabinet sliding?
52. || Your assignment in lab is to measure the coefficient of kinetic friction between a 350 g block and a smooth metal table. To do so, you decide to launch the block at various speeds and measure how far it slides; your data are listed in the table. Use a graph to determine the value of  $\mu_k$ .

Speed (m/s)	Distance (cm)
0.5	5
1.0	24
1.5	41
2.0	83
2.5	130

53. || You're driving along at 25 m/s with your aunt's valuable antiques in the back of your pickup truck when suddenly you see a giant hole in the road 55 m ahead of you. Fortunately, your foot is right beside the brake and your reaction time is zero! Will the antiques be as fortunate?
- a. Can you stop the truck before it falls into the hole?
- b. If your answer to part a is yes, can you stop without the antiques sliding and being damaged? Their coefficients of friction are  $\mu_s = 0.60$  and  $\mu_k = 0.30$ .

**Hint:** You're not trying to stop in the shortest possible distance. What's your best strategy for avoiding damage to the antiques?

54. || The 2.0 kg wood box in FIGURE P6.54 slides down a vertical wood wall while you push on it at a  $45^\circ$  angle. What magnitude of force should you apply to cause the box to slide down at a constant speed?

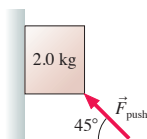


FIGURE P6.54

55. || A 1.0 kg wood block is pressed against a vertical wood wall by the 12 N force shown in FIGURE P6.55. If the block is initially at rest, will it move upward, move downward, or stay at rest?

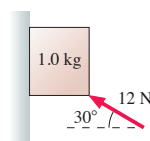


FIGURE P6.55

56. || BIO A person with compromised pinch strength in his fingers can exert a force of only 6.0 N to either side of a pinch-held object, such as the book shown in FIGURE P6.56. What is the heaviest book he can hold vertically before it slips out of his fingers? The coefficient of static friction between his fingers and the book cover is 0.80.

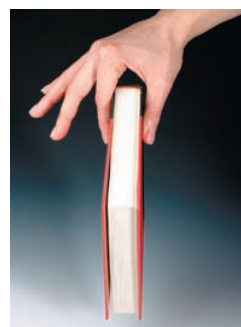


FIGURE P6.56

57. || What is the terminal speed for an 80 kg skier going down a  $40^\circ$  snow-covered slope on wooden skis? Assume that the skier is 1.8 m tall and 0.40 m wide.
58. || A ball is shot from a compressed-air gun at twice its terminal speed.
- a. What is the ball's initial acceleration, as a multiple of  $g$ , if it is shot straight up?
- b. What is the ball's initial acceleration, as a multiple of  $g$ , if it is shot straight down?
59. || An artist friend of yours needs help hanging a 500 lb sculpture from the ceiling. For artistic reasons, she wants to use just two ropes. One will be  $30^\circ$  from vertical, the other  $60^\circ$ . She needs you to determine the smallest diameter rope that can safely support this expensive piece of art. On a visit to the hardware store you find that rope is sold in increments of  $\frac{1}{8}$ -inch diameter and that the safety rating is 4000 pounds per square inch of cross section. What diameter rope should you buy?
60. || You've entered a "slow ski race" where the winner is the skier who takes the *longest* time to go down a  $15^\circ$  slope without ever stopping. You need to choose the best wax to apply to your skis. Red wax has a coefficient of kinetic friction 0.25, yellow is 0.20, green is 0.15, and blue is 0.10. Having just finished taking physics, you realize that a wax too slippery will cause you to accelerate down the slope and lose the race. But a wax that's too sticky will cause you to stop and be disqualified. You know that a strong headwind will apply a 50 N horizontal force against you as you ski, and you know that your mass is 82 kg. Which wax do you choose?
61. || Astronauts in space "weigh" themselves by oscillating on a spring. Suppose the position of an oscillating 75 kg astronaut is given by  $x = (0.30 \text{ m})\sin((\pi \text{ rad/s}) \cdot t)$ , where  $t$  is in s. What force does the spring exert on the astronaut at (a)  $t = 1.0 \text{ s}$  and (b)  $1.5 \text{ s}$ ? Note that the angle of the sine function is in radians.
62. || A particle of mass  $m$  moving along the  $x$ -axis experiences the net force  $F_x = ct$ , where  $c$  is a constant. The particle has velocity  $v_{0x}$  at  $t = 0$ . Find an algebraic expression for the particle's velocity  $v_x$  at a later time  $t$ .



63. || At  $t = 0$ , an object of mass  $m$  is at rest at  $x = 0$  on a horizontal, frictionless surface. A horizontal force  $F_x = F_0(1 - t/T)$ , which decreases from  $F_0$  at  $t = 0$  to zero at  $t = T$ , is exerted on the object. Find an expression for the object's (a) velocity and (b) position at time  $T$ .
64. || At  $t = 0$ , an object of mass  $m$  is at rest at  $x = 0$  on a horizontal, frictionless surface. Starting at  $t = 0$ , a horizontal force  $F_x = F_0 e^{-t/T}$  is exerted on the object.
- Find and graph an expression for the object's velocity at an arbitrary later time  $t$ .
  - What is the object's velocity after a very long time has elapsed?
65. || Large objects have inertia and tend to keep moving—  
 BIO Newton's first law. Life is very different for small microorganisms that swim through water. For them, drag forces are so large that they instantly stop, without coasting, if they cease their swimming motion. To swim at constant speed, they must exert a constant propulsion force by rotating corkscrew-like flagella or beating hair-like cilia. The quadratic model of drag of Equation 6.16 fails for very small particles. Instead, a small object moving in a liquid experiences a *linear* drag force,  $\vec{D} = (bv, \text{direction opposite the motion})$ , where  $b$  is a constant. For a sphere of radius  $R$ , the drag constant can be shown to be  $b = 6\pi\eta R$ , where  $\eta$  is the *viscosity* of the liquid. Water at  $20^\circ\text{C}$  has viscosity  $1.0 \times 10^{-3} \text{ N s/m}^2$ .
- A *paramecium* is about  $100 \mu\text{m}$  long. If it's modeled as a sphere, how much propulsion force must it exert to swim at a typical speed of  $1.0 \text{ mm/s}$ ? How about the propulsion force of a  $2.0\text{-}\mu\text{m}$ -diameter *E. coli* bacterium swimming at  $30 \mu\text{m/s}$ ?
  - The propulsion forces are very small, but so are the organisms. To judge whether the propulsion force is large or small *relative to the organism*, compute the acceleration that the propulsion force could give each organism if there were no drag. The density of both organisms is the same as that of water,  $1000 \text{ kg/m}^3$ .
66. || Very small objects, such as dust particles, experience a *linear* drag force,  $\vec{D} = (bv, \text{direction opposite the motion})$ , where  $b$  is a constant. That is, the quadratic model of drag of Equation 6.16 fails for very small particles. For a sphere of radius  $R$ , the drag constant can be shown to be  $b = 6\pi\eta R$ , where  $\eta$  is the *viscosity* of the gas.
- Find an expression for the terminal speed  $v_{\text{term}}$  of a spherical particle of radius  $R$  and mass  $m$  falling through a gas of viscosity  $\eta$ .
  - Suppose a gust of wind has carried a  $50\text{-}\mu\text{m}$ -diameter dust particle to a height of  $300 \text{ m}$ . If the wind suddenly stops, how long will it take the dust particle to settle back to the ground? Dust has a density of  $2700 \text{ kg/m}^3$ , the viscosity of  $25^\circ\text{C}$  air is  $2.0 \times 10^{-5} \text{ N s/m}^2$ , and you can assume that the falling dust particle reaches terminal speed almost instantly.

Problems 67 and 68 show a free-body diagram. For each:

- Write a realistic dynamics problem for which this is the correct free-body diagram. Your problem should ask a question that can be answered with a value of position or velocity (such as "How far?" or "How fast?"), and should give sufficient information to allow a solution.
- Solve your problem!

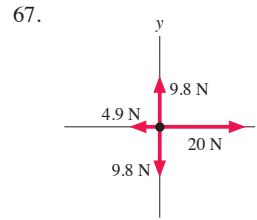


FIGURE P6.67

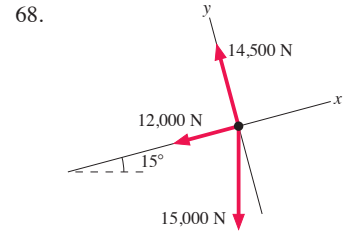


FIGURE P6.68

In Problems 69 through 71 you are given the dynamics equations that are used to solve a problem. For each of these, you are to

- Write a realistic problem for which these are the correct equations.
  - Draw the free-body diagram and the pictorial representation for your problem.
  - Finish the solution of the problem.
69.  $-0.80n = (1500 \text{ kg})a_x$   
 $n - (1500 \text{ kg})(9.80 \text{ m/s}^2) = 0$
70.  $T - 0.20n - (20 \text{ kg})(9.80 \text{ m/s}^2) \sin 20^\circ = (20 \text{ kg})(2.0 \text{ m/s}^2)$   
 $n - (20 \text{ kg})(9.80 \text{ m/s}^2) \cos 20^\circ = 0$
71.  $(100 \text{ N}) \cos 30^\circ - f_k = (20 \text{ kg})a_x$   
 $n + (100 \text{ N}) \sin 30^\circ - (20 \text{ kg})(9.80 \text{ m/s}^2) = 0$   
 $f_k = 0.20n$

### Challenge Problems

72. A block of mass  $m$  is at rest at the origin at  $t = 0$ . It is pushed with constant force  $F_0$  from  $x = 0$  to  $x = L$  across a horizontal surface whose coefficient of kinetic friction is  $\mu_k = \mu_0(1 - x/L)$ . That is, the coefficient of friction decreases from  $\mu_0$  at  $x = 0$  to zero at  $x = L$ .
- Use what you've learned in calculus to prove that

$$a_x = v_x \frac{dv_x}{dx}$$

- Find an expression for the block's speed as it reaches position  $L$ .

73. The machine in **FIGURE CP6.73** has an  $800 \text{ g}$  steel shuttle that is pulled along a square steel rail by an elastic cord. The shuttle is released when the elastic cord has  $20 \text{ N}$  tension at a  $45^\circ$  angle. What is the initial acceleration of the shuttle?

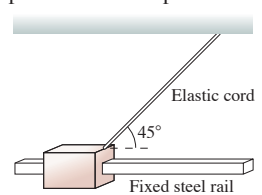


FIGURE CP6.73

74. **FIGURE CP6.74** shows an *accelerometer*, a device for measuring the horizontal acceleration of cars and airplanes. A ball is free to roll on a parabolic track described by the equation  $y = x^2$ , where both  $x$  and  $y$  are in meters. A scale along the bottom is used to measure the ball's horizontal position  $x$ .

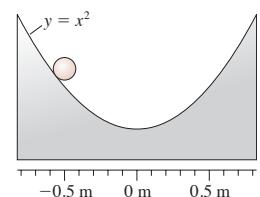


FIGURE CP6.74

- Find an expression that allows you to use a measured position  $x$  (in m) to compute the acceleration  $a_x$  (in  $\text{m/s}^2$ ). (For example,  $a_x = 3x$  is a possible expression.)
- What is the acceleration if  $x = 20 \text{ cm}$ ?

75. An object moving in a liquid experiences a *linear* drag force:  $\vec{D} = (bv, \text{direction opposite the motion})$ , where  $b$  is a constant called the *drag coefficient*. For a sphere of radius  $R$ , the drag constant can be computed as  $b = 6\pi\eta R$ , where  $\eta$  is the *viscosity* of the liquid.

- Find an algebraic expression for  $v_x(t)$ , the  $x$ -component of velocity as a function of time, for a spherical particle of radius  $R$  and mass  $m$  that is shot horizontally with initial speed  $v_0$  through a liquid of viscosity  $\eta$ .
- Water at  $20^\circ\text{C}$  has viscosity  $\eta = 1.0 \times 10^{-3} \text{ N s/m}^2$ . Suppose a 4.0-cm-diameter, 33 g ball is shot horizontally into a tank of  $20^\circ\text{C}$  water. How long will it take for the horizontal speed to decrease to 50% of its initial value?

76. An object moving in a liquid experiences a *linear* drag force:  $\vec{D} = (bv, \text{direction opposite the motion})$ , where  $b$  is a constant called the *drag coefficient*. For a sphere of radius  $R$ , the drag constant can be computed as  $b = 6\pi\eta R$ , where  $\eta$  is the *viscosity* of the liquid.

- Use what you've learned in calculus to prove that

$$a_x = v_x \frac{dv_x}{dx}$$

b. Find an algebraic expression for  $v_x(x)$ , the  $x$ -component of velocity as a function of distance traveled, for a spherical particle of radius  $R$  and mass  $m$  that is shot horizontally with initial speed  $v_0$  through a liquid of viscosity  $\eta$ .

- Water at  $20^\circ\text{C}$  has viscosity  $\eta = 1.0 \times 10^{-3} \text{ N s/m}^2$ . Suppose a 1.0-cm-diameter, 1.0 g marble is shot horizontally into a tank of  $20^\circ\text{C}$  water at 10 cm/s. How far will it travel before stopping?

77. An object with cross section  $A$  is shot horizontally across frictionless ice. Its initial velocity is  $v_{0x}$  at  $t_0 = 0 \text{ s}$ . Air resistance is not negligible.

- Show that the velocity at time  $t$  is given by the expression

$$v_x = \frac{v_{0x}}{1 + C\rho A v_{0x} t / 2m}$$

- A 1.6-m-wide, 1.4-m-high, 1500 kg car with a drag coefficient of 0.35 hits a very slick patch of ice while going 20 m/s. If friction is neglected, how long will it take until the car's speed drops to 10 m/s? To 5 m/s?

c. Assess whether or not it is reasonable to neglect kinetic friction.

### STOP TO THINK ANSWERS

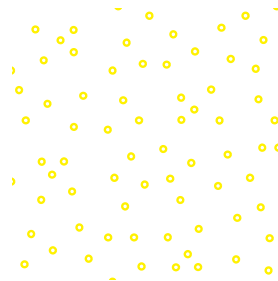
**Stop to Think 6.1:** a. The lander is descending and slowing. The acceleration vector points upward, and so  $\vec{F}_{\text{net}}$  points upward. This can be true only if the thrust has a larger magnitude than the weight.

**Stop to Think 6.2:** a. You are descending and slowing, so your acceleration vector points upward and there is a net upward force on you. The floor pushes up against your feet harder than gravity pulls down.

**Stop to Think 6.3:**  $f_b > f_c = f_d = f_e > f_a$ . Situations c, d, and e are all kinetic friction, which does not depend on either velocity or

acceleration. Kinetic friction is smaller than the maximum static friction that is exerted in b.  $f_a = 0$  because no friction is needed to keep the object at rest.

**Stop to Think 6.4:** d. The ball is shot *down* at 30 m/s, so  $v_{0y} = -30 \text{ m/s}$ . This exceeds the terminal speed, so the upward drag force is *larger* than the downward weight force. Thus the ball *slows down* even though it is “falling.” It will slow until  $v_y = -15 \text{ m/s}$ , the terminal velocity, then maintain that velocity.



# SUMMARY

The goal of Chapter 7 has been to use Newton's third law to understand how objects interact.

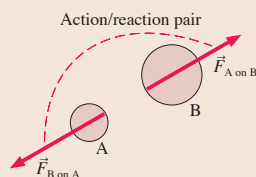
## General Principles

### Newton's Third Law

Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:

- Act on two *different* objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$



### Solving Interacting-Objects Problems

**MODEL** Choose the objects of interest.

#### VISUALIZE

- Draw a pictorial representation.
- Sketch and define coordinates.
- Identify acceleration constraints.
- Draw an interaction diagram.
- Draw a separate free-body diagram for each object.
- Connect action/reaction pairs with dashed lines.

**SOLVE** Write Newton's second law for each object.

- Include *all* forces acting *on* each object.
- Use Newton's third law to equate the magnitudes of action/reaction pairs.
- Include acceleration constraints and friction.

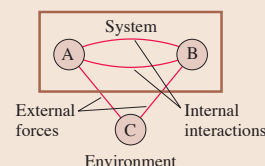
**ASSESS** Is the result reasonable?

## Important Concepts

### Objects, systems, and the environment

Objects whose motion is of interest are the system. Objects whose motion is not of interest form the environment. The objects of interest interact with the environment, but those interactions can be considered external forces.

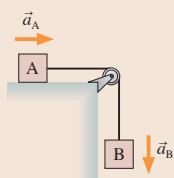
### Interaction diagram



## Applications

### Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude:  $a_A = a_B$ . This must be expressed in terms of components, such as  $a_{Ax} = -a_{By}$ .

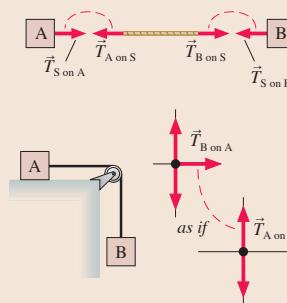


### Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium

Objects connected by massless strings passing over massless, frictionless pulleys act *as if* they interact via an action/reaction pair of forces.



## Terms and Notation

interaction  
action/reaction pair  
system

environment  
interaction diagram  
external force

propulsion  
Newton's third law

acceleration constraint  
massless string approximation

# CONCEPTUAL QUESTIONS

1. You find yourself in the middle of a frozen lake with a surface so slippery ( $\mu_s = \mu_k = 0$ ) you cannot walk. However, you happen to have several rocks in your pocket. The ice is extremely hard. It cannot be chipped, and the rocks slip on it just as much as your feet do. Can you think of a way to get to shore? Use pictures, forces, and Newton's laws to explain your reasoning.
2. How do you paddle a canoe in the forward direction? Explain. Your explanation should include diagrams showing forces on the water and forces on the paddle.
3. How does a rocket take off? What is the upward force on it? Your explanation should include diagrams showing forces on the rocket and forces on the parcel of hot gas that was just expelled from the rocket's exhaust.
4. How do basketball players jump straight up into the air? Your explanation should include pictures showing forces on the player and forces on the ground.
5. A mosquito collides head-on with a car traveling 60 mph. Is the force of the mosquito on the car larger than, smaller than, or equal to the force of the car on the mosquito? Explain.
6. A mosquito collides head-on with a car traveling 60 mph. Is the magnitude of the mosquito's acceleration larger than, smaller than, or equal to the magnitude of the car's acceleration? Explain.
7. A small car is pushing a large truck. They are speeding up. Is the force of the truck on the car larger than, smaller than, or equal to the force of the car on the truck?
8. A very smart 3-year-old child is given a wagon for her birthday. She refuses to use it. "After all," she says, "Newton's third law says that no matter how hard I pull, the wagon will exert an equal but opposite force on me. So I will never be able to get it to move forward." What would you say to her in reply?
9. Teams red and blue are having a tug-of-war. According to Newton's third law, the force with which the red team pulls on the blue team exactly equals the force with which the blue team pulls on the red team. How can one team ever win? Explain.
10. Will hanging a magnet in front of the iron cart in **FIGURE Q7.10** make it go? Explain.

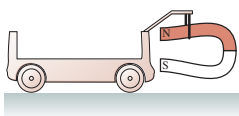


FIGURE Q7.10

11. **FIGURE Q7.11** shows two masses at rest. The string is massless and the pulley is frictionless. The spring scale reads in kg. What is the reading of the scale?

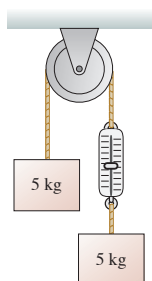


FIGURE Q7.11

12. **FIGURE Q7.12** shows two masses at rest. The string is massless and the pulley is frictionless. The spring scale reads in kg. What is the reading of the scale?

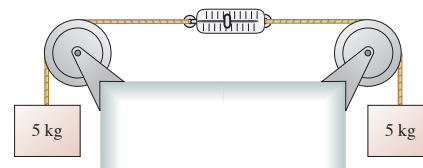


FIGURE Q7.12

13. The hand in **FIGURE Q7.13** is pushing on the back of block A. Blocks A and B, with  $m_B > m_A$ , are connected by a massless string and slide on a frictionless surface. Is the force of the string on B larger than, smaller than, or equal to the force of the hand on A? Explain.

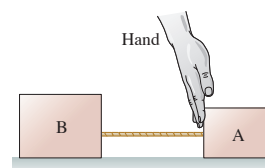


FIGURE Q7.13

14. Blocks A and B in **FIGURE Q7.14** are connected by a massless string over a massless, frictionless pulley. The blocks have just been released from rest. Will the pulley rotate clockwise, counterclockwise, or not at all? Explain.

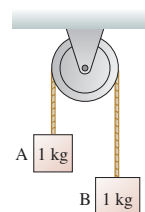


FIGURE Q7.14

15. In case a in **FIGURE Q7.15**, block A is accelerated across a frictionless table by a hanging 10 N weight (1.02 kg). In case b, block A is accelerated across a frictionless table by a steady 10 N tension in the string. The string is massless, and the pulley is massless and frictionless. Is A's acceleration in case b greater than, less than, or equal to its acceleration in case a? Explain.

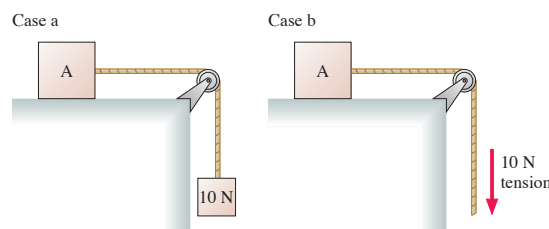


FIGURE Q7.15

# EXERCISES AND PROBLEMS

## Exercises

### Section 7.2 Analyzing Interacting Objects

Exercises 1 through 6 describe a situation. For each:

- Draw an interaction diagram, following the steps of Tactics Box 7.1.
  - Identify the “system” on your interaction diagram.
  - Draw a free-body diagram for each object in the system. Use dashed lines to connect the members of an action/reaction pair.
- A weightlifter stands up at constant speed from a squatting position while holding a heavy barbell across his shoulders.
  - A soccer ball and a bowling ball have a head-on collision at this instant. Rolling friction is negligible.
  - A mountain climber is using a rope to pull a bag of supplies up a  $45^\circ$  slope. The rope is not massless.
  - A battery-powered toy car pushes a stuffed rabbit across the floor.
  - Block A in **FIGURE EX7.5** is heavier than block B and is sliding down the incline. All surfaces have friction. The rope is massless, and the massless pulley turns on frictionless bearings. The rope and the pulley are among the interacting objects, but you'll have to decide if they're part of the system.

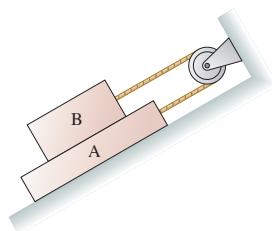


FIGURE EX7.5

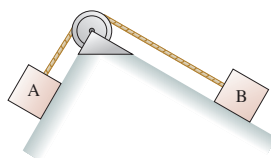


FIGURE EX7.6

- Block A in **FIGURE EX7.6** is sliding down the incline. The rope is massless, and the massless pulley turns on frictionless bearings, but the surface is not frictionless. The rope and the pulley are among the interacting objects, but you'll have to decide if they're part of the system.

### Section 7.3 Newton's Third Law

- How much force does an 80 kg astronaut exert on his chair while sitting at rest on the launch pad?
  - How much force does the astronaut exert on his chair while accelerating straight up at  $10 \text{ m/s}^2$ ?
- Block B in **FIGURE EX7.8** rests on a surface for which the static and kinetic coefficients of friction are 0.60 and 0.40, respectively. The ropes are massless. What is the maximum mass of block A for which the system is in equilibrium?

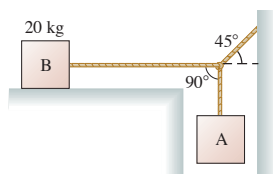


FIGURE EX7.8

- A 1000 kg car pushes a 2000 kg truck that has a dead battery. When the driver steps on the accelerator, the drive wheels of the car push against the ground with a force of 4500 N. Rolling friction can be neglected.
  - What is the magnitude of the force of the car on the truck?
  - What is the magnitude of the force of the truck on the car?
- Blocks with masses of 1 kg, 2 kg, and 3 kg are lined up in a row on a frictionless table. All three are pushed forward by a 12 N force applied to the 1 kg block.
  - How much force does the 2 kg block exert on the 3 kg block?
  - How much force does the 2 kg block exert on the 1 kg block?
- A massive steel cable drags a 20 kg block across a horizontal, frictionless surface. A 100 N force applied to the cable causes the block to reach a speed of 4.0 m/s in a distance of 2.0 m. What is the mass of the cable?

### Section 7.4 Ropes and Pulleys

- What is the tension in the rope of **FIGURE EX7.12**?

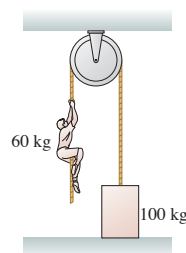
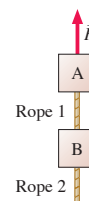


FIGURE EX7.12

- FIGURE EX7.13** shows two 1.0 kg blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upward at  $3.0 \text{ m/s}^2$  by force  $\vec{F}$ .
  - What is  $F$ ?
  - What is the tension at the top end of rope 1?
  - What is the tension at the bottom end of rope 1?
  - What is the tension at the top end of rope 2?
- Jimmy has caught two fish in Yellow Creek. He has tied the line holding the 3.0 kg steelhead trout to the tail of the 1.5 kg carp. To show the fish to a friend, he lifts upward on the carp with a force of 60 N.
  - Draw separate free-body diagrams for the trout and the carp. Label all forces, then use dashed lines to connect action/reaction pairs or forces that act as if they are a pair.
  - Rank in order, from largest to smallest, the magnitudes of all the forces shown on your free-body diagrams. Explain your reasoning.
- A 2.0-m-long, 500 g rope pulls a 10 kg block of ice across a horizontal, frictionless surface. The block accelerates at  $2.0 \text{ m/s}^2$ . How much force pulls forward on (a) the ice, (b) the rope?
- The cable cars in San Francisco are pulled along their tracks by an underground steel cable that moves along at 9.5 mph. The cable is driven by large motors at a central power station and extends, via an intricate pulley arrangement, for several miles beneath the city streets. The length of a cable stretches by up



to 100 ft during its lifetime. To keep the tension constant, the cable passes around a 1.5-m-diameter “tensioning pulley” that rolls back and forth on rails, as shown in **FIGURE EX7.16**. A 2000 kg block is attached to the tensioning pulley’s cart, via a rope and pulley, and is suspended in a deep hole. What is the tension in the cable car’s cable?

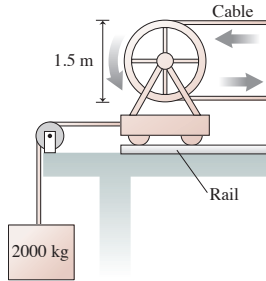


FIGURE EX7.16

17. || A 2.0 kg rope hangs from the ceiling. What is the tension at the midpoint of the rope?
18. || A mobile at the art museum has a 2.0 kg steel cat and a 4.0 kg steel dog suspended from a lightweight cable, as shown in **FIGURE EX7.18**. It is found that  $\theta_1 = 20^\circ$  when the center rope is adjusted to be perfectly horizontal. What are the tension and the angle of rope 3?

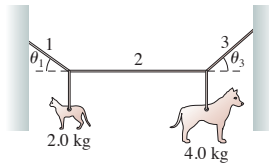


FIGURE EX7.18

### Problems

19. || **FIGURE P7.19** shows two strong magnets on opposite sides of a small table. The long-range attractive force between the magnets keeps the lower magnet in place.
- Draw an interaction diagram and draw free-body diagrams for both magnets and the table. Use dashed lines to connect the members of an action/reaction pair.
  - Suppose the weight of the table is 20 N, the weight of each magnet is 2.0 N, and the magnetic force on the lower magnet is three times its weight. Find the magnitude of each of the forces shown on your free-body diagrams.
20. || An 80 kg spacewalking astronaut pushes off a 640 kg satellite, exerting a 100 N force for the 0.50 s it takes him to straighten his arms. How far apart are the astronaut and the satellite after 1.0 min?
21. || A massive steel cable drags a 20 kg block across a horizontal, frictionless surface. A 100 N force applied to the cable causes the block to reach a speed of 4.0 m/s in 2.0 s. What is the difference in tension between the two ends of the cable?
22. || **FIGURE P7.22** shows a 6.0 N force pushing two gliders along an air track. The 200 g spring between the gliders is compressed. How much force does the spring exert on (a) glider A and (b) glider B?

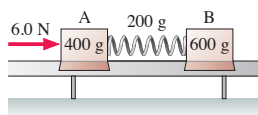


FIGURE P7.22

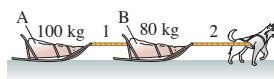


FIGURE P7.23

23. || The sled dog in **FIGURE P7.23** drags sleds A and B across the snow. The coefficient of friction between the sleds and the snow is 0.10. If the tension in rope 1 is 150 N, what is the tension in rope 2?
24. || A rope of length  $L$  and mass  $m$  is suspended from the ceiling. Find an expression for the tension in the rope at position  $y$ , measured upward from the free end of the rope.
25. || While driving to work last year, I was holding my coffee mug in my left hand while changing the CD with my right hand. Then the cell phone rang, so I placed the mug on the flat part of my dashboard. Then, believe it or not, a deer ran out of the woods and on to the road right in front of me. Fortunately, my reaction time was zero, and I was able to stop from a speed of 20 m/s in a mere 50 m, just barely avoiding the deer. Later tests revealed that the static and kinetic coefficients of friction of the coffee mug on the dash are 0.50 and 0.30, respectively; the coffee and mug had a mass of 0.50 kg; and the mass of the deer was 120 kg. Did my coffee mug slide?
26. || Two-thirds of the weight of a 1500 kg car rests on the drive wheels. What is the maximum acceleration of this car on a concrete surface?
27. || A Federation starship ( $2.0 \times 10^6$  kg) uses its tractor beam to pull a shuttlecraft ( $2.0 \times 10^4$  kg) aboard from a distance of 10 km away. The tractor beam exerts a constant force of  $4.0 \times 10^4$  N on the shuttlecraft. Both spacecraft are initially at rest. How far does the starship move as it pulls the shuttlecraft aboard?
28. || Your forehead can withstand a force of about 6.0 kN before fracturing, while your cheekbone can withstand only about 1.3 kN. Suppose a 140 g baseball traveling at 30 m/s strikes your head and stops in 1.5 ms.
- What is the magnitude of the force that stops the baseball?
  - What force does the baseball exert on your head? Explain.
  - Are you in danger of a fracture if the ball hits you in the forehead? On the cheek?
29. || Bob, who has a mass of 75 kg, can throw a 500 g rock with a speed of 30 m/s. The distance through which his hand moves as he accelerates the rock from rest until he releases it is 1.0 m.
- What constant force must Bob exert on the rock to throw it with this speed?
  - If Bob is standing on frictionless ice, what is his recoil speed after releasing the rock?
30. || You see the boy next door trying to push a crate down the sidewalk. He can barely keep it moving, and his feet occasionally slip. You start to wonder how heavy the crate is. You call to ask the boy his mass, and he replies “50 kg.” From your recent physics class you estimate that the static and kinetic coefficients of friction are 0.8 and 0.4 for the boy’s shoes, and 0.5 and 0.2 for the crate. Estimate the mass of the crate.
31. || Two packages at UPS start sliding down the  $20^\circ$  ramp shown in **FIGURE P7.31**. Package A has a mass of 5.0 kg and a coefficient of friction of 0.20. Package B has a mass of 10 kg and a coefficient of friction of 0.15. How long does it take package A to reach the bottom?

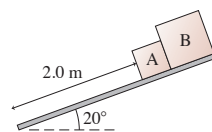


FIGURE P7.31

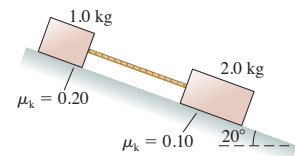


FIGURE P7.32

32. || The two blocks in **FIGURE P7.32** are sliding down the incline. What is the tension in the massless string?



33. || The 1.0 kg block in **FIGURE P7.33** is tied to the wall with a rope. It sits on top of the 2.0 kg block. The lower block is pulled to the right with a tension force of 20 N. The coefficient of kinetic friction at both the lower and upper surfaces of the 2.0 kg block is  $\mu_k = 0.40$ .

- What is the tension in the rope holding the 1.0 kg block to the wall?
- What is the acceleration of the 2.0 kg block?

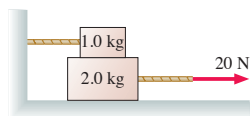


FIGURE P7.33

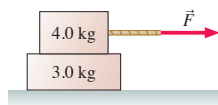


FIGURE P7.34

34. || The coefficient of static friction is 0.60 between the two blocks in **FIGURE P7.34**. The coefficient of kinetic friction between the lower block and the floor is 0.20. Force  $\vec{F}$  causes both blocks to cross a distance of 5.0 m, starting from rest. What is the least amount of time in which this motion can be completed without the top block sliding on the lower block?

35. || The lower block in **FIGURE P7.35** is pulled on by a rope with a tension force of 20 N. The coefficient of kinetic friction between the lower block and the surface is 0.30. The coefficient of kinetic friction between the lower block and the upper block is also 0.30. What is the acceleration of the 2.0 kg block?

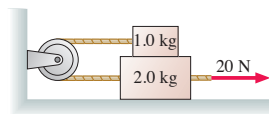


FIGURE P7.35

36. || The block of mass  $M$  in **FIGURE P7.36** slides on a frictionless surface. Find an expression for the tension in the string.

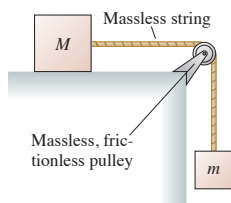


FIGURE P7.36

37. || A rope attached to a 20 kg wood sled pulls the sled up a  $20^\circ$  snow-covered hill. A 10 kg wood box rides on top of the sled. If the tension in the rope steadily increases, at what value of the tension does the box slip?
38. || The 100 kg block in **FIGURE P7.38** takes 6.0 s to reach the floor after being released from rest. What is the mass of the block on the left? The pulley is massless and frictionless.

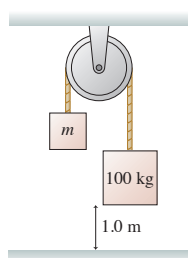


FIGURE P7.38

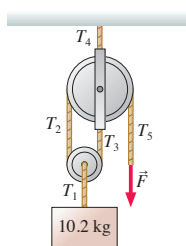


FIGURE P7.39

39. || The 10.2 kg block in **FIGURE P7.39** is held in place by a force applied to a rope passing over two massless, frictionless pulleys. Find the tensions  $T_1$  to  $T_5$  and the magnitude of force  $\vec{F}$ .

40. || The coefficient of kinetic friction between the 2.0 kg block in **FIGURE P7.40** and the table is 0.30. What is the acceleration of the 2.0 kg block?

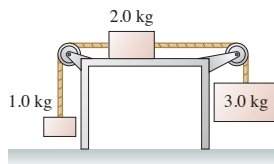


FIGURE P7.40

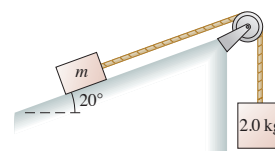


FIGURE P7.41

41. || **FIGURE P7.41** shows a block of mass  $m$  resting on a  $20^\circ$  slope. The block has coefficients of friction  $\mu_s = 0.80$  and  $\mu_k = 0.50$  with the surface. It is connected via a massless string over a massless, frictionless pulley to a hanging block of mass 2.0 kg.
- What is the minimum mass  $m$  that will stick and not slip?
  - If this minimum mass is nudged ever so slightly, it will start being pulled up the incline. What acceleration will it have?

42. || A 4.0 kg box is on a frictionless  $35^\circ$  slope and is connected via a massless string over a massless, frictionless pulley to a hanging 2.0 kg weight. The picture for this situation is similar to **FIGURE P7.41**.
- What is the tension in the string if the 4.0 kg box is held in place, so that it cannot move?
  - If the box is then released, which way will it move on the slope?
  - What is the tension in the string once the box begins to move?

43. || The 1.0 kg physics book in **FIGURE P7.43** is connected by a string to a 500 g coffee cup. The book is given a push up the slope and released with a speed of 3.0 m/s. The coefficients of friction are  $\mu_s = 0.50$  and  $\mu_k = 0.20$ .

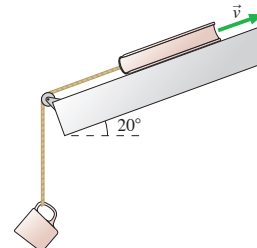


FIGURE P7.43

- How far does the book slide?
- At the highest point, does the book stick to the slope, or does it slide back down?

44. || The 2000 kg cable car shown in **FIGURE P7.44** descends a 200-m-high hill. In addition to its brakes, the cable car controls its speed by pulling an 1800 kg counterweight up the other side of the hill. The rolling friction of both the cable car and the counterweight are negligible.

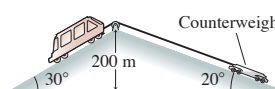


FIGURE P7.44

- How much braking force does the cable car need to descend at constant speed?
- One day the brakes fail just as the cable car leaves the top on its downward journey. What is the runaway car's speed at the bottom of the hill?

45. || The century-old *ascensores* in Valparaiso, Chile, are small cable cars that go up and down the steep hillsides. As **FIGURE P7.45** shows, one car ascends as the other descends. The cars use a two-cable arrangement to compensate for friction; one cable passing around a large pulley connects the cars, the second is pulled by a small motor. Suppose the mass of both cars (with passengers) is 1500 kg, the coefficient of rolling friction is 0.020, and the cars move at constant speed. What is the tension in (a) the connecting cable and (b) the cable to the motor?

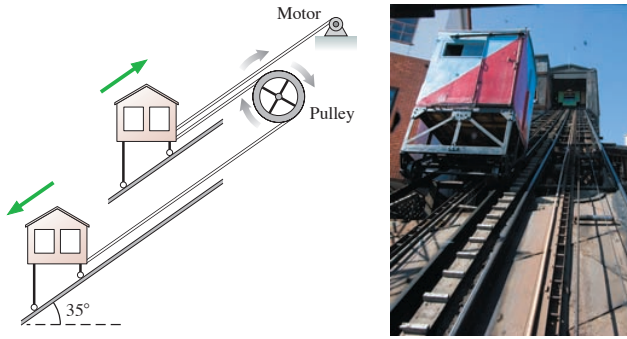


FIGURE P7.45

46. ■ A house painter uses the chair-and-pulley arrangement of FIGURE P7.46 to lift himself up the side of a house. The painter's mass is 70 kg and the chair's mass is 10 kg. With what force must he pull down on the rope in order to accelerate upward at  $0.20 \text{ m/s}^2$ ?

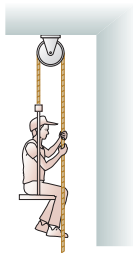


FIGURE P7.46

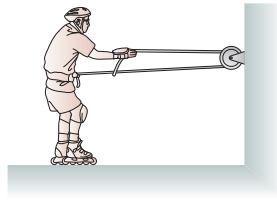


FIGURE P7.47

47. ■ Jorge, with mass  $m$ , is wearing roller skates whose coefficient of friction with the floor is  $\mu_s$ . He ties a massless rope around his waist, passes it around a frictionless pulley, and grabs hold of the other end, as shown in FIGURE P7.47. Jorge then pulls hand over hand on the rope with a constant force  $F$ . Find an expression for Jorge's acceleration toward the wall.
48. ■ A 70 kg tightrope walker stands at the center of a rope. The rope supports are 10 m apart and the rope sags  $10^\circ$  at each end. The tightrope walker crouches down, then leaps straight up with an acceleration of  $8.0 \text{ m/s}^2$  to catch a passing trapeze. What is the tension in the rope as he jumps?
49. ■ Find an expression for the magnitude of the horizontal force  $F$  in FIGURE P7.49 for which  $m_1$  does not slip either up or down along the wedge. All surfaces are frictionless.

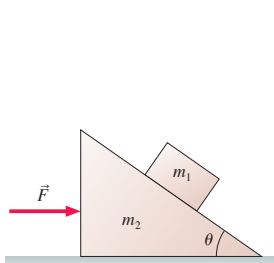


FIGURE P7.49

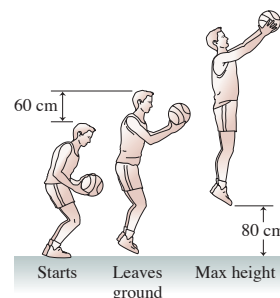


FIGURE P7.50

50. ■ A 100 kg basketball player can leap straight up in the air to a height of 80 cm, as shown in FIGURE P7.50. You can understand how by analyzing the situation as follows:

- The player bends his legs until the upper part of his body has dropped by 60 cm, then he begins his jump. Draw separate free-body diagrams for the player and for the floor *as* he is jumping, but before his feet leave the ground.
- Is there a net force on the player as he jumps (before his feet leave the ground)? How can that be? Explain.
- With what speed must the player leave the ground to reach a height of 80 cm?
- What was his acceleration, assumed to be constant, as he jumped?
- Suppose the player jumps while standing on a bathroom scale that reads in newtons. What does the scale read before he jumps, as he is jumping, and after his feet leave the ground?

Problems 51 and 52 show the free-body diagrams of two interacting systems. For each of these, you are to

- Write a realistic problem for which these are the correct free-body diagrams. Be sure that the answer your problem requests is consistent with the diagrams shown.
- Finish the solution of the problem.

51.

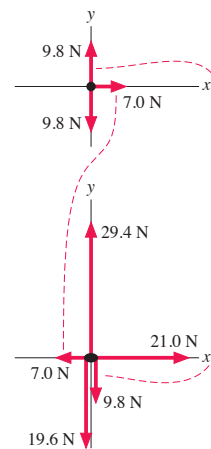


FIGURE P7.51

52.

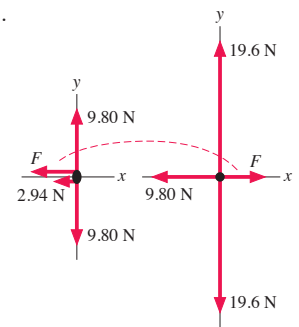


FIGURE P7.52

### Challenge Problems

53. A 100 g ball of clay is thrown horizontally with a speed of 10 m/s toward a 900 g block resting on a frictionless surface. It hits the block and sticks. The clay exerts a constant force on the block during the 10 ms it takes the clay to come to rest relative to the block. After 10 ms, the block and the clay are sliding along the surface as a single system.
- What is their speed after the collision?
  - What is the force of the clay on the block during the collision?
  - What is the force of the block on the clay?

**NOTE** ▶ This problem can be worked using the conservation laws you will be learning in the next few chapters. However, here you're asked to solve the problem using Newton's laws. ◀

54. In FIGURE CP7.54, find an expression for the acceleration of  $m_1$ . The pulleys are massless and frictionless.  
**Hint:** Think carefully about the acceleration constraint.

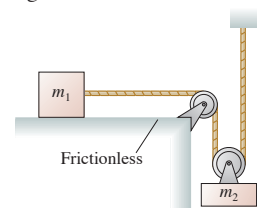
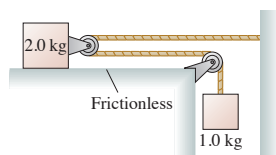


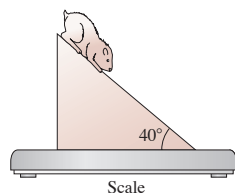
FIGURE CP7.54

55. What is the acceleration of the 2.0 kg block in **FIGURE CP7.55** across the frictionless table?

**Hint:** Think carefully about the acceleration constraint.



**FIGURE CP7.55**



**FIGURE CP7.56**

56. **FIGURE CP7.56** shows a 200 g hamster sitting on an 800 g wedge-shaped block. The block, in turn, rests on a spring scale. An extra-fine lubricating oil having  $\mu_s = \mu_k = 0$  is sprayed on the top surface of the block, causing the hamster to slide down. Friction between the block and the scale is large enough that the block does *not* slip on the scale. What does the scale read, in grams, as the hamster slides down?

57. **FIGURE CP7.57** shows three hanging masses connected by massless strings over two massless, frictionless pulleys.

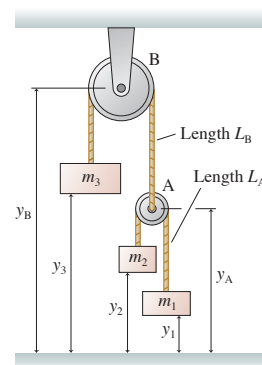
- a. Find the acceleration constraint for this system. It is a single equation relating  $a_{1y}$ ,  $a_{2y}$ , and  $a_{3y}$ .

**Hint:**  $y_A$  isn't constant.

- b. Find an expression for the tension in string A.

**Hint:** You should be able to write four second-law equations. These, plus the acceleration constraint, are five equations in five unknowns.

- c. Suppose:  $m_1 = 2.5$  kg,  $m_2 = 1.5$  kg, and  $m_3 = 4.0$  kg. Find the acceleration of each.
- d. The 4.0 kg mass would appear to be in equilibrium. Explain why it accelerates.



**FIGURE CP7.57**

### STOP TO THINK ANSWERS

**Stop to Think 7.1:** The crate's gravitational force and the normal force are incorrectly identified as an action/reaction pair. The normal force should be paired with a downward force of the crate on the ground. Gravity is the pull of the entire earth, so  $\vec{F}_G$  should be paired with a force pulling up on the entire earth.

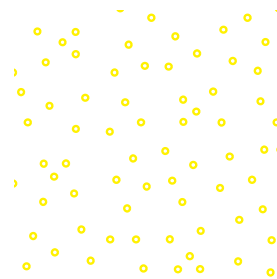
**Stop to Think 7.2:** c. Newton's third law says that the force of A on B is *equal* and opposite to the force of B on A. This is always true. The speed of the objects isn't relevant.

**Stop to Think 7.3:** b.  $F_{B \text{ on } H} = F_{H \text{ on } B}$  and  $F_{A \text{ on } B} = F_{B \text{ on } A}$  because these are action/reaction pairs. Box B is slowing down and therefore must have a net force to the left. So from Newton's second law we also know that  $F_{H \text{ on } B} > F_{A \text{ on } B}$ .

**Stop to Think 7.4:** Equal to. Each block is hanging in equilibrium, with no net force, so the upward tension force is  $mg$ .

**Stop to Think 7.5:** Less than. Block B is *accelerating* downward, so the net force on B must point down. The only forces acting on B are the tension and gravity, so  $T_{S \text{ on } B} < (F_G)_B$ .

**Stop to Think 7.6:** c. Newton's third law says that the force of A on B is *equal* and opposite to the force of B on A. This is always true. The mass of the objects isn't relevant.



# SUMMARY

The goal of Chapter 8 has been to learn how to solve problems about motion in a plane.

## General Principles

### Newton's Second Law

Expressed in  $x$ - and  $y$ -component form:

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$(F_{\text{net}})_y = \sum F_y = ma_y$$

Expressed in  $rtz$ -component form:

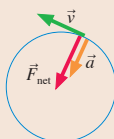
$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

$$(F_{\text{net}})_t = \sum F_t = \begin{cases} 0 & \text{uniform circular motion} \\ ma_t & \text{nonuniform circular motion} \end{cases}$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

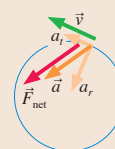
### Uniform Circular Motion

- $v$  is constant.
- $\vec{F}_{\text{net}}$  points toward the center of the circle.
- The **centripetal acceleration**  $\vec{a}$  points toward the center of the circle. It changes the particle's direction but not its speed.



### Nonuniform Circular Motion

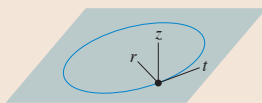
- $v$  changes.
- $\vec{a}$  is parallel to  $\vec{F}_{\text{net}}$ .
- The radial component  $a_r$  changes the particle's direction.
- The tangential component  $a_t$  changes the particle's speed.



## Important Concepts

### $rtz$ -coordinates

- The  $r$ -axis points toward the center of the circle.
- The  $t$ -axis is tangent, pointing counterclockwise.



### Angular velocity

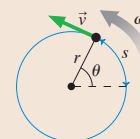
$$\omega = d\theta/dt$$

$$v_t = \omega r$$

### Angular acceleration

$$\alpha = d\omega/dt$$

$$a_t = \alpha r$$

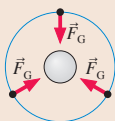


## Applications

### Orbits

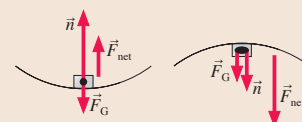
A circular orbit has radius  $r$  if

$$v = \sqrt{rg}$$



### Hills and valleys

Circular motion requires a net force pointing to the center.  $n$  must be  $> 0$  for the object to be in contact with a surface.



## Terms and Notation

orbit  
fictitious force

## CONCEPTUAL QUESTIONS

1. In uniform circular motion, which of the following are constant: speed, velocity, angular velocity, centripetal acceleration, magnitude of the net force?
2. A car runs out of gas while driving down a hill. It rolls through the valley and starts up the other side. At the very bottom of the valley, which of the free-body diagrams in **FIGURE Q8.2** is correct? The car is moving to the right, and drag and rolling friction are negligible.

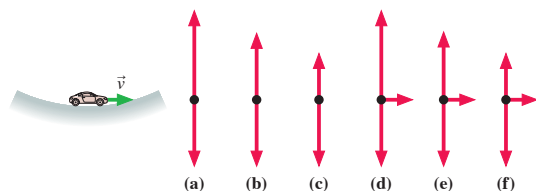


FIGURE Q8.2

3. **FIGURE Q8.3** is a bird's-eye view of particles moving in horizontal circles on a tabletop. All are moving at the same speed. Rank in order, from largest to smallest, the tensions  $T_a$  to  $T_d$ . Give your answer in the form  $a > b = c > d$  and explain your ranking.

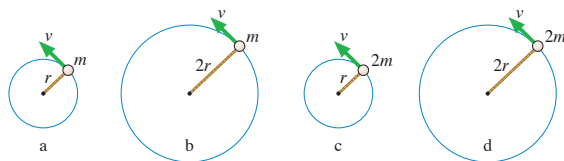


FIGURE Q8.3

4. Tarzan swings through the jungle on a vine. At the lowest point of his swing, is the tension in the vine greater than, less than, or equal to the gravitational force on Tarzan? Explain.

5. **FIGURE Q8.5** shows two balls of equal mass moving in vertical circles. Is the tension in string A greater than, less than, or equal to the tension in string B if the balls travel over the top of the circle (a) with equal speed and (b) with equal angular velocity?

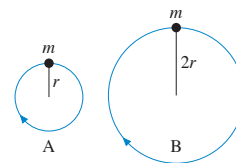


FIGURE Q8.5

6. Ramon and Sally are observing a toy car speed up as it goes around a circular track. Ramon says, "The car's speeding up, so there must be a net force parallel to the track." "I don't think so," replies Sally. "It's moving in a circle, and that requires centripetal acceleration. The net force has to point to the center of the circle." Do you agree with Ramon, Sally, or neither? Explain.
7. A jet plane is flying on a level course at constant speed. The engines are at full throttle.
  - a. What is the net force on the plane? Explain.
  - b. Draw a free-body diagram of the plane as seen from the side with the plane flying to the right. Name (don't just label) any and all forces shown on your diagram.
  - c. Airplanes bank when they turn. Draw a free-body diagram of the plane as seen from behind as it makes a right turn.
  - d. Why do planes bank as they turn? Explain.
8. A small projectile is launched parallel to the ground at height  $h = 1$  m with sufficient speed to orbit a completely smooth, airless planet. A bug rides inside a small hole inside the projectile. Is the bug weightless? Explain.
9. You can swing a ball on a string in a vertical circle if you swing it fast enough. But if you swing too slowly, the string goes slack as the ball nears the top. Explain *why* there's a minimum speed to keep the ball moving in a circle.
10. A golfer starts with the club over her head and swings it to reach maximum speed as it contacts the ball. Halfway through her swing, when the golf club is parallel to the ground, does the acceleration vector of the club head point (a) straight down, (b) parallel to the ground, approximately toward the golfer's shoulders, (c) approximately toward the golfer's feet, or (d) toward a point above the golfer's head? Explain.

## EXERCISES AND PROBLEMS

Problems labeled   integrate material from earlier chapters.

### Exercises

#### Section 8.1 Dynamics in Two Dimensions

1. || As a science fair project, you want to launch an 800 g model rocket straight up and hit a horizontally moving target as it passes 30 m above the launch point. The rocket engine provides a constant thrust of 15.0 N. The target is approaching at a speed of 15 m/s. At what horizontal distance between the target and the rocket should you launch?
2. || A 500 g model rocket is on a cart that is rolling to the right at a speed of 3.0 m/s. The rocket engine, when it is fired, exerts an 8.0 N thrust on the rocket. Your goal is to have the rocket pass

through a small horizontal hoop that is 20 m above the launch point. At what horizontal distance left of the hoop should you launch?

3. || A  $4.0 \times 10^{10}$  kg asteroid is heading directly toward the center of the earth at a steady 20 km/s. To save the planet, astronauts strap a giant rocket to the asteroid perpendicular to its direction of travel. The rocket generates  $5.0 \times 10^9$  N of thrust. The rocket is fired when the asteroid is  $4.0 \times 10^6$  km away from earth. You can ignore the earth's gravitational force on the asteroid and their rotation about the sun.
  - a. If the mission fails, how many hours is it until the asteroid impacts the earth?
  - b. The radius of the earth is 6400 km. By what minimum angle must the asteroid be deflected to just miss the earth?
  - c. The rocket fires at full thrust for 300 s before running out of fuel. Is the earth saved?

### Section 8.2 Uniform Circular Motion

4. **|** A 1500 kg car drives around a flat 200-m-diameter circular track at 25 m/s. What are the magnitude and direction of the net force on the car? What causes this force?
5. **|** A 1500 kg car takes a 50-m-radius unbanked curve at 15 m/s. What is the size of the friction force on the car?
6. **||** A 200 g block on a 50-cm-long string swings in a circle on a horizontal, frictionless table at 75 rpm.
  - a. What is the speed of the block?
  - b. What is the tension in the string?
7. **||** In the Bohr model of the hydrogen atom, an electron (mass  $m = 9.1 \times 10^{-31}$  kg) orbits a proton at a distance of  $5.3 \times 10^{-11}$  m. The proton pulls on the electron with an electric force of  $8.2 \times 10^{-8}$  N. How many revolutions per second does the electron make?
8. **||** A highway curve of radius 500 m is designed for traffic moving at a speed of 90 km/h. What is the correct banking angle of the road?
9. **||** Suppose the moon were held in its orbit not by gravity but by a massless cable attached to the center of the earth. What would be the tension in the cable? Use the table of astronomical data inside the back cover of the book.
10. **|** It is proposed that future space stations create an artificial gravity by rotating. Suppose a space station is constructed as a 1000-m-diameter cylinder that rotates about its axis. The inside surface is the deck of the space station. What rotation period will provide “normal” gravity?

### Section 8.3 Circular Orbits

11. **|** A satellite orbiting the moon very near the surface has a period of 110 min. What is free-fall acceleration on the surface of the moon?
12. **||** What is free-fall acceleration toward the sun at the distance of the earth’s orbit? Astronomical data are inside the back cover of the book.

### Section 8.4 Fictitious Forces

13. **|** A car drives over the top of a hill that has a radius of 50 m. What maximum speed can the car have at the top without flying off the road?
14. **||** The weight of passengers on a roller coaster increases by 50% as the car goes through a dip with a 30 m radius of curvature. What is the car’s speed at the bottom of the dip?
15. **||** A roller coaster car crosses the top of a circular loop-the-loop at twice the critical speed. What is the ratio of the normal force to the gravitational force?
16. **||** The normal force equals the magnitude of the gravitational force as a roller coaster car crosses the top of a 40-m-diameter loop-the-loop. What is the car’s speed at the top?
17. **||** A student has 65-cm-long arms. What is the minimum angular velocity (in rpm) for swinging a bucket of water in a vertical circle without spilling any? The distance from the handle to the bottom of the bucket is 35 cm.
18. **|** While at the county fair, you decide to ride the Ferris wheel. Having eaten too many candy apples and elephant ears, you find the motion somewhat unpleasant. To take your mind off your stomach, you wonder about the motion of the ride. You estimate the radius of the big wheel to be 15 m, and you use your watch to find that each loop around takes 25 s.
  - a. What are your speed and the magnitude of your acceleration?
  - b. What is the ratio of your weight at the top of the ride to your weight while standing on the ground?
  - c. What is the ratio of your weight at the bottom of the ride to your weight while standing on the ground?

### Section 8.5 Nonuniform Circular Motion

19. **||** A new car is tested on a 200-m-diameter track. If the car speeds up at a steady  $1.5 \text{ m/s}^2$ , how long after starting is the magnitude of its centripetal acceleration equal to the tangential acceleration?
20. **||** A toy train rolls around a horizontal 1.0-m-diameter track. The coefficient of rolling friction is 0.10.
  - a. What is the magnitude of the train’s angular acceleration after it is released?
  - b. How long does it take the train to stop if it’s released with an angular speed of 30 rpm?

### Problems

21. **||** A popular pastime is to see who can push an object closest to the edge of a table without its going off. You push the 100 g object and release it 2.0 m from the table edge. Unfortunately, you push a little too hard. The object slides across, sails off the edge, falls 1.0 m to the floor, and lands 30 cm from the edge of the table. If the coefficient of kinetic friction is 0.50, what was the object’s speed as you released it?
22. **||** A motorcycle daredevil plans to ride up a 2.0-m-high,  $20^\circ$  ramp, sail across a 10-m-wide pool filled with hungry crocodiles, and land at ground level on the other side. He has done this stunt many times and approaches it with confidence. Unfortunately, the motorcycle engine dies just as he starts up the ramp. He is going 11 m/s at that instant, and the rolling friction of his rubber tires (coefficient 0.02) is not negligible. Does he survive, or does he become crocodile food?
23. **||** Sam (75 kg) takes off up a 50-m-high,  $10^\circ$  frictionless slope on his jet-powered skis. The skis have a thrust of 200 N. He keeps his skis tilted at  $10^\circ$  after becoming airborne, as shown in **FIGURE P8.23**. How far does Sam land from the base of the cliff?

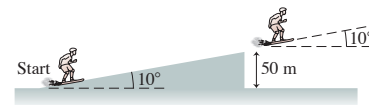


FIGURE P8.23

24. **||** Derive Equations 8.4 for the acceleration of a projectile subject to drag.
25. **||** A 5000 kg interceptor rocket is launched at an angle of  $44.7^\circ$ . The thrust of the rocket motor is 140,700 N.
  - a. Find an equation  $y(x)$  that describes the rocket’s trajectory.
  - b. What is the shape of the trajectory?
  - c. At what elevation does the rocket reach the speed of sound, 330 m/s?
26. **||** A rocket-powered hockey puck has a thrust of 2.0 N and a total mass of 1.0 kg. It is released from rest on a frictionless table, 4.0 m from the edge of a 2.0 m drop. The front of the rocket is pointed directly toward the edge. How far does the puck land from the base of the table?



27. || A 500 g model rocket is resting horizontally at the top edge of a 40-m-high wall when it is accidentally bumped. The bump pushes it off the edge with a horizontal speed of 0.5 m/s and at the same time causes the engine to ignite. When the engine fires, it exerts a constant 20 N horizontal thrust away from the wall.
- How far from the base of the wall does the rocket land?
  - Describe the trajectory of the rocket while it travels to the ground.
28. || An experimental aircraft begins its takeoff at  $t = 0$  s. Every second, an onboard GPS measures and records the plane's distances east and north of a reference marker. The following data are downloaded to your computer:

Time (s)	East (m)	North (m)
0.0	91	0
1.0	86	4
2.0	77	18
3.0	65	39
4.0	39	63
5.0	19	101

Analyze these data to determine the magnitude of the aircraft's takeoff acceleration.

29. || Communications satellites are placed in circular orbits where they stay directly over a fixed point on the equator as the earth rotates. These are called *geosynchronous orbits*. The altitude of a geosynchronous orbit is  $3.58 \times 10^7$  m ( $\approx 22,000$  miles).
- What is the period of a satellite in a geosynchronous orbit?
  - Find the value of  $g$  at this altitude.
  - What is the weight of a 2000 kg satellite in a geosynchronous orbit?
30. || A 75 kg man weighs himself at the north pole and at the equator. Which scale reading is higher? By how much?
31. || A 500 g ball swings in a vertical circle at the end of a 1.5-m-long string. When the ball is at the bottom of the circle, the tension in the string is 15 N. What is the speed of the ball at that point?
32. || A concrete highway curve of radius 70 m is banked at a  $15^\circ$  angle. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding?
33. || a. An object of mass  $m$  swings in a horizontal circle on a string of length  $L$  that tilts downward at angle  $\theta$ . Find an expression for the angular velocity  $\omega$ .  
b. A student ties a 500 g rock to a 1.0-m-long string and swings it around her head in a horizontal circle. At what angular speed, in rpm, does the string tilt down at a  $10^\circ$  angle?
34. || A 5.0 g coin is placed 15 cm from the center of a turntable. The coin has static and kinetic coefficients of friction with the turntable surface of  $\mu_s = 0.80$  and  $\mu_k = 0.50$ . The turntable very slowly speeds up to 60 rpm. Does the coin slide off?
35. || You've taken your neighbor's young child to the carnival to ride the rides. She wants to ride The Rocket. Eight rocket-shaped cars hang by chains from the outside edge of a large steel disk. A vertical axle through the center of the ride turns the disk, causing the cars to revolve in a circle. You've just finished taking physics, so you decide to figure out the speed of the cars while you wait. You estimate that the disk is 5 m in diameter and the chains are 6 m long. The ride takes 10 s to reach full speed, then the cars swing out until the chains are  $20^\circ$  from vertical. What is the cars' speed?

36. || A conical pendulum is formed by attaching a ball of mass  $m$  to a string of length  $L$ , then allowing the ball to move in a horizontal circle of radius  $r$ . FIGURE P8.36 shows that the string traces out the surface of a cone, hence the name.
- Find an expression for the tension  $T$  in the string.
  - Find an expression for the ball's angular speed  $\omega$ .
  - What are the tension and angular speed (in rpm) for a 500 g ball swinging in a 20-cm-radius circle at the end of a 1.0-m-long string?

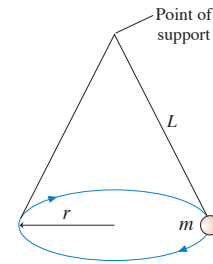


FIGURE P8.36

37. || Two wires are tied to the 2.0 kg sphere shown in FIGURE P8.37. The sphere revolves in a horizontal circle at constant speed.
- For what speed is the tension the same in both wires?
  - What is the tension?

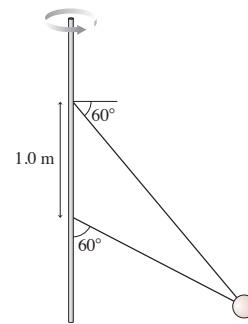


FIGURE P8.37

38. || In an old-fashioned amusement park ride, passengers stand inside a 5.0-m-diameter hollow steel cylinder with their backs against the wall. The cylinder begins to rotate about a vertical axis. Then the floor on which the passengers are standing suddenly drops away! If all goes well, the passengers will "stick" to the wall and not slide. Clothing has a static coefficient of friction against steel in the range 0.60 to 1.0 and a kinetic coefficient in the range 0.40 to 0.70. A sign next to the entrance says "No children under 30 kg allowed." What is the minimum angular speed, in rpm, for which the ride is safe?
39. || A 10 g steel marble is spun so that it rolls at 150 rpm around the *inside* of a vertically oriented steel tube. The tube, shown in FIGURE P8.39, is 12 cm in diameter. Assume that the rolling resistance is small enough for the marble to maintain 150 rpm for several seconds. During this time, will the marble spin in a horizontal circle, at constant height, or will it spiral down the inside of the tube?

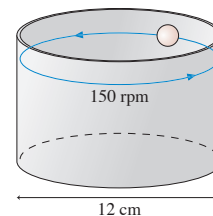


FIGURE P8.39

40. **BIO** The ultracentrifuge is an important tool for separating and analyzing proteins. Because of the enormous centripetal accelerations, the centrifuge must be carefully balanced, with each sample matched by a sample of identical mass on the opposite side. Any difference in the masses of opposing samples creates a net force on the shaft of the rotor, potentially leading to a catastrophic failure of the apparatus. Suppose a scientist makes a slight error in sample preparation and one sample has a mass 10 mg larger than the opposing sample. If the samples are 12 cm from the axis of the rotor and the ultracentrifuge spins at 70,000 rpm, what is the magnitude of the net force on the rotor due to the unbalanced samples?
41. **BIO** Three cars are driving at 25 m/s along the road shown in **FIGURE P8.41**. Car B is at the bottom of a hill and car C is at the top. Both hills have a 200 m radius of curvature. Suppose each car suddenly brakes hard and starts to skid. What is the tangential acceleration (i.e., the acceleration parallel to the road) of each car? Assume  $\mu_k = 1.0$ .

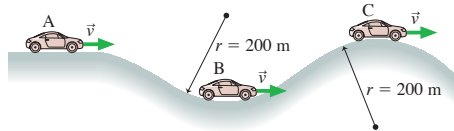


FIGURE P8.41

42. **BIO** A 500 g ball moves in a vertical circle on a 102-cm-long string. If the speed at the top is 4.0 m/s, then the speed at the bottom will be 7.5 m/s. (You'll learn how to show this in Chapter 10.)
- What is the gravitational force acting on the ball?
  - What is the tension in the string when the ball is at the top?
  - What is the tension in the string when the ball is at the bottom?
43. **BIO** In an amusement park ride called The Roundup, passengers stand inside a 16-m-diameter rotating ring. After the ring has acquired sufficient speed, it tilts into a vertical plane, as shown in **FIGURE P8.43**.
- Suppose the ring rotates once every 4.5 s. If a rider's mass is 55 kg, with how much force does the ring push on her at the top of the ride? At the bottom?
  - What is the longest rotation period of the wheel that will prevent the riders from falling off at the top?
44. **BIO** You have a new job designing rides for an amusement park. In one ride, the rider's chair is attached by a 9.0-m-long chain to the top of a tall rotating tower. The tower spins the chair and rider around at the rate of 1.0 rev every 4.0 s. In your design, you've assumed that the maximum possible combined weight of the chair and rider is 150 kg. You've found a great price for chain at the local discount store, but your supervisor wonders if the chain is strong enough. You contact the manufacturer and learn that the chain is rated to withstand a tension of 3000 N. Will this chain be strong enough for the ride?
45. **BIO** Suppose you swing a ball of mass  $m$  in a vertical circle on a string of length  $L$ . As you probably know from experience, there is a minimum angular velocity  $\omega_{\min}$  you must maintain if you

want the ball to complete the full circle without the string going slack at the top.

- Find an expression for  $\omega_{\min}$ .
  - Evaluate  $\omega_{\min}$  in rpm for a 65 g ball tied to a 1.0-m-long string.
46. **BIO** A heavy ball with a weight of 100 N ( $m = 10.2$  kg) is hung from the ceiling of a lecture hall on a 4.5-m-long rope. The ball is pulled to one side and released to swing as a pendulum, reaching a speed of 5.5 m/s as it passes through the lowest point. What is the tension in the rope at that point?
47. **BIO** A 30 g ball rolls around a 40-cm-diameter L-shaped track, shown in **FIGURE P8.47**, at 60 rpm. What is the magnitude of the *net* force that the track exerts on the ball? Rolling friction can be neglected.



FIGURE P8.47

**Hint:** The track exerts more than one force on the ball.

48. **BIO** Mass  $m_1$  on the frictionless table of **FIGURE P8.48** is connected by a string through a hole in the table to a hanging mass  $m_2$ . With what speed must  $m_1$  rotate in a circle of radius  $r$  if  $m_2$  is to remain hanging at rest?

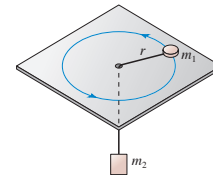


FIGURE P8.48

49. **BIO** The physics of circular motion sets an upper limit to the speed of human walking. (If you need to go faster, your gait changes from a walk to a run.) If you take a few steps and watch what's happening, you'll see that your body pivots in circular motion over your forward foot as you bring your rear foot forward for the next step. As you do so, the normal force of the ground on your foot decreases and your body tries to "lift off" from the ground.
- A person's center of mass is very near the hips, at the top of the legs. Model a person as a particle of mass  $m$  at the top of a leg of length  $L$ . Find an expression for the person's maximum walking speed  $v_{\max}$ .
  - Evaluate your expression for the maximum walking speed of a 70 kg person with a typical leg length of 70 cm. Give your answer in both m/s and mph, then comment, based on your experience, as to whether this is a reasonable result. A "normal" walking speed is about 3 mph.
50. **BIO** A 100 g ball on a 60-cm-long string is swung in a vertical circle about a point 200 cm above the floor. The tension in the string when the ball is at the very bottom of the circle is 5.0 N. A very sharp knife is suddenly inserted, as shown in **FIGURE P8.50**, to cut the string directly below the point of support. How far to the right of where the string was cut does the ball hit the floor?

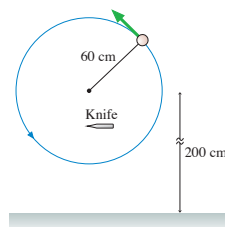


FIGURE P8.50

51. || A 60 g ball is tied to the end of a 50-cm-long string and swung in a vertical circle. The center of the circle, as shown in **FIGURE P8.51**, is 150 cm above the floor. The ball is swung at the minimum speed necessary to make it over the top without the string going slack. If the string is released at the instant the ball is at the top of the loop, how far to the right does the ball hit the ground?

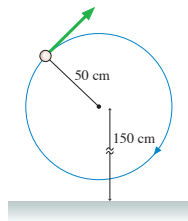


FIGURE P8.51

52. || Elm Street has a pronounced dip at the bottom of a steep hill before going back uphill on the other side. Your science teacher has asked everyone in the class to measure the radius of curvature of the dip. Some of your classmates are using surveying equipment, but you decide to base your measurement on what you've learned in physics. To do so, you sit on a spring scale, drive through the dip at different speeds, and for each speed record the scale's reading as you pass through the bottom of the dip. Your data are as follows:

Speed (m/s)	Scale reading (N)
5	599
10	625
15	674
20	756
25	834

Sitting on the scale while the car is parked gives a reading of 588 N. Analyze your data, using a graph, to determine the dip's radius of curvature.

53. || A 100 g ball on a 60-cm-long string is swung in a vertical circle about a point 200 cm above the floor. The string suddenly breaks when it is parallel to the ground and the ball is moving upward. The ball reaches a height 600 cm above the floor. What was the tension in the string an instant before it broke?
54. || A 1500 kg car starts from rest and drives around a flat 50-m-diameter circular track. The forward force provided by the car's drive wheels is a constant 1000 N.
- What are the magnitude and direction of the car's acceleration at  $t = 10$  s? Give the direction as an angle from the  $r$ -axis.
  - If the car has rubber tires and the track is concrete, at what time does the car begin to slide out of the circle?

55. || A 500 g steel block rotates on a steel table while attached to a 2.0-m-long massless rod. Compressed air fed through the rod is ejected from a nozzle on the back of the block, exerting a thrust force of 3.5 N. The nozzle is  $70^\circ$  from the radial line, as shown in **FIGURE P8.55**. The block starts from rest.
- What is the block's angular velocity after 10 rev?
  - What is the tension in the rod after 10 rev?

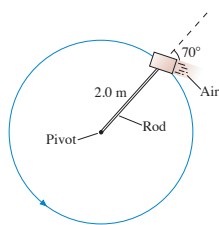


FIGURE P8.55

56. || A 2.0 kg ball swings in a vertical circle on the end of an 80-cm-long string. The tension in the string is 20 N when its angle from the highest point on the circle is  $\theta = 30^\circ$ .
- What is the ball's speed when  $\theta = 30^\circ$ ?
  - What are the magnitude and direction of the ball's acceleration when  $\theta = 30^\circ$ ?

In Problems 57 and 58 you are given the equation used to solve a problem. For each of these, you are to

- Write a realistic problem for which this is the correct equation. Be sure that the answer your problem requests is consistent with the equation given.
  - Finish the solution of the problem.
57.  $60 \text{ N} = (0.30 \text{ kg})\omega^2(0.50 \text{ m})$
58.  $(1500 \text{ kg})(9.8 \text{ m/s}^2) - 11,760 \text{ N} = (1500 \text{ kg}) v^2/(200 \text{ m})$

### Challenge Problems

59. In the absence of air resistance, a projectile that lands at the elevation from which it was launched achieves maximum range when launched at a  $45^\circ$  angle. Suppose a projectile of mass  $m$  is launched with speed  $v_0$  into a headwind that exerts a constant, horizontal retarding force  $\vec{F}_{\text{wind}} = -F_{\text{wind}}\hat{i}$ .
- Find an expression for the angle at which the range is maximum.
  - By what percentage is the maximum range of a 0.50 kg ball reduced if  $F_{\text{wind}} = 0.60 \text{ N}$ ?
60. The father of Example 8.3 stands at the summit of a conical hill as he spins his 20 kg child around on a 5.0 kg cart with a 2.0-m-long rope. The sides of the hill are inclined at  $20^\circ$ . He again keeps the rope parallel to the ground, and friction is negligible. What rope tension will allow the cart to spin with the same 14 rpm it had in the example?
61. A small ball rolls around a horizontal circle at height  $y$  inside the cone shown in **FIGURE CP8.61**. Find an expression for the ball's speed in terms of  $a$ ,  $h$ ,  $y$ , and  $g$ .

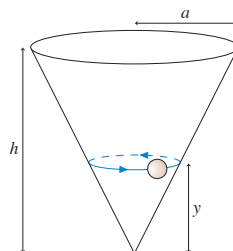


FIGURE CP8.61

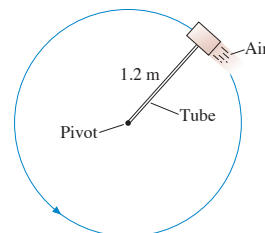


FIGURE CP8.62

62. A 500 g steel block rotates on a steel table while attached to a 1.2-m-long hollow tube as shown in **FIGURE CP8.62**. Compressed air fed through the tube and ejected from a nozzle on the back of the block exerts a thrust force of 4.0 N perpendicular to the tube. The maximum tension the tube can withstand without breaking is 50 N. If the block starts from rest, how many revolutions does it make before the tube breaks?
63. Two wires are tied to the 300 g sphere shown in **FIGURE CP8.63**. The sphere revolves in a horizontal circle at a constant speed of 7.5 m/s. What is the tension in each of the wires?

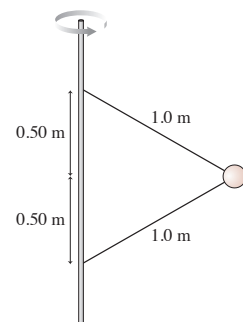


FIGURE CP8.63

64. A small ball rolls around a horizontal circle at height  $y$  inside a frictionless hemispherical bowl of radius  $R$ , as shown in **FIGURE CP8.64**.

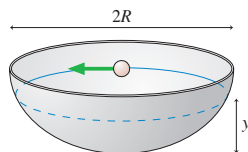


FIGURE CP8.64

- Find an expression for the ball's angular velocity in terms of  $R$ ,  $y$ , and  $g$ .
  - What is the minimum value of  $\omega$  for which the ball can move in a circle?
  - What is  $\omega$  in rpm if  $R = 20$  cm and the ball is halfway up?
65. You are flying to New York. You've been reading the in-flight magazine, which has an article about the physics of flying. You learned that the airflow over the wings creates a *lift force* that is always perpendicular to the wings. In level flight, the upward lift force exactly balances the downward gravitational force. The pilot comes on to say that, because of heavy traffic, the plane is going to circle the airport for a while. She says that you'll maintain a speed of 400 mph at an altitude of 20,000 ft. You start to wonder what the diameter of the plane's circle around the airport is. You notice that the pilot has banked the plane so that the wings are  $10^\circ$  from horizontal. The safety card in the seatback pocket informs you that the plane's wing span is 250 ft. What can you learn about the diameter?

66. If a vertical cylinder of water (or any other liquid) rotates about its axis, as shown in **FIGURE CP8.66**, the surface forms a smooth curve. Assuming that the water rotates as a unit (i.e., all the water rotates with the same angular velocity), show that the shape of the surface is a parabola described by the equation  $z = (\omega^2/2g)r^2$ . **Hint:** Each particle of water on the surface is subject to only two forces: gravity and the normal force due to the water underneath it. The normal force, as always, acts perpendicular to the surface.

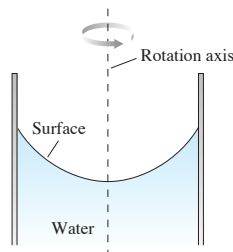


FIGURE CP8.66



FIGURE CP8.67

67. **FIGURE CP8.67** shows a small block of mass  $m$  sliding around the inside of an L-shaped track of radius  $r$ . The bottom of the track is frictionless; the coefficient of kinetic friction between the block and the wall of the track is  $\mu_k$ . The block's speed is  $v_0$  at  $t_0 = 0$ . Find an expression for the block's speed at a later time  $t$ .

## STOP TO THINK ANSWERS

**Stop to Think 8.1: d.** The parallel component of  $\vec{a}$  is opposite  $\vec{v}$  and will cause the particle to slow down. The perpendicular component of  $\vec{a}$  will cause the particle to change directions in a downward direction.

**Stop to Think 8.2:**  $(a_r)_b > (a_r)_e > (a_r)_a = (a_r)_c > (a_r)_d$ . Centripetal acceleration is  $v^2/r$ . Doubling  $r$  decreases  $a_r$  by a factor of 2. Doubling  $v$  increases  $a_r$  by a factor of 4. Reversing direction doesn't change  $a_r$ .

**Stop to Think 8.3:**  $T_d > T_b = T_e > T_c > T_a$ . The center-directed force is  $m\omega^2 r$ . Changing  $r$  by a factor of 2 changes the tension by a

factor of 2, but changing  $\omega$  by a factor of 2 changes the tension by a factor of 4.

**Stop to Think 8.4: b.** The car is moving in a circle, so there must be a net force toward the center of the circle. The circle is below the car, so the net force must point downward. This can be true only if  $F_G > n$ .

**Stop to Think 8.5: c.** The ball does not have a "memory" of its previous motion. The velocity  $\vec{v}$  is straight up at the instant the string breaks. The only force on the ball after the string breaks is the gravitational force, straight down. This is just like tossing a ball straight up.

# SUMMARY

The goals of Chapter 9 have been to understand and apply the new concepts of impulse and momentum.

## General Principles

### Law of Conservation of Momentum

The total momentum  $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$  of an isolated system is a constant. Thus

$$\vec{P}_i = \vec{P}_f$$

### Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

### Solving Momentum Conservation Problems

**MODEL** Choose an isolated system or a system that is isolated during at least part of the problem.

**VISUALIZE** Draw a pictorial representation of the system before and after the interaction.

**SOLVE** Write the law of conservation of momentum in terms of vector components:

$$(p_{ix})_1 + (p_{ix})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$

$$(p_{iy})_1 + (p_{iy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

**ASSESS** Is the result reasonable?

## Important Concepts

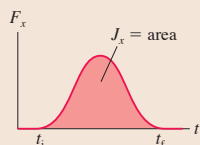
**Momentum**  $\vec{p} = m\vec{v}$

**Impulse**  $J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve}$

Impulse and momentum are related by the **impulse-momentum theorem**

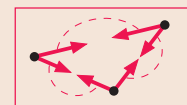
$$\Delta p_x = J_x$$

The impulse delivered to a particle causes the particle's momentum to change. This is an alternative statement of Newton's second law.



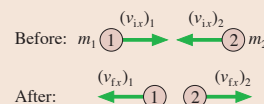
**System** A group of interacting particles.

**Isolated system** A system on which there are no external forces or the net external force is zero.



### Before-and-after pictorial representation

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.

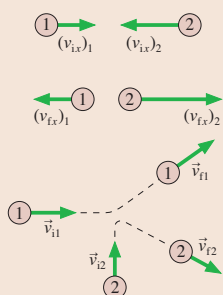


## Applications

**Collisions** Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.

**Explosions** Two or more particles move away from each other.

**Two dimensions** No new ideas, but both the  $x$ - and  $y$ -components of  $\vec{P}$  must be conserved, giving two simultaneous equations.



**Momentum bar charts** display the impulse-momentum theorem  $p_{fx} = p_{ix} + J_x$  in graphical form.



## Terms and Notation

collision  
impulsive force  
momentum,  $\vec{p}$   
impulse,  $J_x$

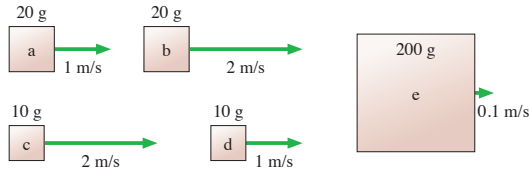
impulse-momentum theorem  
momentum bar chart  
before-and-after pictorial  
representation

impulse approximation  
total momentum,  $\vec{P}$   
isolated system

law of conservation of momentum  
perfectly inelastic collision  
explosion

# CONCEPTUAL QUESTIONS

1. Rank in order, from largest to smallest, the momenta  $(p_x)_a$  to  $(p_x)_e$  of the objects in **FIGURE Q9.1**.



**FIGURE Q9.1**

- Explain the concept of *impulse* in nonmathematical language. That is, don't simply put the equation in words to say that "impulse is the time integral of force." Explain it in terms that would make sense to an educated person who had never heard of it.
- Explain the concept of *isolated system* in nonmathematical language that would make sense to an educated person who had never heard of it.
- A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a time of 1 s, starting from rest. After the force is removed at  $t = 1$  s, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.
- A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a distance of 1 m, starting from rest. After traveling 1 m, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.
- Angie, Brad, and Carlos are discussing a physics problem in which two identical bullets are fired with equal speeds at equal-mass wood and steel blocks resting on a frictionless table. One bullet bounces off the steel block while the second becomes embedded in the wood block. "All the masses and speeds are the same," says Angie, "so I think the blocks will have equal speeds after the collisions." "But what about momentum?" asks Brad.
 

"The bullet hitting the wood block transfers all its momentum and energy to the block, so the wood block should end up going faster than the steel block." "I think the bounce is an important factor," replies Carlos. "The steel block will be faster because the bullet bounces off it and goes back the other direction." Which of these three do you agree with, and why?
- It feels better to catch a hard ball while wearing a padded glove than to catch it bare handed. Use the ideas of this chapter to explain why.
- Automobiles are designed with "crumple zones" intended to collapse in a collision. Use the ideas of this chapter to explain why.
- A 2 kg object is moving to the right with a speed of 1 m/s when it experiences an impulse of 4 N·s. What are the object's speed and direction after the impulse?
- A 2 kg object is moving to the right with a speed of 1 m/s when it experiences an impulse of  $-4$  N·s. What are the object's speed and direction after the impulse?
- A golf club continues forward after hitting the golf ball. Is momentum conserved in the collision? Explain, making sure you are careful to identify "the system."
- Suppose a rubber ball collides head-on with a steel ball of equal mass traveling in the opposite direction with equal speed. Which ball, if either, receives the larger impulse? Explain.
- Two particles collide, one of which was initially moving and the other initially at rest.
  - Is it possible for *both* particles to be at rest after the collision? Give an example in which this happens, or explain why it can't happen.
  - Is it possible for *one* particle to be at rest after the collision? Give an example in which this happens, or explain why it can't happen.
- Two ice skaters, Paula and Ricardo, push off from each other. Ricardo weighs more than Paula.
  - Which skater, if either, has the greater momentum after the push-off? Explain.
  - Which skater, if either, has the greater speed after the push-off? Explain.

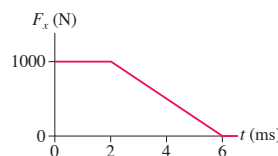
# EXERCISES AND PROBLEMS

Problems labeled   integrate material from earlier chapters.

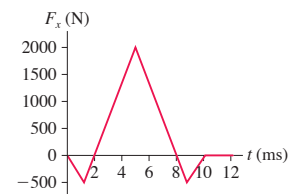
## Exercises

### Section 9.1 Momentum and Impulse

- What is the magnitude of the momentum of
  - A 3000 kg truck traveling at 15 m/s?
  - A 200 g baseball thrown at 40 m/s?
- At what speed do a bicycle and its rider, with a combined mass of 100 kg, have the same momentum as a 1500 kg car traveling at 5.0 m/s?
- What impulse does the force shown in **FIGURE EX9.3** exert on a 250 g particle?



**FIGURE EX9.3**



**FIGURE EX9.4**

- What is the impulse on a 3.0 kg particle that experiences the force shown in **FIGURE EX9.4**?



5. || In **FIGURE EX9.5**, what value of  $F_{\max}$  gives an impulse of 6.0 N·s?

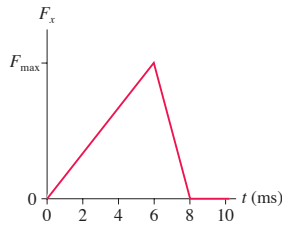


FIGURE EX9.5

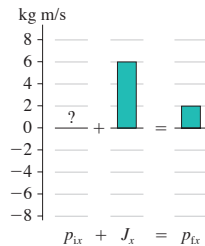


FIGURE EX9.6

6. || **FIGURE EX9.6** is an incomplete momentum bar chart for a 50 g particle that experiences an impulse lasting 10 ms. What were the speed and direction of the particle before the impulse?

7. || **FIGURE EX9.7** is an incomplete momentum bar chart for a collision that lasts 10 ms. What are the magnitude and direction of the average collision force exerted on the object?

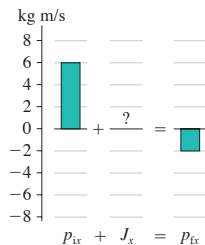


FIGURE EX9.7

### Section 9.2 Solving Impulse and Momentum Problems

8. | A 2.0 kg object is moving to the right with a speed of 1.0 m/s when it experiences the force shown in **FIGURE EX9.8**. What are the object's speed and direction after the force ends?

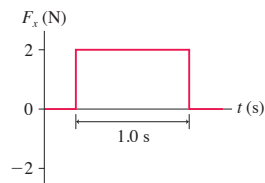


FIGURE EX9.8

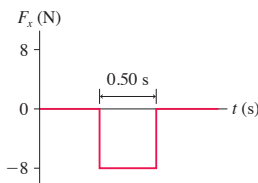


FIGURE EX9.9

9. | A 2.0 kg object is moving to the right with a speed of 1.0 m/s when it experiences the force shown in **FIGURE EX9.9**. What are the object's speed and direction after the force ends?
10. | A sled slides along a horizontal surface on which the coefficient of kinetic friction is 0.25. Its velocity at point A is 8.0 m/s and at point B is 5.0 m/s. Use the impulse-momentum theorem to find how long the sled takes to travel from A to B.
11. | Far in space, where gravity is negligible, a 425 kg rocket traveling at 75 m/s fires its engines. **FIGURE EX9.11** shows the thrust force as a function of time. The mass lost by the rocket during these 30 s is negligible.

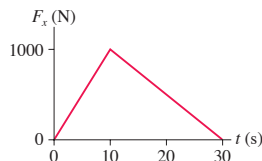


FIGURE EX9.11

- a. What impulse does the engine impart to the rocket?
- b. At what time does the rocket reach its maximum speed? What is the maximum speed?

12. || A 250 g ball collides with a wall. **FIGURE EX9.12** shows the ball's velocity and the force exerted on the ball by the wall. What is  $v_{fx}$ , the ball's rebound velocity?

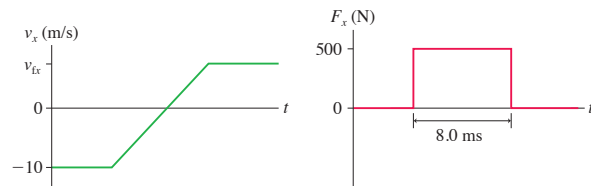


FIGURE EX9.12

13. || A 600 g air-track glider collides with a spring at one end of the track. **FIGURE EX9.13** shows the glider's velocity and the force exerted on the glider by the spring. How long is the glider in contact with the spring?

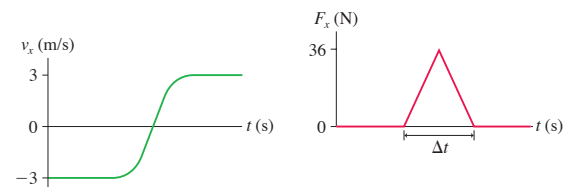


FIGURE EX9.13

### Section 9.3 Conservation of Momentum

14. | A 10,000 kg railroad car is rolling at 2.0 m/s when a 4000 kg load of gravel is suddenly dropped in. What is the car's speed just after the gravel is loaded?
15. | A 5000 kg open train car is rolling on frictionless rails at 22 m/s when it starts pouring rain. A few minutes later, the car's speed is 20 m/s. What mass of water has collected in the car?
16. || A 10-m-long glider with a mass of 680 kg (including the passengers) is gliding horizontally through the air at 30 m/s when a 60 kg skydiver drops out by releasing his grip on the glider. What is the glider's velocity just after the skydiver lets go?

### Section 9.4 Inelastic Collisions

17. | A 300 g bird flying along at 6.0 m/s sees a 10 g insect heading straight toward it with a speed of 30 m/s. The bird opens its mouth wide and enjoys a nice lunch. What is the bird's speed immediately after swallowing?
18. | The parking brake on a 2000 kg Cadillac has failed, and it is rolling slowly, at 1.0 mph, toward a group of small children. Seeing the situation, you realize you have just enough time to drive your 1000 kg Volkswagen head-on into the Cadillac and save the children. With what speed should you impact the Cadillac to bring it to a halt?
19. | A 1500 kg car is rolling at 2.0 m/s. You would like to stop the car by firing a 10 kg blob of sticky clay at it. How fast should you fire the clay?

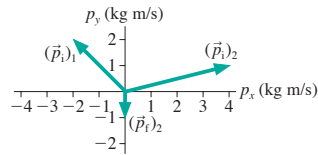
### Section 9.5 Explosions

20. | A 50 kg archer, standing on frictionless ice, shoots a 100 g arrow at a speed of 100 m/s. What is the recoil speed of the archer?

21. || Dan is gliding on his skateboard at 4.0 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8.0 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5.0 kg?
22. || A 70.0 kg football player is gliding across very smooth ice at 2.00 m/s. He throws a 0.450 kg football straight forward. What is the player's speed afterward if the ball is thrown at
- 15.0 m/s relative to the ground?
  - 15.0 m/s relative to the player?

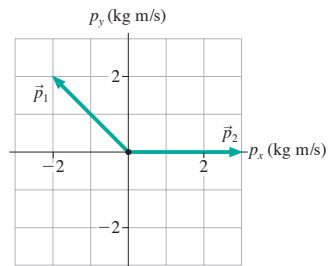
### Section 9.6 Momentum in Two Dimensions

23. || Two particles collide and bounce apart. **FIGURE EX9.23** shows the initial momenta of both and the final momentum of particle 2. What is the final momentum of particle 1? Write your answer in component form.



**FIGURE EX9.23**

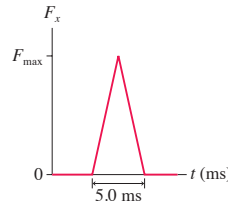
24. || An object at rest explodes into three fragments. **FIGURE EX9.24** shows the momentum vectors of two of the fragments. What are  $p_x$  and  $p_y$  of the third fragment?



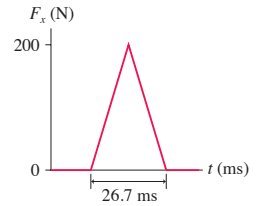
**FIGURE EX9.24**

25. || A 20 g ball of clay traveling east at 3.0 m/s collides with a 30 g ball of clay traveling north at 2.0 m/s. What are the speed and the direction of the resulting 50 g ball of clay?

28. || A 200 g ball is dropped from a height of 2.0 m, bounces on a hard floor, and rebounds to a height of 1.5 m. **FIGURE P9.28** shows the impulse received from the floor. What maximum force does the floor exert on the ball?



**FIGURE P9.28**



**FIGURE P9.29**

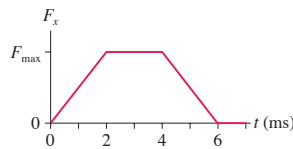
29. || A 500 g cart is released from rest 1.00 m from the bottom of a frictionless,  $30.0^\circ$  ramp. The cart rolls down the ramp and bounces off a rubber block at the bottom. **FIGURE P9.29** shows the force during the collision. After the cart bounces, how far does it roll back up the ramp?
30. || One week in lab, you're given a spring-loaded bar that can be used to strike a metal ball. Your assignment is to measure what size impulse the bar delivers to the ball. You and your lab partner decide to place several balls of different mass on the edge of the lab table, use the striker to launch them horizontally, and measure the horizontal distance to where each ball hits the floor.
- Let the table height be  $h$  and the horizontal distance traveled by the ball be its range  $R$ . Find an expression for the range. The range depends on  $h$ , the ball's mass  $m$ , and the impulse  $J$ .
  - What should you graph the measured range against to get a linear graph whose slope is related to  $J$ ?
  - After measuring the table height to be 1.5 m, you and your partner acquire the following data:

Mass (g)	Range (cm)
100	247
150	175
200	129
250	98

Draw an appropriate graph of the data and, from the slope of the best-fit line, determine the impulse.

### Problems

26. || A 60 g tennis ball with an initial speed of 32 m/s hits a wall and rebounds with the same speed. **FIGURE P9.26** shows the force of the wall on the ball during the collision. What is the value of  $F_{\max}$ , the maximum value of the contact force during the collision?
27. || A tennis player swings her 1000 g racket with a speed of 10 m/s. She hits a 60 g tennis ball that was approaching her at a speed of 20 m/s. The ball rebounds at 40 m/s.
- How fast is her racket moving immediately after the impact? You can ignore the interaction of the racket with her hand for the brief duration of the collision.
  - If the tennis ball and racket are in contact for 10 ms, what is the average force that the racket exerts on the ball? How does this compare to the gravitational force on the ball?



**FIGURE P9.26**

31. | The flowers of the bunchberry plant open with astonishing BIO force and speed, causing the pollen grains to be ejected out of the flower in a mere 0.30 ms at an acceleration of  $2.5 \times 10^4 \text{ m/s}^2$ . If the acceleration is constant, what impulse is delivered to a pollen grain with a mass of  $1.0 \times 10^{-7} \text{ g}$ ?
32. || A particle of mass  $m$  is at rest at  $t = 0$ . Its momentum for  $t > 0$  is given by  $p_x = 6t^2 \text{ kg m/s}$ , where  $t$  is in s. Find an expression for  $F_x(t)$ , the force exerted on the particle as a function of time.
33. || A small rocket to gather weather data is launched straight up. Several seconds into the flight, its velocity is 120 m/s and it is accelerating at  $18 \text{ m/s}^2$ . At this instant, the rocket's mass is 48 kg and it is losing mass at the rate of 0.50 kg/s as it burns fuel. What is the net force on the rocket? **Hint:** Newton's second law was presented in a new form in this chapter.
34. | Three identical train cars, coupled together, are rolling east at speed  $v_0$ . A fourth car traveling east at  $2v_0$  catches up with the three and couples to make a four-car train. A moment later, the train cars hit a fifth car that was at rest on the tracks, and it couples to make a five-car train. What is the speed of the five-car train?

35. || A clay blob of mass  $m_1$ , initially at rest, is pushed across a frictionless surface with constant force  $F$  for a distance  $d$ . It then hits and sticks to a second clay blob of mass  $m_2$  that is at rest. Find an expression for their speed after the collision.
36. || Air-track gliders with masses 300 g, 400 g, and 200 g are lined up and held in place with lightweight springs compressed between them. All three are released at once. The 200 g glider flies off to the right while the 300 g glider goes left. Their position-versus-time graphs, as measured by motion detectors, are shown in **FIGURE P9.36**. What are the direction (right or left) and speed of the 400 g glider that was in the middle?

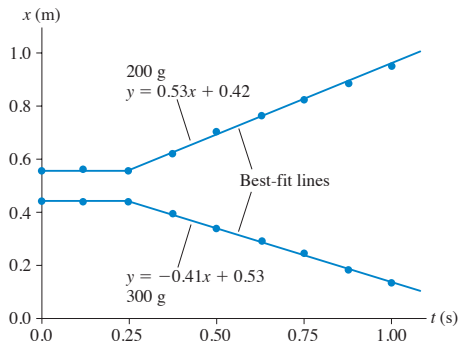


FIGURE P9.36

37. || Most geologists believe that the dinosaurs became extinct 65 million years ago when a large comet or asteroid struck the earth, throwing up so much dust that the sun was blocked out for a period of many months. Suppose an asteroid with a diameter of 2.0 km and a mass of  $1.0 \times 10^{13}$  kg hits the earth with an impact speed of  $4.0 \times 10^4$  m/s.
- What is the earth's recoil speed after such a collision? (Use a reference frame in which the earth was initially at rest.)
  - What percentage is this of the earth's speed around the sun? (Use the astronomical data inside the back cover.)
38. || At the center of a 50-m-diameter circular ice rink, a 75 kg skater traveling north at 2.5 m/s collides with and holds onto a 60 kg skater who had been heading west at 3.5 m/s.
- How long will it take them to glide to the edge of the rink?
  - Where will they reach it? Give your answer as an angle north of west.
39. || Squids rely on jet propulsion to move around. A 1.5 kg squid drifting at 0.40 m/s suddenly expels 0.10 kg of water backward to quickly get itself moving forward at 2.5 m/s. If drag is ignored over the small interval of time needed to expel the water (the impulse approximation), with what speed relative to itself does the squid eject the water?
40. | Two ice skaters, with masses of 50 kg and 75 kg, are at the center of a 60-m-diameter circular rink. The skaters push off against each other and glide to opposite edges of the rink. If the heavier skater reaches the edge in 20 s, how long does the lighter skater take to reach the edge?
41. || A firecracker in a coconut blows the coconut into three pieces. Two pieces of equal mass fly off south and west, perpendicular to each other, at speed  $v_0$ . The third piece has twice the mass as the other two. What are the speed and direction of the third piece? Give the direction as an angle east of north.
42. || One billiard ball is shot east at 2.0 m/s. A second, identical billiard ball is shot west at 1.0 m/s. The balls have a glancing collision, not a head-on collision, deflecting the second ball by 90° and sending it north at 1.41 m/s. What are the speed and direction of the first ball after the collision? Give the direction as an angle south of east.
43. || a. A bullet of mass  $m$  is fired into a block of mass  $M$  that is at rest. The block, with the bullet embedded, slides distance  $d$  across a horizontal surface. The coefficient of kinetic friction is  $\mu_k$ . Find an expression for the bullet's speed  $v_{\text{bullet}}$ .  
b. What is the speed of a 10 g bullet that, when fired into a 10 kg stationary wood block, causes the block to slide 5.0 cm across a wood table?
44. || Fred (mass 60 kg) is running with the football at a speed of 6.0 m/s when he is met head-on by Brutus (mass 120 kg), who is moving at 4.0 m/s. Brutus grabs Fred in a tight grip, and they fall to the ground. Which way do they slide, and how far? The coefficient of kinetic friction between football uniforms and Astroturf is 0.30.
45. | You are part of a search-and-rescue mission that has been called out to look for a lost explorer. You've found the missing explorer, but, as **FIGURE P9.45** shows, you're separated from him by a 200-m-high cliff and a 30-m-wide raging river. To save his life, you need to get a 5.0 kg package of emergency supplies across the river. Unfortunately, you can't throw the package hard enough to make it across. Fortunately, you happen to have a 1.0 kg rocket intended for launching flares. Improvising quickly, you attach a sharpened stick to the front of the rocket, so that it will impale itself into the package of supplies, then fire the rocket at ground level toward the supplies. What minimum speed must the rocket have just before impact in order to save the explorer's life?
46. || An object at rest on a flat, horizontal surface explodes into two fragments, one seven times as massive as the other. The heavier fragment slides 8.2 m before stopping. How far does the lighter fragment slide? Assume that both fragments have the same coefficient of kinetic friction.
47. || A 1500 kg weather rocket accelerates upward at  $10 \text{ m/s}^2$ . It explodes 2.0 s after liftoff and breaks into two fragments, one twice as massive as the other. Photos reveal that the lighter fragment traveled straight up and reached a maximum height of 530 m. What were the speed and direction of the heavier fragment just after the explosion?
48. || In a ballistics test, a 25 g bullet traveling horizontally at 1200 m/s goes through a 30-cm-thick 350 kg stationary target and emerges with a speed of 900 m/s. The target is free to slide on a smooth horizontal surface. What is the target's speed just after the bullet emerges?
49. | Two 500 g blocks of wood are 2.0 m apart on a frictionless table. A 10 g bullet is fired at 400 m/s toward the blocks. It passes all the way through the first block, then embeds itself in the second block. The speed of the first block immediately afterward is 6.0 m/s. What is the speed of the second block after the bullet stops in it?
50. || The skiing duo of Brian (80 kg) and Ashley (50 kg) is always a crowd pleaser. In one routine, Brian, wearing wood skis, starts at the top of a 200-m-long,  $20^\circ$  slope. Ashley waits for him halfway down. As he skis past, she leaps into his arms and he carries her

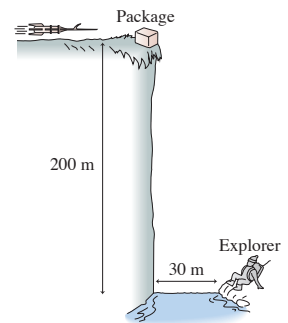


FIGURE P9.45

the rest of the way down. What is their speed at the bottom of the slope?

51. || The stoplight had just changed and a 2000 kg Cadillac had entered the intersection, heading north at 3.0 m/s, when it was struck by a 1000 kg eastbound Volkswagen. The cars stuck together and slid to a halt, leaving skid marks angled  $35^\circ$  north of east. How fast was the Volkswagen going just before the impact?
52. || Ann (mass 50 kg) is standing at the left end of a 15-m-long, 500 kg cart that has frictionless wheels and rolls on a frictionless track. Initially both Ann and the cart are at rest. Suddenly, Ann starts running along the cart at a speed of 5.0 m/s relative to the cart. How far will Ann have run *relative to the ground* when she reaches the right end of the cart?
53. || A ball of mass  $m$  and another ball of mass  $3m$  are placed inside a smooth metal tube with a massless spring compressed between them. When the spring is released, the heavier ball flies out of one end of the tube with speed  $v_0$ . With what speed does the lighter ball emerge from the other end?
54. || Force  $F_x = (10 \text{ N}) \sin(2\pi t/4.0 \text{ s})$  is exerted on a 250 g particle during the interval  $0 \text{ s} \leq t \leq 2.0 \text{ s}$ . If the particle starts from rest, what is its speed at  $t = 2.0 \text{ s}$ ?
55. || A 500 g particle has velocity  $v_x = -5.0 \text{ m/s}$  at  $t = -2 \text{ s}$ . Force  $F_x = (4 - t^2) \text{ N}$  is exerted on the particle between  $t = -2 \text{ s}$  and  $t = 2 \text{ s}$ . This force increases from 0 N at  $t = -2 \text{ s}$  to 4 N at  $t = 0 \text{ s}$  and then back to 0 N at  $t = 2 \text{ s}$ . What is the particle's velocity at  $t = 2 \text{ s}$ ?
56. || A 30 ton rail car and a 90 ton rail car, initially at rest, are connected together with a giant but massless compressed spring between them. When released, the 30 ton car is pushed away at a speed of 4.0 m/s relative to the 90 ton car. What is the speed of the 30 ton car relative to the ground?
57. || A 75 kg shell is fired with an initial speed of 125 m/s at an angle  $55^\circ$  above horizontal. Air resistance is negligible. At its highest point, the shell explodes into two fragments, one four times more massive than the other. The heavier fragment lands directly below the point of the explosion. If the explosion exerts forces only in the horizontal direction, how far from the launch point does the lighter fragment land?
58. || A proton (mass 1 u) is shot at a speed of  $5.0 \times 10^7 \text{ m/s}$  toward a gold target. The nucleus of a gold atom (mass 197 u) repels the proton and deflects it straight back toward the source with 90% of its initial speed. What is the recoil speed of the gold nucleus?
59. || A proton (mass 1 u) is shot toward an unknown target nucleus at a speed of  $2.50 \times 10^6 \text{ m/s}$ . The proton rebounds with its speed reduced by 25% while the target nucleus acquires a speed of  $3.12 \times 10^5 \text{ m/s}$ . What is the mass, in atomic mass units, of the target nucleus?
60. || The nucleus of the polonium isotope  $^{214}\text{Po}$  (mass 214 u) is radioactive and decays by emitting an alpha particle (a helium nucleus with mass 4 u). Laboratory experiments measure the speed of the alpha particle to be  $1.92 \times 10^7 \text{ m/s}$ . Assuming the polonium nucleus was initially at rest, what is the recoil speed of the nucleus that remains after the decay?
61. || A neutron is an electrically neutral subatomic particle with a mass just slightly greater than that of a proton. A free neutron is radioactive and decays after a few minutes into other subatomic particles. In one experiment, a neutron at rest was observed to decay into a proton (mass  $1.67 \times 10^{-27} \text{ kg}$ ) and an electron (mass  $9.11 \times 10^{-31} \text{ kg}$ ). The proton and electron were shot out back-to-back. The proton speed was measured to be

$1.0 \times 10^5 \text{ m/s}$  and the electron speed was  $3.0 \times 10^7 \text{ m/s}$ . No other decay products were detected.

a. Was momentum conserved in the decay of this neutron?

**NOTE** ▶ Experiments such as this were first performed in the 1930s and seemed to indicate a failure of the law of conservation of momentum. In 1933, Wolfgang Pauli postulated that the neutron might have a *third* decay product that is virtually impossible to detect. Even so, it can carry away just enough momentum to keep the total momentum conserved. This proposed particle was named the *neutrino*, meaning “little neutral one.” Neutrinos were, indeed, discovered nearly 20 years later. ◀

b. If a neutrino was emitted in the above neutron decay, in which direction did it travel? Explain your reasoning.

c. How much momentum did this neutrino “carry away” with it?

62. || A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay traveling  $30^\circ$  south of west at 1.0 m/s. What are the speed and direction of the resulting 50 g blob of clay?

63. || **FIGURE P9.63** shows a collision between three balls of clay. The three hit simultaneously and stick together. What are the speed and direction of the resulting blob of clay?

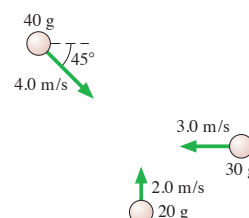


FIGURE P9.63

64. || A 2100 kg truck is traveling east through an intersection at 2.0 m/s when it is hit simultaneously from the side and the rear. (Some people have all the luck!) One car is a 1200 kg compact traveling north at 5.0 m/s. The other is a 1500 kg midsize traveling east at 10 m/s. The three vehicles become entangled and slide as one body. What are their speed and direction just after the collision?
65. || The carbon isotope  $^{14}\text{C}$  is used for carbon dating of archaeological artifacts.  $^{14}\text{C}$  (mass  $2.34 \times 10^{-26} \text{ kg}$ ) decays by the process known as *beta decay* in which the nucleus emits an electron (the beta particle) and a subatomic particle called a neutrino. In one such decay, the electron and the neutrino are emitted at right angles to each other. The electron (mass  $9.11 \times 10^{-31} \text{ kg}$ ) has a speed of  $5.0 \times 10^7 \text{ m/s}$  and the neutrino has a momentum of  $8.0 \times 10^{-24} \text{ kg m/s}$ . What is the recoil speed of the nucleus?

In Problems 66 through 69 you are given the equation used to solve a problem. For each of these, you are to

- a. Write a realistic problem for which this is the correct equation.
  - b. Finish the solution of the problem, including a pictorial representation.
66.  $(0.10 \text{ kg})(40 \text{ m/s}) - (0.10 \text{ kg})(-30 \text{ m/s}) = \frac{1}{2}(1400 \text{ N}) \Delta t$
  67.  $(600 \text{ g})(4.0 \text{ m/s}) = (400 \text{ g})(3.0 \text{ m/s}) + (200 \text{ g})(v_{ix})_2$
  68.  $(3000 \text{ kg})v_{ix} = (2000 \text{ kg})(5.0 \text{ m/s}) + (1000 \text{ kg})(-4.0 \text{ m/s})$
  69.  $(50 \text{ g})(v_{ix})_1 + (100 \text{ g})(7.5 \text{ m/s}) = (150 \text{ g})(1.0 \text{ m/s})$

## Challenge Problems

70. A 1000 kg cart is rolling to the right at 5.0 m/s. A 70 kg man is standing on the right end of the cart. What is the speed of the cart if the man suddenly starts running to the left with a speed of 10 m/s relative to the cart?

71. A spaceship of mass  $2.0 \times 10^6$  kg is cruising at a speed of  $5.0 \times 10^6$  m/s when the antimatter reactor fails, blowing the ship into three pieces. One section, having a mass of  $5.0 \times 10^5$  kg, is blown straight backward with a speed of  $2.0 \times 10^6$  m/s. A second piece, with mass  $8.0 \times 10^5$  kg, continues forward at  $1.0 \times 10^6$  m/s. What are the direction and speed of the third piece?
72. A 20 kg wood ball hangs from a 2.0-m-long wire. The maximum tension the wire can withstand without breaking is 400 N. A 1.0 kg projectile traveling horizontally hits and embeds itself in the wood ball. What is the greatest speed this projectile can have without causing the cable to break?
73. A two-stage rocket is traveling at 1200 m/s with respect to the earth when the first stage runs out of fuel. Explosive bolts release the first stage and push it backward with a speed of 35 m/s relative to the second stage. The first stage is three times as massive as the second stage. What is the speed of the second stage after the separation?
74. You are the ground-control commander of a 2000 kg scientific rocket that is approaching Mars at a speed of 25,000 km/h. It needs to quickly slow to 15,000 km/h to begin a controlled descent to the surface. If the rocket enters the Martian atmosphere too fast it will burn up, and if it enters too slowly, it will use up its maneuvering fuel before reaching the surface and will crash. The rocket has a new braking system: Several 5.0 kg “bullets” on the front of the rocket can be fired straight ahead. Each has a high-explosive charge that fires it at a speed of 139,000 m/s relative to the rocket. You need to send the rocket an instruction to tell it how many bullets to fire. Success will bring you fame and glory, but failure of this \$500,000,000 mission will ruin your career. How many bullets will you tell the rocket to fire?
75. You are a world-famous physicist-lawyer defending a client who has been charged with murder. It is alleged that your client,

Mr. Smith, shot the victim, Mr. Wesson. The detective who investigated the scene of the crime found a second bullet, from a shot that missed Mr. Wesson, that had embedded itself into a chair. You arise to cross-examine the detective.

You: In what type of chair did you find the bullet?

Det: A wooden chair.

You: How massive was this chair?

Det: It had a mass of 20 kg.

You: How did the chair respond to being struck with a bullet?

Det: It slid across the floor.

You: How far?

Det: A good three centimeters. The slide marks on the dusty floor are quite distinct.

You: What kind of floor was it?

Det: A wood floor, very nice oak planks.

You: What was the mass of the bullet you retrieved from the chair?

Det: Its mass was 10 g.

You: And how far had it penetrated into the chair?

Det: A distance of 1.5 cm.

You: Have you tested the gun you found in Mr. Smith's possession?

Det: I have.

You: What is the muzzle velocity of bullets fired from that gun?

Det: The muzzle velocity is 450 m/s.

You: And the barrel length?

Det: The gun has a barrel length of 16 cm.

With only a slight hesitation, you turn confidently to the jury and proclaim, “My client's gun did not fire these shots!” How are you going to convince the jury and the judge?

### STOP TO THINK ANSWERS

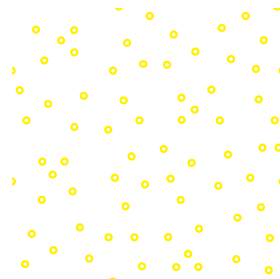
**Stop to Think 9.1: f.** The cart is initially moving in the negative  $x$ -direction, so  $p_{ix} = -20$  kg m/s. After it bounces,  $p_{fx} = 10$  kg m/s. Thus  $\Delta p = (10 \text{ kg m/s}) - (-20 \text{ kg m/s}) = 30 \text{ kg m/s}$ .

**Stop to Think 9.2: b.** The clay ball goes from  $v_{ix} = v$  to  $v_{fx} = 0$ , so  $J_{\text{clay}} = \Delta p_x = -mv$ . The rubber ball rebounds, going from  $v_{ix} = v$  to  $v_{fx} = -v$  (same speed, opposite direction). Thus  $J_{\text{rubber}} = \Delta p_x = -2mv$ . The rubber ball has a larger momentum change, and this requires a larger impulse.

**Stop to Think 9.3: Less than.** The ball's momentum  $m_B v_B$  is the same in both cases. Momentum is conserved, so the *total* momentum is the same after both collisions. The ball that rebounds from C has *negative* momentum, so C must have a larger momentum than A.

**Stop to Think 9.4: c.** Momentum conservation requires  $(m_1 + m_2) \times v_f = m_1 v_1 + m_2 v_2$ . Because  $v_1 > v_2$ , it must be that  $(m_1 + m_2) \times v_f = m_1 v_1 + m_2 v_2 > m_1 v_2 + m_2 v_2 = (m_1 + m_2) v_2$ . Thus  $v_f > v_2$ . Similarly,  $v_2 < v_1$  so  $(m_1 + m_2) v_f = m_1 v_1 + m_2 v_2 < m_1 v_1 + m_2 v_1 = (m_1 + m_2) v_1$ . Thus  $v_f < v_1$ . The collision causes  $m_1$  to slow down and  $m_2$  to speed up.

**Stop to Think 9.5: Right end.** The pieces started at rest, so the total momentum of the system is zero. It's an isolated system, so the total momentum after the explosion is still zero. The 6 g piece has momentum  $6v$ . The 4 g piece, with velocity  $-2v$ , has momentum  $-8v$ . The combined momentum of these two pieces is  $-2v$ . In order for  $P$  to be zero, the third piece must have a *positive* momentum ( $+2v$ ) and thus a positive velocity.





## SUMMARY

The goals of Chapter 10 have been to introduce the concept of energy and the basic energy model.

### General Principles

#### Law of Conservation of Mechanical Energy

If a system is isolated and frictionless, then the mechanical energy  $E_{\text{mech}} = K + U$  of the system is conserved. Thus

$$K_f + U_f = K_i + U_i$$

- $K$  is the sum of the kinetic energies of all particles.
- $U$  is the sum of all potential energies.

#### Solving Energy Conservation Problems

**MODEL** Choose an isolated system without friction or other losses of mechanical energy.

**VISUALIZE** Draw a before-and-after pictorial representation.

**SOLVE** Use the law of conservation of energy:

$$K_f + U_f = K_i + U_i$$

**ASSESS** Is the result reasonable?

### Important Concepts

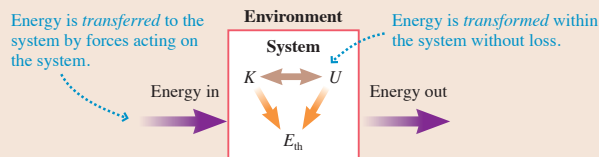
**Kinetic energy** is an energy of motion:  $K = \frac{1}{2}mv^2$ .

**Potential energy** is an energy of position.

- **Gravitational:**  $U_g = mgy$
- **Elastic:**  $U_s = \frac{1}{2}k(\Delta s)^2$

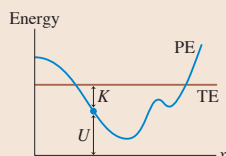
**Thermal energy** is due to atomic motions. Hotter objects have more thermal energy.

#### Basic Energy Model



#### Energy diagrams

These diagrams show the potential-energy curve PE and the total mechanical energy line TE.



- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a **turning point**.
- Minima in the PE curve are points of **stable equilibrium**. Maxima are points of **unstable equilibrium**.
- Regions where PE is greater than TE are forbidden.

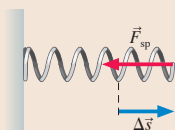
### Applications

#### Hooke's law

The restoring force of an ideal spring is

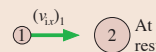
$$(F_{\text{sp}})_s = -k \Delta s$$

where  $k$  is the spring constant and  $\Delta s = s - s_e$  is the displacement from equilibrium.



#### Perfectly elastic collisions

Both mechanical energy and momentum are conserved.



$$(v_{1x})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{1x})_1 \quad (v_{1x})_2 = \frac{2m_1}{m_1 + m_2} (v_{1x})_1$$

If ball 2 is moving, transform to a reference frame in which ball 2 is at rest.

### Terms and Notation

energy  
basic energy model  
kinetic energy,  $K$   
gravitational potential energy,  $U_g$   
joule, J  
mechanical energy  
law of conservation of mechanical energy

restoring force  
elastic  
equilibrium length,  $L_0$   
displacement from equilibrium,  $\Delta s$   
spring constant,  $k$   
Hooke's law

elastic potential energy,  $U_s$   
energy diagram  
stable equilibrium  
unstable equilibrium  
perfectly elastic collision



# CONCEPTUAL QUESTIONS

1. Upon what basic quantity does kinetic energy depend? Upon what basic quantity does potential energy depend?
2. Can kinetic energy ever be negative? Can gravitational potential energy ever be negative? For each, give a plausible *reason* for your answer without making use of any equations.
3. If a particle's speed increases by a factor of 3, by what factor does its kinetic energy change?
4. Particle A has half the mass and eight times the kinetic energy of particle B. What is the speed ratio  $v_A/v_B$ ?
5. A roller-coaster car rolls down a frictionless track, reaching speed  $v_0$  at the bottom. If you want the car to go twice as fast at the bottom, by what factor must you increase the height of the track? Explain.
6. The three balls in **FIGURE Q10.6**, which have equal masses, are fired with equal speeds from the same height above the ground. Rank in order, from largest to smallest, their speeds  $v_a$ ,  $v_b$ , and  $v_c$  as they hit the ground. Explain.

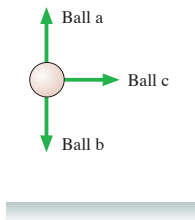


FIGURE Q10.6

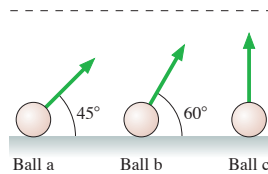


FIGURE Q10.7

7. The three balls in **FIGURE Q10.7**, which have equal masses, are fired with equal speeds at the angles shown. Rank in order, from largest to smallest, their speeds  $v_a$ ,  $v_b$ , and  $v_c$  as they cross the dashed horizontal line. Explain. (All three are fired with sufficient speed to reach the line.)
8. A spring has an unstretched length of 10 cm. It exerts a restoring force  $F$  when stretched to a length of 11 cm.
  - a. For what length of the spring is its restoring force  $3F$ ?
  - b. At what compressed length is the restoring force  $2F$ ?
9. The left end of a spring is attached to a wall. When Bob pulls on the right end with a 200 N force, he stretches the spring by 20 cm. The same spring is then used for a tug-of-war between Bob and Carlos. Each pulls on his end of the spring with a 200 N force. How far does the spring stretch? Explain.

10. Rank in order, from most to least, the elastic potential energy  $(U_s)_a$  to  $(U_s)_d$  stored in the springs of **FIGURE Q10.10**. Explain.

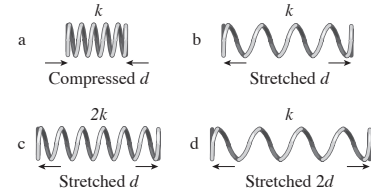


FIGURE Q10.10

11. A spring is compressed 1.0 cm. How far must you compress a spring with twice the spring constant to store the same amount of energy?
12. A spring gun shoots out a plastic ball at speed  $v_0$ . The spring is then compressed twice the distance it was on the first shot. By what factor is the ball's speed increased? Explain.
13. A particle with the potential energy shown in **FIGURE Q10.13** is moving to the right at  $x = 5$  m with total energy  $E$ .
  - a. At what value or values of  $x$  is this particle's speed a maximum?
  - b. Does this particle have a turning point or points in the range of  $x$  covered by the graph? If so, where?
  - c. If  $E$  is changed appropriately, could the particle remain at rest at any point or points in the range of  $x$  covered by the graph? If so, where?

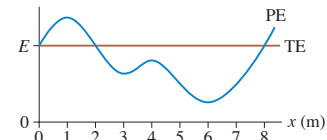


FIGURE Q10.13

14. Two balls of clay of known masses hang from the ceiling on massless strings of equal length. They barely touch when both hang at rest. One ball is pulled back until its string is at  $45^\circ$ , then released. It swings down, collides with the second ball, and they stick together. To determine the angle to which the balls swing on the opposite side, would you invoke (a) conservation of momentum, (b) conservation of mechanical energy, (c) both, (d) either but not both, or (e) these laws alone are not sufficient to find the angle? Explain.

# EXERCISES AND PROBLEMS

Problems labeled   integrate material from earlier chapters.

## Exercises

### Section 10.2 Kinetic Energy and Gravitational Potential Energy

1. Which has the larger kinetic energy, a 10 g bullet fired at 500 m/s or a 75 kg student running at 5.5 m/s?
2. The lowest point in Death Valley is 85 m below sea level. The summit of nearby Mt. Whitney has an elevation of 4420 m. What is the change in potential energy of an energetic 65 kg hiker who makes it from the floor of Death Valley to the top of Mt. Whitney?
3. At what speed does a 1000 kg compact car have the same kinetic energy as a 20,000 kg truck going 25 km/h?

4. I a. What is the kinetic energy of a 1500 kg car traveling at a speed of 30 m/s ( $\approx 65$  mph)?  
b. From what height would the car have to be dropped to have this same amount of kinetic energy just before impact?  
c. Does your answer to part b depend on the car's mass?
5. I A boy reaches out of a window and tosses a ball straight up with a speed of 10 m/s. The ball is 20 m above the ground as he releases it. Use energy to find  
a. The ball's maximum height above the ground.  
b. The ball's speed as it passes the window on its way down.  
c. The speed of impact on the ground.
6. I a. With what minimum speed must you toss a 100 g ball straight up to just touch the 10-m-high roof of the gymnasium if you release the ball 1.5 m above the ground? Solve this problem using energy.  
b. With what speed does the ball hit the ground?
7. II A mother has four times the mass of her young son. Both are running with the same kinetic energy. What is the ratio  $v_{\text{son}}/v_{\text{mother}}$  of their speeds?

### Section 10.3 A Closer Look at Gravitational Potential Energy

8. I A 55 kg skateboarder wants to just make it to the upper edge of a "quarter pipe," a track that is one-quarter of a circle with a radius of 3.0 m. What speed does he need at the bottom?
9. II What minimum speed does a 100 g puck need to make it to the top of a 3.0-m-long,  $20^\circ$  frictionless ramp?
10. II A pendulum is made by tying a 500 g ball to a 75-cm-long string. The pendulum is pulled  $30^\circ$  to one side, then released.  
a. What is the ball's speed at the lowest point of its trajectory?  
b. To what angle does the pendulum swing on the other side?
11. II A 20 kg child is on a swing that hangs from 3.0-m-long chains. What is her maximum speed if she swings out to a  $45^\circ$  angle?
12. II A 1500 kg car traveling at 10 m/s suddenly runs out of gas while approaching the valley shown in **FIGURE EX10.12**. The alert driver immediately puts the car in neutral so that it will roll. What will be the car's speed as it coasts into the gas station on the other side of the valley?

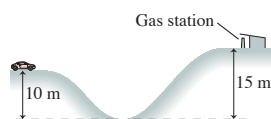


FIGURE EX10.12

### Section 10.4 Restoring Forces and Hooke's Law

13. I You need to make a spring scale for measuring mass. You want each 1.0 cm length along the scale to correspond to a mass difference of 100 g. What should be the value of the spring constant?
14. II A 10-cm-long spring is attached to the ceiling. When a 2.0 kg mass is hung from it, the spring stretches to a length of 15 cm.  
a. What is the spring constant  $k$ ?  
b. How long is the spring when a 3.0 kg mass is suspended from it?
15. II A 60 kg student is standing atop a spring in an elevator as it accelerates upward at  $3.0 \text{ m/s}^2$ . The spring constant is 2500 N/m. By how much is the spring compressed?
16. II A spring hanging from the ceiling has equilibrium length  $L_0$ . Hanging mass  $m$  from the spring stretches its length to  $L_1$ . Find an expression for the spring's length  $L_3$  when mass  $3m$  hangs from it.

17. II A 5.0 kg mass hanging from a spring scale is slowly lowered onto a vertical spring, as shown in **FIGURE EX10.17**. The scale reads in newtons.  
a. What does the spring scale read just before the mass touches the lower spring?  
b. The scale reads 20 N when the lower spring has been compressed by 2.0 cm. What is the value of the spring constant for the lower spring?  
c. At what compression length will the scale read zero?

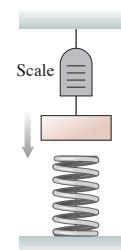


FIGURE EX10.17

### Section 10.5 Elastic Potential Energy

18. I How far must you stretch a spring with  $k = 1000 \text{ N/m}$  to store 200 J of energy?
19. I A stretched spring stores 2.0 J of energy. How much energy will be stored if the spring is stretched three times as far?
20. II A student places her 500 g physics book on a frictionless table. She pushes the book against a spring, compressing the spring by 4.0 cm, then releases the book. What is the book's speed as it slides away? The spring constant is 1250 N/m.
21. I A block sliding along a horizontal frictionless surface with speed  $v$  collides with a spring and compresses it by 2.0 cm. What will be the compression if the same block collides with the spring at a speed of  $2v$ ?
22. II A 10 kg runaway grocery cart runs into a spring with spring constant 250 N/m and compresses it by 60 cm. What was the speed of the cart just before it hit the spring?
23. I The desperate contestants on a TV survival show are very hungry. The only food they can see is some fruit hanging on a branch high in a tree. Fortunately, they have a spring they can use to launch a rock. The spring constant is 1000 N/m, and they can compress the spring a maximum of 30 cm. All the rocks on the island seem to have a mass of 400 g.  
a. With what speed does the rock leave the spring?  
b. If the fruit hangs 15 m above the ground, will they feast or go hungry?
24. II As a 15,000 kg jet plane lands on an aircraft carrier, its tail hook snags a cable to slow it down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, what was the plane's landing speed?

### Section 10.6 Energy Diagrams

25. I **FIGURE EX10.25** is the potential-energy diagram for a 20 g particle that is released from rest at  $x = 1.0 \text{ m}$ .  
a. Will the particle move to the right or to the left? How can you tell?  
b. What is the particle's maximum speed? At what position does it have this speed?  
c. Where are the turning points of the motion?

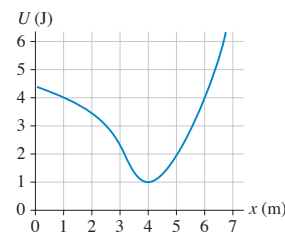
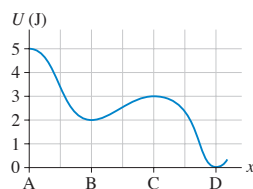


FIGURE EX10.25

26. || FIGURE EX10.26 is the potential-energy diagram for a 500 g particle that is released from rest at A. What are the particle's speeds at B, C, and D?

FIGURE EX10.26



27. | a. In FIGURE EX10.27, what minimum speed does a 100 g particle need at point A to reach point B?  
b. What minimum speed does a 100 g particle need at point B to reach point A?

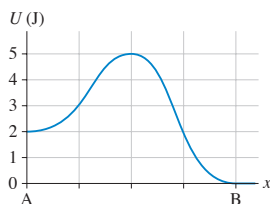


FIGURE EX10.27

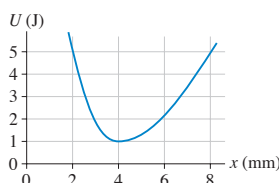


FIGURE EX10.28

28. || In FIGURE EX10.28, what is the maximum speed of a 2.0 g particle that oscillates between  $x = 2.0$  mm and  $x = 8.0$  mm?

### Section 10.7 Elastic Collisions

29. | A 50 g marble moving at 2.0 m/s strikes a 20 g marble at rest. What is the speed of each marble immediately after the collision?
30. | A proton is traveling to the right at  $2.0 \times 10^7$  m/s. It has a head-on perfectly elastic collision with a carbon atom. The mass of the carbon atom is 12 times the mass of the proton. What are the speed and direction of each after the collision?
31. | Ball 1, with a mass of 100 g and traveling at 10 m/s, collides head-on with ball 2, which has a mass of 300 g and is initially at rest. What is the final velocity of each ball if the collision is (a) perfectly elastic? (b) perfectly inelastic?
32. || A 50 g ball of clay traveling at speed  $v_0$  hits and sticks to a 1.0 kg brick sitting at rest on a frictionless surface.  
a. What is the speed of the brick after the collision?  
b. What percentage of the mechanical energy is lost in this collision?

### Problems

33. | BIO The maximum energy a bone can absorb without breaking is surprisingly small. Experimental data show that the leg bones of a healthy, 60 kg human can absorb about 200 J.  
a. From what maximum height could a 60 kg person jump and land rigidly upright on both feet without breaking his legs? Assume that all energy is absorbed by the leg bones in a rigid landing.  
b. People jump safely from much greater heights than this. Explain how this is possible.
34. | You're driving at 35 km/h when the road suddenly descends 15 m into a valley. You take your foot off the accelerator and coast down the hill. Just as you reach the bottom you see the policeman hiding behind the speed limit sign that reads "70 km/h." Are you going to get a speeding ticket?

35. || A cannon tilted up at a  $30^\circ$  angle fires a cannon ball at 80 m/s from atop a 10-m-high fortress wall. What is the ball's impact speed on the ground below?
36. || You have a ball of unknown mass, a spring with spring constant 950 N/m, and a meter stick. You use various compressions of the spring to launch the ball vertically, then use the meter stick to measure the ball's maximum height above the launch point. Your data are as follows:

Compression (cm)	Height (cm)
2.0	32
3.0	65
4.0	115
5.0	189

Use an appropriate graph of the data to determine the ball's mass.

37. || A very slippery ice cube slides in a *vertical* plane around the inside of a smooth, 20-cm-diameter horizontal pipe. The ice cube's speed at the bottom of the circle is 3.0 m/s.  
a. What is the ice cube's speed at the top?  
b. Find an algebraic expression for the ice cube's speed when it is at angle  $\theta$ , where the angle is measured counterclockwise from the bottom of the circle. Your expression should give 3.0 m/s for  $\theta = 0^\circ$  and your answer to part a for  $\theta = 180^\circ$ .
38. | A 50 g rock is placed in a slingshot and the rubber band is stretched. The force of the rubber band on the rock is shown by the graph in FIGURE P10.38.  
a. Is the rubber band stretched to the right or to the left? How can you tell?  
b. Does this rubber band obey Hooke's law? Explain.  
c. What is the rubber band's spring constant  $k$ ?  
d. The rubber band is stretched 30 cm and then released. What is the speed of the rock?

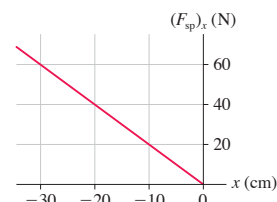


FIGURE P10.38

39. | BIO The elastic energy stored in your tendons can contribute up to 35% of your energy needs when running. Sports scientists find that (on average) the knee extensor tendons in sprinters stretch 41 mm while those of nonathletes stretch only 33 mm. The spring constant of the tendon is the same for both groups, 33 N/mm. What is the difference in maximum stored energy between the sprinters and the nonathletes?
40. || The spring in FIGURE P10.40a is compressed by  $\Delta x$ . It launches the block across a frictionless surface with speed  $v_0$ . The two springs in FIGURE P10.40b are identical to the spring of Figure P10.40a. They are compressed by the same  $\Delta x$  and used to launch the same block. What is the block's speed now?

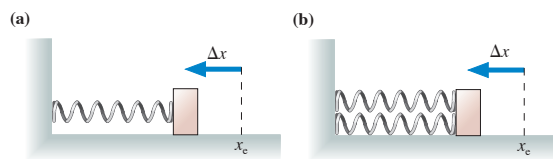


FIGURE P10.40

41. || The spring in **FIGURE P10.41a** is compressed by  $\Delta x$ . It launches the block across a frictionless surface with speed  $v_0$ . The two springs in **FIGURE P10.41b** are identical to the spring of **FIGURE P10.41a**. They are compressed the same *total*  $\Delta x$  and used to launch the same block. What is the block's speed now?

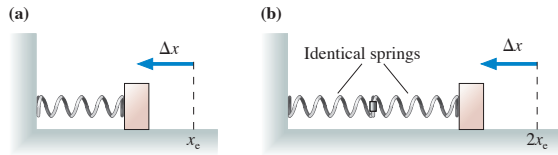


FIGURE P10.41

42. || a. A block of mass  $m$  can slide up and down a frictionless slope tilted at angle  $\theta$ . The block is pressed against a spring at the bottom of the slope, compressing the spring (with spring constant  $k$ ) by  $\Delta x$ , then released. Find an expression for the block's maximum height  $h$  above its starting point.  
 b. A 50 g ice cube can slide up and down a frictionless  $30^\circ$  slope. At the bottom, a spring with spring constant 25 N/m is compressed 10 cm and used to launch the ice cube up the slope. How high does it go?
43. || A package of mass  $m$  is released from rest at a warehouse loading dock and slides down the 3.0-m-high, frictionless chute of **FIGURE P10.43** to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass  $2m$ , from the bottom of the chute.  
 a. Suppose the packages stick together. What is their common speed after the collision?  
 b. Suppose the collision between the packages is perfectly elastic. To what height does the package of mass  $m$  rebound?



FIGURE P10.43

44. || A 100 g granite cube slides down a  $40^\circ$  frictionless ramp. At the bottom, just as it exits onto a horizontal table, it collides with a 200 g steel cube at rest. How high above the table should the granite cube be released to give the steel cube a speed of 150 cm/s?
45. || A 1000 kg safe is 2.0 m above a heavy-duty spring when the rope holding the safe breaks. The safe hits the spring and compresses it 50 cm. What is the spring constant of the spring?
46. || A vertical spring with  $k = 490$  N/m is standing on the ground. You are holding a 5.0 kg block just above the spring, not quite touching it.  
 a. How far does the spring compress if you let go of the block suddenly?  
 b. How far does the spring compress if you slowly lower the block to the point where you can remove your hand without disturbing it?  
 c. Why are your two answers different?

47. || You have been hired to design a spring-launched roller coaster that will carry two passengers per car. The car goes up a 10-m-high hill, then descends 15 m to the track's lowest point. You've determined that the spring can be compressed a maximum of 2.0 m and that a loaded car will have a maximum mass of 400 kg. For safety reasons, the spring constant should be 10% larger than the minimum needed for the car to just make it over the top.

- a. What spring constant should you specify?  
 b. What is the maximum speed of a 350 kg car if the spring is compressed the full amount?

48. || It's been a great day of new, frictionless snow. Julie starts at the top of the  $60^\circ$  slope shown in **FIGURE P10.48**. At the bottom, a circular arc carries her through a  $90^\circ$  turn, and she then launches off a 3.0-m-high ramp. How far horizontally is her touchdown point from the end of the ramp?

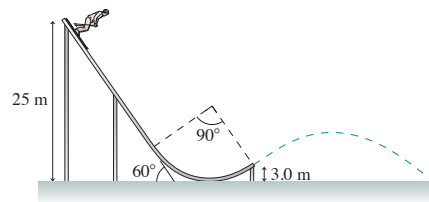


FIGURE P10.48

49. || A 100 g block on a frictionless table is firmly attached to one end of a spring with  $k = 20$  N/m. The other end of the spring is anchored to the wall. A 20 g ball is thrown horizontally toward the block with a speed of 5.0 m/s.  
 a. If the collision is perfectly elastic, what is the ball's speed immediately after the collision?  
 b. What is the maximum compression of the spring?  
 c. Repeat parts a and b for the case of a perfectly inelastic collision.

50. || You have been asked to design a "ballistic spring system" to measure the speed of bullets. A bullet of mass  $m$  is fired into a block of mass  $M$ . The block, with the embedded bullet, then slides across a frictionless table and collides with a horizontal spring whose spring constant is  $k$ . The opposite end of the spring is anchored to a wall. The spring's maximum compression  $d$  is measured.

- a. Find an expression for the bullet's speed  $v_B$  in terms of  $m$ ,  $M$ ,  $k$ , and  $d$ .  
 b. What was the speed of a 5.0 g bullet if the block's mass is 2.0 kg and if the spring, with  $k = 50$  N/m, was compressed by 10 cm?

- c. What fraction of the bullet's energy is "lost"? Where did it go?

51. || You have been asked to design a "ballistic spring system" to measure the speed of bullets. A spring whose spring constant is  $k$  is suspended from the ceiling. A block of mass  $M$  hangs from the spring. A bullet of mass  $m$  is fired vertically upward into the bottom of the block and stops in the block. The spring's maximum compression  $d$  is measured.

- a. Find an expression for the bullet's speed  $v_B$  in terms of  $m$ ,  $M$ ,  $k$ , and  $d$ .  
 b. What was the speed of a 10 g bullet if the block's mass is 2.0 kg and if the spring, with  $k = 50$  N/m, was compressed by 45 cm?

52. || In FIGURE P10.52, a block of mass  $m$  slides along a frictionless track with speed  $v_m$ . It collides with a stationary block of mass  $M$ . Find an expression for the minimum value of  $v_m$  that will allow the second block to circle the loop-the-loop without falling off if the collision is (a) perfectly inelastic or (b) perfectly elastic.

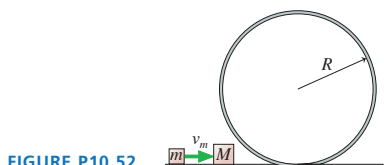


FIGURE P10.52

53. || A block of mass  $m$  slides down a frictionless track, then around the inside of a circular loop-the-loop of radius  $R$ . From what minimum height  $h$  must the block start to make it around without falling off? Give your answer as a multiple of  $R$ .
54. || A new event has been proposed for the Winter Olympics. As seen in FIGURE P10.54, an athlete will sprint 100 m, starting from rest, then leap onto a 20 kg bobsled. The person and bobsled will then slide down a 50-m-long ice-covered ramp, sloped at  $20^\circ$ , and into a spring with a carefully calibrated spring constant of 2000 N/m. The athlete who compresses the spring the farthest wins the gold medal. Lisa, whose mass is 40 kg, has been training for this event. She can reach a maximum speed of 12 m/s in the 100 m dash.
- How far will Lisa compress the spring?
  - The Olympic committee has very exact specifications about the shape and angle of the ramp. Is this necessary? What factors about the ramp are important?

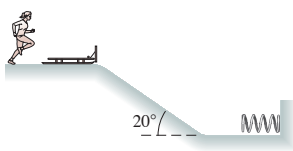


FIGURE P10.54

55. || A 20 g ball is fired horizontally with speed  $v_0$  toward a 100 g ball hanging motionless from a 1.0-m-long string. The balls undergo a head-on, perfectly elastic collision, after which the 100 g ball swings out to a maximum angle  $\theta_{\max} = 50^\circ$ . What was  $v_0$ ?
56. || A 100 g ball moving to the right at 4.0 m/s collides head-on with a 200 g ball that is moving to the left at 3.0 m/s.
- If the collision is perfectly elastic, what are the speed and direction of each ball after the collision?
  - If the collision is perfectly inelastic, what are the speed and direction of the combined balls after the collision?
57. || A 100 g ball moving to the right at 4.0 m/s catches up and collides with a 400 g ball that is moving to the right at 1.0 m/s. If the collision is perfectly elastic, what are the speed and direction of each ball after the collision?
58. || FIGURE P10.58 shows the potential energy of a 500 g particle as it moves along the  $x$ -axis. Suppose the particle's mechanical energy is 12 J.
- Where are the particle's turning points?
  - What is the particle's speed when it is at  $x = 6.0$  m?
  - What is the particle's maximum speed? At what position or positions does this occur?
  - Write a description of the motion of the particle as it moves from the left turning point to the right turning point.
  - Suppose the particle's energy is lowered to 4.0 J. Describe the possible motions.

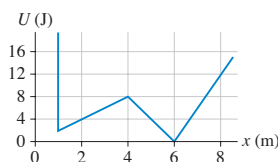


FIGURE P10.58

59. || A particle has potential energy

$$U(x) = x + \sin((2 \text{ rad/m})x)$$

over the range  $0 \text{ m} \leq x \leq \pi \text{ m}$ .

- Where are the equilibrium positions in this range?
  - For each, is it a point of stable or unstable equilibrium?
60. || Protons and neutrons (together called *nucleons*) are held together in the nucleus of an atom by a force called the *strong force*. At very small separations, the strong force between two nucleons is larger than the repulsive electrical force between two protons—hence its name. But the strong force quickly weakens as the distance between the protons increases. A well-established model for the potential energy of two nucleons interacting via the strong force is

$$U = U_0[1 - e^{-x/x_0}]$$

where  $x$  is the distance between the centers of the two nucleons,  $x_0$  is a constant having the value  $x_0 = 2.0 \times 10^{-15} \text{ m}$ , and  $U_0 = 6.0 \times 10^{-11} \text{ J}$ .

- Calculate and draw an accurate potential-energy curve from  $x = 0 \text{ m}$  to  $x = 10 \times 10^{-15} \text{ m}$ . Either use your calculator to compute the value at several points or use computer software.
  - Quantum effects are essential for a proper understanding of how nucleons behave. Nonetheless, let us innocently consider two neutrons *as if* they were small, hard, electrically neutral spheres of mass  $1.67 \times 10^{-27} \text{ kg}$  and diameter  $1.0 \times 10^{-15} \text{ m}$ . (We will consider neutrons rather than protons so as to avoid complications from the electric forces between protons.) You are going to hold two neutrons  $5.0 \times 10^{-15} \text{ m}$  apart, measured between their centers, then release them. Draw the total energy line for this situation on your diagram of part a.
  - What is the speed of each neutron as they crash together? Keep in mind that *both* neutrons are moving.
61. || A 50 g air-track glider is repelled by a post fixed at one end of the track. It is hypothesized that the glider's potential energy is  $U = c/x$ , where  $x$  is the distance from the post and  $c$  is an unknown constant. To test this hypothesis, you launch the glider with the same speed at various distances from the post and then use a motion detector to measure its speed when it is 1.0 m from the post. Your data are as follows:

Initial distance (cm)	Speed at 1.0 m (m/s)
2.0	1.40
4.0	0.98
6.0	0.79
8.0	0.68

- Do the data support the hypothesis? To find out, you'll need to compare the shape of an appropriate graph to a theoretical prediction.
  - Find an experimental value for  $c$ . Don't forget to determine the appropriate units.
- Hint:** Both the slope *and* the  $y$ -intercept of the graph are important.



62. Write a realistic problem for which the energy bar chart shown in **FIGURE P10.62** correctly shows the energy at the beginning and end of the problem.

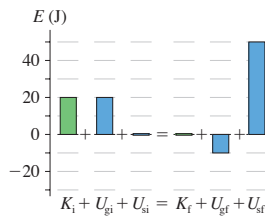


FIGURE P10.62

In Problems 63 through 66 you are given the equation used to solve a problem. For each of these, you are to

- Write a realistic problem for which this is the correct equation.
- Draw the before-and-after pictorial representation.
- Finish the solution of the problem.

$$63. \frac{1}{2}(1500 \text{ kg})(5.0 \text{ m/s})^2 + (1500 \text{ kg})(9.80 \text{ m/s}^2)(10 \text{ m}) = \frac{1}{2}(1500 \text{ kg})(v_f)^2 + (1500 \text{ kg})(9.80 \text{ m/s}^2)(0 \text{ m})$$

$$64. \frac{1}{2}(0.20 \text{ kg})(2.0 \text{ m/s})^2 + \frac{1}{2}k(0 \text{ m})^2 = \frac{1}{2}(0.20 \text{ kg})(0 \text{ m/s})^2 + \frac{1}{2}k(-0.15 \text{ m})^2$$

$$65. (0.10 \text{ kg} + 0.20 \text{ kg})v_{1x} = (0.10 \text{ kg})(3.0 \text{ m/s})$$

$$\frac{1}{2}(0.30 \text{ kg})(0 \text{ m/s})^2 + \frac{1}{2}(3.0 \text{ N/m})(\Delta x_2)^2 = \frac{1}{2}(0.30 \text{ kg})(v_{1x})^2 + \frac{1}{2}(3.0 \text{ N/m})(0 \text{ m})^2$$

$$66. \frac{1}{2}(0.50 \text{ kg})(v_f)^2 + (0.50 \text{ kg})(9.80 \text{ m/s}^2)(0 \text{ m}) + \frac{1}{2}(400 \text{ N/m})(0 \text{ m})^2 = \frac{1}{2}(0.50 \text{ kg})(0 \text{ m/s})^2 + (0.50 \text{ kg})(9.80 \text{ m/s}^2)((-0.10 \text{ m}) \sin 30^\circ) + \frac{1}{2}(400 \text{ N/m})(-0.10 \text{ m})^2$$

### Challenge Problems

67. A massless pan hangs from a spring that is suspended from the ceiling. When empty, the pan is 50 cm below the ceiling. If a 100 g clay ball is placed gently on the pan, the pan hangs 60 cm below the ceiling. Suppose the clay ball is dropped from the ceiling onto an empty pan. What is the pan's distance from the ceiling when the spring reaches its maximum length?

68. A pendulum is formed from a small ball of mass  $m$  on a string of length  $L$ . As **FIGURE CP10.68** shows, a peg is height  $h = L/3$  above the pendulum's lowest point. From what minimum angle  $\theta$  must the pendulum be released in order for the ball to go over the top of the peg without the string going slack?

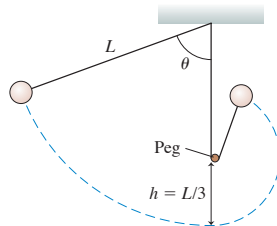


FIGURE CP10.68

69. In a physics lab experiment, a compressed spring launches a 20 g metal ball at a  $30^\circ$  angle. Compressing the spring 20 cm causes the ball to hit the floor 1.5 m below the point at which it leaves the spring after traveling 5.0 m horizontally. What is the spring constant?
70. It's your birthday, and to celebrate you're going to make your first bungee jump. You stand on a bridge 100 m above a raging river and attach a 30-m-long bungee cord to your harness. A bungee cord, for practical purposes, is just a long spring, and this cord has a spring constant of 40 N/m. Assume that your mass is 80 kg. After a long hesitation, you dive off the bridge. How far are you above the water when the cord reaches its maximum elongation?
71. A 10 kg box slides 4.0 m down the frictionless ramp shown in **FIGURE CP10.71**, then collides with a spring whose spring constant is 250 N/m.
- What is the maximum compression of the spring?
  - At what compression of the spring does the box have its maximum speed?

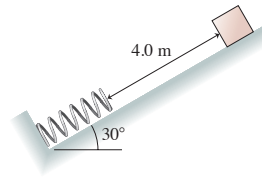


FIGURE CP10.71

72. Old naval ships fired 10 kg cannon balls from a 200 kg cannon. It was very important to stop the recoil of the cannon, since otherwise the heavy cannon would go careening across the deck of the ship. In one design, a large spring with spring constant 20,000 N/m was placed behind the cannon. The other end of the spring braced against a post that was firmly anchored to the ship's frame. What was the speed of the cannon ball if the spring compressed 50 cm when the cannon was fired?
73. A 2.0 kg cart has a spring with  $k = 5000 \text{ N/m}$  attached to its front, parallel to the ground. This cart rolls at 4.0 m/s toward a stationary 1.0 kg cart.
- What is the maximum compression of the spring during the collision?
  - What is the speed of each cart after the collision?
74. The air-track carts in **FIGURE CP10.74** are sliding to the right at 1.0 m/s. The spring between them has a spring constant of 120 N/m and is compressed 4.0 cm. The carts slide past a flame that burns through the string holding them together. Afterward, what are the speed and direction of each cart?

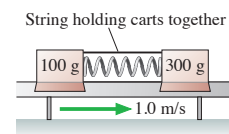


FIGURE CP10.74

75. A 100 g steel ball and a 200 g steel ball each hang from 1.0-m-long strings. At rest, the balls hang side by side, barely touching. The 100 g ball is pulled to the left until the angle between its string and vertical is  $45^\circ$ . The 200 g ball is pulled to a  $45^\circ$  angle on the right. The balls are released so as to collide at the very bottom of their swings. To what angle does each ball rebound?



76. A sled starts from rest at the top of the frictionless, hemispherical, snow-covered hill shown in **FIGURE CP10.76**.
- Find an expression for the sled's speed when it is at angle  $\phi$ .
  - Use Newton's laws to find the maximum speed the sled can have at angle  $\phi$  without leaving the surface.
  - At what angle  $\phi_{\max}$  does the sled "fly off" the hill?

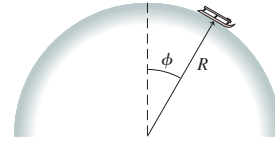


FIGURE CP10.76

## STOP TO THINK ANSWERS

**Stop to Think 10.1:**  $(U_g)_c > (U_g)_b = (U_g)_d > (U_g)_a$ . Gravitational potential energy depends only on height, not on speed.

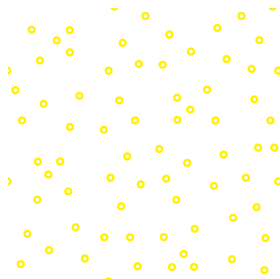
**Stop to Think 10.2:**  $v_a = v_b = v_c = v_d$ . Her increase in kinetic energy depends only on the vertical height through which she falls, not the shape of the slide.

**Stop to Think 10.3: b.** Mechanical energy is conserved on a frictionless surface. Because  $K_i = 0$  and  $K_f = 0$ , it must be true that  $U_f = U_i$  and thus  $y_f = y_i$ . The final height matches the initial height.

**Stop to Think 10.4:**  $k_a > k_b > k_c$ . The spring constant is the slope of the force-versus-displacement graph.

**Stop to Think 10.5: c.**  $U_s$  depends on  $(\Delta s)^2$ , so doubling the compression increases  $U_s$  by a factor of 4. All the potential energy is converted to kinetic energy, so  $K$  increases by a factor of 4. But  $K$  depends on  $v^2$ , so  $v$  increases by only a factor of  $(4)^{1/2} = 2$ .

**Stop to Think 10.6:  $x = 6$  m.** From the graph, the particle's potential energy at  $x = 1$  m is  $U = 3$  J. Its total energy is thus  $E = K + U = 4$  J. A TE line at 4 J crosses the PE curve at  $x = 6$  m.



# SUMMARY

The goal of Chapter 11 has been to develop a more complete understanding of energy and its conservation.

## General Principles

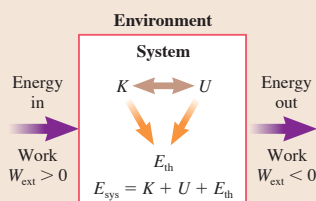
### Basic Energy Model

- Energy is *transferred* to or from the system by work.
- Energy is *transformed* within the system.

Two versions of the energy equation are

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}}$$

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i + W_{\text{ext}}$$



### Solving Energy Problems

**MODEL** Identify objects in the system.

**VISUALIZE** Draw a before-and-after pictorial representation and an energy bar chart.

**SOLVE** Use the energy equation

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i + W_{\text{ext}}$$

**ASSESS** Is the result reasonable?

### Law of Conservation of Energy

- Isolated system:**  $W_{\text{ext}} = 0$ . The total energy  $E_{\text{sys}} = E_{\text{mech}} + E_{\text{th}}$  is conserved.  $\Delta E_{\text{sys}} = 0$ .
- Isolated, nondissipative system:**  $W_{\text{ext}} = 0$  and  $W_{\text{diss}} = 0$ . The mechanical energy  $E_{\text{mech}}$  is conserved.

$$\Delta E_{\text{mech}} = 0 \text{ or } K_f + U_f = K_i + U_i$$

## Important Concepts

The **work-kinetic energy theorem** is

$$\Delta K = W_{\text{net}} = W_c + W_{\text{diss}} + W_{\text{ext}}$$

With  $W_c = -\Delta U$  for conservative forces and  $W_{\text{diss}} = -\Delta E_{\text{th}}$  for dissipative forces, this becomes the energy equation.

The **work** done by a force on a particle as it moves from  $s_i$  to  $s_f$  is

$$W = \int_{s_i}^{s_f} F_s ds = \text{area under the force curve}$$

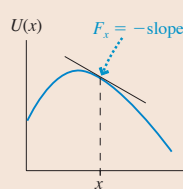
$$= \vec{F} \cdot \Delta \vec{r} \text{ if } \vec{F} \text{ is a constant force}$$

**Conservative forces** are forces for which the work is independent of the path followed. The work done by a conservative force can be represented as a **potential energy**:

$$\Delta U = U_f - U_i = -W_c(i \rightarrow f)$$

A conservative force is found from the potential energy by

$$F_s = -dU/ds = \text{negative of the slope of the PE curve}$$



**Dissipative forces** transform **macroscopic energy** into thermal energy, which is the **microscopic energy** of the atoms and molecules. For friction:

$$\Delta E_{\text{th}} = f_k \Delta s$$

## Applications

**Power** is the rate at which energy is transferred or transformed:

$$P = \frac{dE_{\text{sys}}}{dt}$$

For a particle moving with velocity  $\vec{v}$ , the power delivered to the particle by force  $\vec{F}$  is  $P = \vec{F} \cdot \vec{v} = Fv \cos \theta$ .

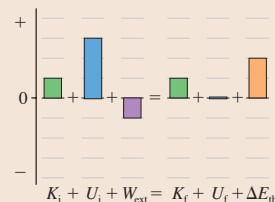
**Dot product**

$$\vec{A} \cdot \vec{B} = AB \cos \alpha = A_x B_x + A_y B_y$$



**Energy bar charts** display the energy equation

$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i + W_{\text{ext}}$  in graphical form.



## Terms and Notation

thermal energy,  $E_{\text{th}}$   
 system energy,  $E_{\text{sys}}$   
 energy transformation  
 energy transfer  
 work,  $W$   
 work-kinetic energy theorem

dot product  
 scalar product  
 conservative force  
 nonconservative force  
 macrophysics  
 microphysics

dissipative force  
 energy equation  
 law of conservation of energy  
 power,  $P$   
 watt,  $W$

## CONCEPTUAL QUESTIONS

1. A process occurs in which a system's potential energy decreases while the system does work on the environment. Does the system's kinetic energy increase, decrease, or stay the same? Or is there not enough information to tell? Explain.
2. A process occurs in which a system's potential energy increases while the environment does work on the system. Does the system's kinetic energy increase, decrease, or stay the same? Or is there not enough information to tell? Explain.
3. The kinetic energy of a system decreases while its potential energy and thermal energy are unchanged. Does the environment do work on the system, or does the system do work on the environment? Explain.
4. You drop a ball from a high balcony and it falls freely. Does the ball's kinetic energy increase by equal amounts in equal time intervals, or by equal amounts in equal distances? Explain.
5. A particle moves in a vertical plane along the *closed* path seen in **FIGURE Q11.5**, starting at A and eventually returning to its starting point. How much work is done on the particle by gravity? Explain.

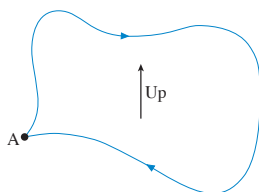



FIGURE Q11.5

6. A 0.2 kg plastic cart and a 20 kg lead cart both roll without friction on a horizontal surface. Equal forces are used to push both carts forward a distance of 1 m, starting from rest. After traveling 1 m, is the kinetic energy of the plastic cart greater than, less than, or equal to the kinetic energy of the lead cart? Explain.
7. You need to raise a heavy block by pulling it with a massless rope. You can either (a) pull the block straight up height  $h$ , or (b) pull it up a long, frictionless plane inclined at a  $15^\circ$  angle until its height has increased by  $h$ . Assume you will move the block at constant speed either way. Will you do more work in case a or case b? Or is the work the same in both cases? Explain.
8. a. If the force on a particle at some point in space is zero, must its potential energy also be zero at that point? Explain.  
 b. If the potential energy of a particle at some point in space is zero, must the force on it also be zero at that point? Explain.
9. A car traveling at 60 mph slams on its brakes and skids to a halt. What happened to the kinetic energy the car had just before stopping?
10. What energy transformations occur as a skier glides down a gentle slope at constant speed?
11. Give a *specific* example of a situation in which  
 a.  $W_{\text{ext}} \rightarrow K$  with  $\Delta U = 0$  and  $\Delta E_{\text{th}} = 0$ .  
 b.  $W_{\text{ext}} \rightarrow E_{\text{th}}$  with  $\Delta K = 0$  and  $\Delta U = 0$ .
12. The motor of a crane uses power  $P$  to lift a steel beam. By what factor must the motor's power increase to lift the beam twice as high in half the time?


## EXERCISES AND PROBLEMS




Problems labeled  integrate material from earlier chapters.

### Exercises

#### Section 11.2 Work and Kinetic Energy

#### Section 11.3 Calculating and Using Work

1.  Evaluate the dot product  $\vec{A} \cdot \vec{B}$  if  
 a.  $\vec{A} = 3\hat{i} + 4\hat{j}$  and  $\vec{B} = 2\hat{i} - 6\hat{j}$ .  
 b.  $\vec{A} = 3\hat{i} - 2\hat{j}$  and  $\vec{B} = 6\hat{i} + 4\hat{j}$ .

2.  Evaluate the dot product  $\vec{A} \cdot \vec{B}$  if  
 a.  $\vec{A} = 4\hat{i} - 2\hat{j}$  and  $\vec{B} = -2\hat{i} - 3\hat{j}$ .  
 b.  $\vec{A} = -4\hat{i} + 2\hat{j}$  and  $\vec{B} = 2\hat{i} + 4\hat{j}$ .
3.  What is the angle  $\theta$  between vectors  $\vec{A}$  and  $\vec{B}$  in each part of Exercise 1?
4.  What is the angle  $\theta$  between vectors  $\vec{A}$  and  $\vec{B}$  in each part of Exercise 2?

5. | Evaluate the dot product of the three pairs of vectors in **FIGURE EX11.5**.

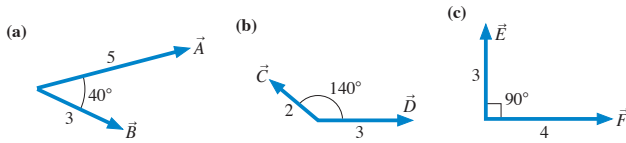


FIGURE EX11.5

6. | Evaluate the dot product of the three pairs of vectors in **FIGURE EX11.6**.

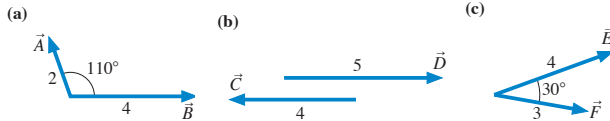


FIGURE EX11.6

7. | How much work is done by the force  $\vec{F} = (-3.0\hat{i} + 6.0\hat{j})$  N on a particle that moves through displacement (a)  $\Delta\vec{r} = 2.0\hat{i}$  m and (b)  $\Delta\vec{r} = 2.0\hat{j}$  m?
8. | How much work is done by the force  $\vec{F} = (-4.0\hat{i} - 6.0\hat{j})$  N on a particle that moves through displacement (a)  $\Delta\vec{r} = -3.0\hat{i}$  m and (b)  $\Delta\vec{r} = (3.0\hat{i} - 2.0\hat{j})$  m?
9. || A 20 g particle is moving to the left at 30 m/s. How much net work must be done on the particle to cause it to move to the right at 30 m/s?
10. | A 2.0 kg book is lying on a 0.75-m-high table. You pick it up and place it on a bookshelf 2.25 m above the floor.
- How much work does gravity do on the book?
  - How much work does your hand do on the book?
11. || The two ropes seen in **FIGURE EX11.11** are used to lower a 255 kg piano 5.00 m from a second-story window to the ground. How much work is done by each of the three forces?

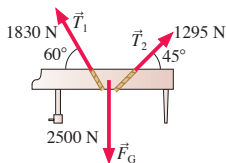


FIGURE EX11.11

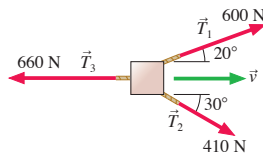


FIGURE EX11.12

12. || The three ropes shown in the bird's-eye view of **FIGURE EX11.12** are used to drag a crate 3.0 m across the floor. How much work is done by each of the three forces?
13. || **FIGURE EX11.13** is the velocity-versus-time graph for a 2.0 kg object moving along the  $x$ -axis. Determine the work done on the object during each of the four intervals AB, BC, CD, and DE.

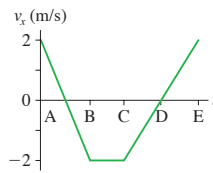


FIGURE EX11.13

#### Section 11.4 The Work Done by a Variable Force

14. | **FIGURE EX11.14** is the force-versus-position graph for a particle moving along the  $x$ -axis. Determine the work done on the particle during each of the three intervals 0–1 m, 1–2 m, and 2–3 m.

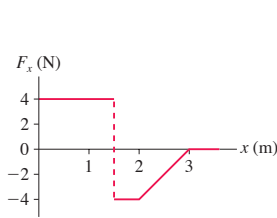


FIGURE EX11.14

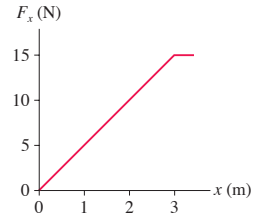


FIGURE EX11.15

15. || A 500 g particle moving along the  $x$ -axis experiences the force shown in **FIGURE EX11.15**. The particle's velocity is 2.0 m/s at  $x = 0$  m. What is its velocity at  $x = 1$  m, 2 m, and 3 m?
16. || A 2.0 kg particle moving along the  $x$ -axis experiences the force shown in **FIGURE EX11.16**. The particle's velocity is 4.0 m/s at  $x = 0$  m. What is its velocity at  $x = 2$  m and 4 m?

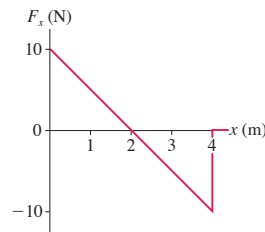


FIGURE EX11.16

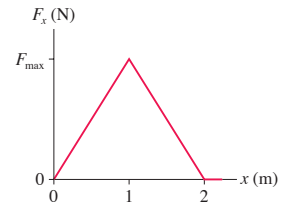


FIGURE EX11.17

17. || A 500 g particle moving along the  $x$ -axis experiences the force shown in **FIGURE EX11.17**. The particle goes from  $v_x = 2.0$  m/s at  $x = 0$  m to  $v_x = 6.0$  m/s at  $x = 2$  m. What is  $F_{\max}$ ?

#### Section 11.5 Work and Potential Energy

##### Section 11.6 Finding Force from Potential Energy

18. || A particle has the potential energy shown in **FIGURE EX11.18**. What is the  $x$ -component of the force on the particle at  $x = 5$ , 15, 25, and 35 cm?

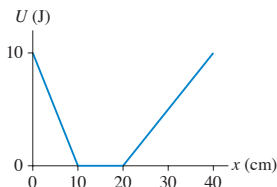


FIGURE EX11.18

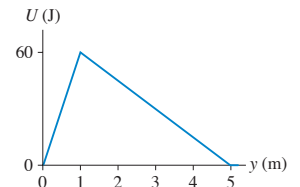


FIGURE EX11.19

19. || A particle has the potential energy shown in **FIGURE EX11.19**. What is the  $y$ -component of the force on the particle at  $y = 0.5$  m and 4 m?
20. || A particle moving along the  $y$ -axis has the potential energy  $U = 4y^3$  J, where  $y$  is in m. What is the  $y$ -component of the force on the particle at  $y = 0$  m, 1 m, and 2 m?
21. || A particle moving along the  $x$ -axis has the potential energy  $U = 10/x$  J, where  $x$  is in m. What is the  $x$ -component of the force on the particle at  $x = 2$  m, 5 m, and 8 m?

#### Section 11.7 Thermal Energy

22. || The mass of a carbon atom is  $2.0 \times 10^{-26}$  kg.
- What is the kinetic energy of a carbon atom moving with a speed of 500 m/s?

- b. Two carbon atoms are joined by a spring-like carbon-carbon bond. The potential energy stored in the bond has the value you calculated in part a if the bond is stretched 0.050 nm. What is the bond's spring constant?
23. || In Part IV you'll learn to calculate that 1 mole ( $6.02 \times 10^{23}$  atoms) of helium atoms in the gas phase has 3700 J of microscopic kinetic energy at room temperature. If we assume that all atoms move with the same speed, what is that speed? The mass of a helium atom is  $6.68 \times 10^{-27}$  kg.
24. || A 20 kg child slides down a 3.0-m-high playground slide. She starts from rest, and her speed at the bottom is 2.0 m/s.
- Describe the energy transfers and transformations occurring during the slide.
  - What is the change in the combined thermal energy of the slide and the seat of her pants?

### Section 11.8 Conservation of Energy

25. || A system loses 400 J of potential energy. In the process, it does 400 J of work on the environment and the thermal energy increases by 100 J. Show this process on an energy bar chart.
26. || A system loses 500 J of kinetic energy while gaining 200 J of potential energy. The thermal energy increases 100 J. Show this process on an energy bar chart.
27. | How much work is done by the environment in the process shown in **FIGURE EX11.27**? Is energy transferred from the environment to the system or from the system to the environment?

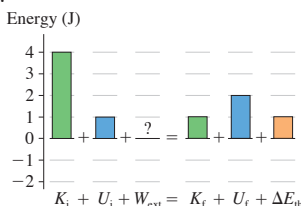


FIGURE EX11.27

28. || A cable with 20.0 N of tension pulls straight up on a 1.02 kg block that is initially at rest. What is the block's speed after being lifted 2.00 m? Solve this problem using work and energy.

### Section 11.9 Power

29. | a. How much work does an elevator motor do to lift a 1000 kg elevator a height of 100 m?  
b. How much power must the motor supply to do this in 50 s at constant speed?
30. | a. How much work must you do to push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s?  
b. What is your power output while doing so?
31. || At midday, solar energy strikes the earth with an intensity of about 1 kW/m<sup>2</sup>. What is the area of a solar collector that could collect 150 MJ of energy in 1 h? This is roughly the energy content of 1 gallon of gasoline.
32. || Which consumes more energy, a 1.2 kW hair dryer used for 10 min or a 10 W night light left on for 24 h?
33. | A 2.0 hp electric motor on a water well pumps water from 10 m below the surface. The density of water is 1.0 kg per liter. How many liters of water does the motor pump in 1 h?
34. || A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at constant acceleration.
- What is the magnitude of the horizontal force acting on the sprinter?
  - What is the sprinter's power output at 2.0 s, 4.0 s, and 6.0 s?

35. | a. Estimate the height in meters of the two flights of stairs that go from the first to the third floor of a building.  
b. Estimate how long it takes you to *run* up these two flights of stairs.  
c. Estimate your power output in both watts and horsepower while running up the stairs.
36. | A 70 kg human sprinter can accelerate from rest to 10 m/s in 3.0 s. During the same time interval, a 30 kg greyhound can go from rest to 20 m/s. What is the average power output of each? Average power over a time interval  $\Delta t$  is  $\Delta E / \Delta t$ .

### Problems

37. | A particle moves from A to D in **FIGURE P11.37** while experiencing force  $\vec{F} = (6\hat{i} + 8\hat{j})$  N. How much work does the force do if the particle follows path (a) ABD, (b) ACD, and (c) AD? Is this a conservative force? Explain.

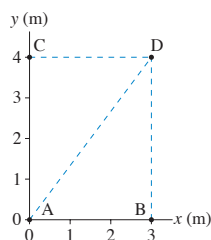


FIGURE P11.37

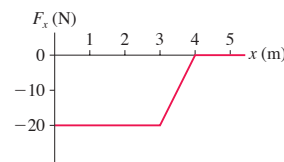


FIGURE P11.38

38. || A 100 g particle experiences the one-dimensional, conservative force  $F_x$  shown in **FIGURE P11.38**.
- Draw a graph of the potential energy  $U$  from  $x = 0$  m to  $x = 5$  m. Let the zero of the potential energy be at  $x = 0$  m.  
**Hint:** Think about the definition of potential energy *and* the geometric interpretation of the work done by a varying force.
  - The particle is shot toward the right from  $x = 1.0$  m with a speed of 25 m/s. What is the particle's mechanical energy?
  - Draw the particle's total energy line on your graph of part a.
  - Where is the particle's turning point?
39. || A 10 g particle has the potential energy shown in **FIGURE P11.39**.
- Draw a force-versus-position graph from  $x = 0$  cm to  $x = 8$  cm.
  - How much work does the force do as the particle moves from  $x = 2$  cm to  $x = 6$  cm?
  - What speed does the particle need at  $x = 2$  cm to arrive at  $x = 6$  cm with a speed of 10 m/s?
40. || a. **FIGURE P11.40a** shows the force  $F_x$  exerted on a particle that moves along the  $x$ -axis. Draw a graph of the particle's potential energy as a function of position  $x$ . Let  $U$  be zero at  $x = 0$  m.  
b. **FIGURE P11.40b** shows the potential energy  $U$  of a particle that moves along the  $x$ -axis. Draw a graph of the force  $F_x$  as a function of position  $x$ .

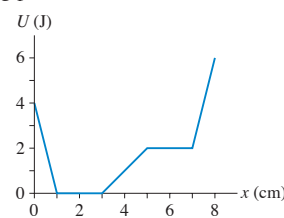


FIGURE P11.39

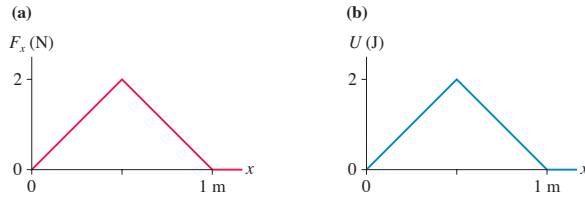


FIGURE P11.40

41. **FIGURE P11.41** is the velocity-versus-time graph of a 500 g particle that starts at  $x = 0$  m and moves along the  $x$ -axis. Draw graphs of the following by calculating and plotting numerical values at  $t = 0, 1, 2, 3$ , and 4 s. Then sketch lines or curves of the appropriate shape between the points. Make sure you include appropriate scales on both axes of each graph.
- Acceleration versus time.
  - Position versus time.
  - Kinetic energy versus time.
  - Force versus time.
  - Use your  $F_x$ -versus- $t$  graph to determine the *impulse* delivered to the particle during the time interval 0–2 s and also the interval 2–4 s.
  - Use the impulse-momentum theorem to determine the particle's velocity at  $t = 2$  s and at  $t = 4$  s. Do your results agree with the velocity graph?
  - Now draw a graph of force versus *position*. This requires no calculations; just think carefully about what you learned in parts a to d.
  - Use your  $F_x$ -versus- $x$  graph to determine the *work* done on the particle during the time interval 0–2 s and also the interval 2–4 s.
  - Use the work-kinetic energy theorem to determine the particle's velocity at  $t = 2$  s and at  $t = 4$  s. Do your results agree with the velocity graph?
42. **II** A 1000 kg elevator accelerates upward at  $1.0 \text{ m/s}^2$  for 10 m, starting from rest.
- How much work does gravity do on the elevator?
  - How much work does the tension in the elevator cable do on the elevator?
  - Use the work-kinetic energy theorem to find the kinetic energy of the elevator as it reaches 10 m.
  - What is the speed of the elevator as it reaches 10 m?
43. **I** Bob can throw a 500 g rock with a speed of 30 m/s. He moves his hand forward 1.0 m while doing so.
- How much work does Bob do on the rock?
  - How much force, assumed to be constant, does Bob apply to the rock?
  - What is Bob's maximum power output as he throws the rock?
44. **II** a. Starting from rest, a crate of mass  $m$  is pushed up a frictionless slope of angle  $\theta$  by a *horizontal* force of magnitude  $F$ . Use work and energy to find an expression for the crate's speed  $v$  when it is at height  $h$  above the bottom of the slope.  
 b. Doug uses a 25 N horizontal force to push a 5.0 kg crate up a 2.0-m-high,  $20^\circ$  frictionless slope. What is the speed of the crate at the top of the slope?
45. **II** Sam, whose mass is 75 kg, straps on his skis and starts down a 50-m-high,  $20^\circ$  frictionless slope. A strong headwind exerts a *horizontal* force of 200 N on him as he skies. Use work and energy to find Sam's speed at the bottom.

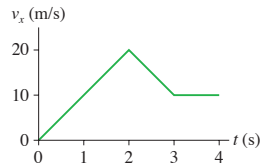


FIGURE P11.41

46. **III** Susan's 10 kg baby brother Paul sits on a mat. Susan pulls the mat across the floor using a rope that is angled  $30^\circ$  above the floor. The tension is a constant 30 N and the coefficient of friction is 0.20. Use work and energy to find Paul's speed after being pulled 3.0 m.
47. **II** A horizontal spring with spring constant 100 N/m is compressed 20 cm and used to launch a 2.5 kg box across a frictionless, horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction of the box on the surface is 0.15. Use work and energy to find how far the box slides across the rough surface before stopping.
48. **II** a. A box of mass  $m$  and initial speed  $v_0$  slides distance  $d$  across a horizontal floor before coming to rest. Use work and energy to find an expression for the coefficient of kinetic friction.  
 b. A baggage handler throws a 15 kg suitcase along the floor of an airplane luggage compartment with a speed of 1.2 m/s. The suitcase slides 2.0 m before stopping. What is the suitcase's coefficient of kinetic friction on the floor?
49. **II** Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at  $6.0^\circ$  and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s ( $\approx 75$  mph).
50. **II** A freight company uses a compressed spring to shoot 2.0 kg packages up a 1.0-m-high frictionless ramp into a truck, as **FIGURE P11.50** shows. The spring constant is 500 N/m and the spring is compressed 30 cm.
- What is the speed of the package when it reaches the truck?
  - A careless worker spills his soda on the ramp. This creates a 50-cm-long sticky spot with a coefficient of kinetic friction 0.30. Will the next package make it into the truck?

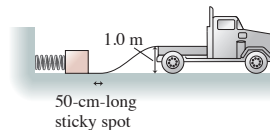


FIGURE P11.50

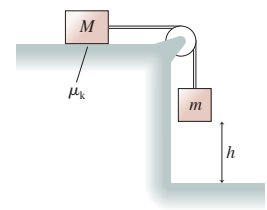


FIGURE P11.51

51. **III** Use work and energy to find an expression for the speed of the block in **FIGURE P11.51** just before it hits the floor if (a) the coefficient of kinetic friction for the block on the table is  $\mu_k$  and (b) the table is frictionless.
52. **III** An 8.0 kg crate is pulled 5.0 m up a  $30^\circ$  incline by a rope angled  $18^\circ$  above the incline. The tension in the rope is 120 N, and the crate's coefficient of kinetic friction on the incline is 0.25.
- How much work is done by tension, by gravity, and by the normal force?
  - What is the increase in thermal energy of the crate and incline?



53. || You've taken a summer job at a water park. In one stunt, a water skier is going to glide up the 2.0-m-high frictionless ramp shown in **FIGURE P11.53**, then sail over a 5.0-m-wide tank filled with hungry sharks. You will be driving the boat that pulls her to the ramp. She'll drop the tow rope at the base of the ramp just as you veer away. What minimum speed must you have as you reach the ramp in order for her to live to do this again tomorrow?

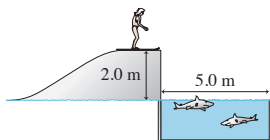


FIGURE P11.53

54. || A 50 kg ice skater is gliding along the ice, heading due north at 4.0 m/s. The ice has a small coefficient of static friction, to prevent the skater from slipping sideways, but  $\mu_k = 0$ . Suddenly, a wind from the northeast exerts a force of 4.0 N on the skater.
- Use work and energy to find the skater's speed after gliding 100 m in this wind.
  - What is the minimum value of  $\mu_s$  that allows her to continue moving straight north?
55. || a. A 50 g ice cube can slide without friction up and down a  $30^\circ$  slope. The ice cube is pressed against a spring at the bottom of the slope, compressing the spring 10 cm. The spring constant is 25 N/m. When the ice cube is released, what total distance will it travel up the slope before reversing direction?
- b. The ice cube is replaced by a 50 g plastic cube whose coefficient of kinetic friction is 0.20. How far will the plastic cube travel up the slope? Use work and energy.
56. || A 5.0 kg box slides down a 5.0-m-high frictionless hill, starting from rest, across a 2.0-m-wide horizontal surface, then hits a horizontal spring with spring constant 500 N/m. The other end of the spring is anchored against a wall. The ground under the spring is frictionless, but the 2.0-m-wide horizontal surface is rough. The coefficient of kinetic friction of the box on this surface is 0.25.
- What is the speed of the box just before reaching the rough surface?
  - What is the speed of the box just before hitting the spring?
  - How far is the spring compressed?
  - Including the first crossing, how many *complete* trips will the box make across the rough surface before coming to rest?
57. || The spring shown in **FIGURE P11.57** is compressed 50 cm and used to launch a 100 kg physics student. The track is frictionless until it starts up the incline. The student's coefficient of kinetic friction on the  $30^\circ$  incline is 0.15.
- What is the student's speed just after losing contact with the spring?
  - How far up the incline does the student go?

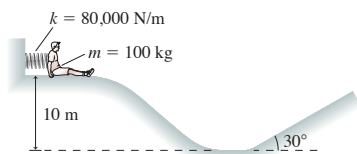


FIGURE P11.57

58. || A block of mass  $m$  starts from rest at height  $h$ . It slides down a frictionless incline, across a rough horizontal surface of length  $L$ , then up a frictionless incline. The coefficient of kinetic friction on the rough surface is  $\mu_k$ .
- What is the block's speed at the bottom of the first incline?
  - How high does the block go on the second incline?
- Give your answers in terms of  $m$ ,  $h$ ,  $L$ ,  $\mu_k$ , and  $g$ .

59. || Show that Hooke's law for an ideal spring is a conservative force. To do so, first calculate the work done by the spring as it expands from A to B. Then calculate the work done by the spring as it expands from A to point C, which is beyond B, then returns from C to B.

60. || A clever engineer designs a "sprong" that obeys the force law  $F_x = -q(x - x_e)^3$ , where  $x_e$  is the equilibrium position of the end of the sprong and  $q$  is the sprong constant. For simplicity, we'll let  $x_e = 0$  m. Then  $F_x = -qx^3$ .

- What are the units of  $q$ ?
  - Find an expression for the potential energy of a stretched or compressed sprong.
  - A sprong-loaded toy gun shoots a 20 g plastic ball. What is the launch speed if the sprong constant is 40,000, with the units you found in part a, and the sprong is compressed 10 cm? Assume the barrel is frictionless.
61. || A particle of mass  $m$  starts from  $x_0 = 0$  m with  $v_0 > 0$  m/s. The particle experiences the variable force  $F_x = F_0 \sin(cx)$  as it moves to the right along the  $x$ -axis, where  $F_0$  and  $c$  are constants.
- What are the units of  $F_0$ ?
  - What are the units of  $c$ ?
  - At what position  $x_{\max}$  does the force first reach a maximum value? Your answer will be in terms of the constants  $F_0$  and  $c$  and perhaps other numerical constants.
  - Sketch a graph of  $F$  versus  $x$  from  $x_0$  to  $x_{\max}$ .
  - What is the particle's velocity as it reaches  $x_{\max}$ ? Give your answer in terms of  $m$ ,  $v_0$ ,  $F_0$ , and  $c$ .

62. || A 5.0 kg cat leaps from the floor to the top of a 95-cm-high table. If the cat pushes against the floor for 0.20 s to accomplish this feat, what was her average power output during the pushoff period?

63. || **BIO** The human heart pumps the average adult's 6.0 L (6000 cm<sup>3</sup>) of blood through the body every minute. The heart must do work to overcome frictional forces that resist blood flow. The average adult blood pressure is  $1.3 \times 10^4$  N/m<sup>2</sup>.

- How much work does the heart do to move the 6.0 L of blood completely through the body?
- What power output must the heart have to do this task once a minute?

**Hint:** When the heart contracts, it applies force to the blood. Pressure is force/area. Model the circulatory system as a single closed tube, with cross-section area  $A$  and volume  $V = 6.0$  L, filled with blood to which the heart applies a force.

64. || **BIO** When you ride a bicycle at constant speed, nearly all the energy you expend goes into the work you do against the drag force of the air. Model a cyclist as having cross-section area 0.45 m<sup>2</sup> and, because the human body is not aerodynamically shaped, a drag coefficient of 0.90.

- What is the cyclist's power output while riding at a steady 7.3 m/s (16 mph)?
  - Metabolic power* is the rate at which your body "burns" fuel to power your activities. For many activities, your body is roughly 25% efficient at converting the chemical energy of food into mechanical energy. What is the cyclist's metabolic power while cycling at 7.3 m/s?
  - The food calorie is equivalent to 4190 J. How many calories does the cyclist burn if he rides over level ground at 7.3 m/s for 1 h?
65. || In a hydroelectric dam, water falls 25 m and then spins a turbine to generate electricity.

- a. What is  $\Delta U$  of 1.0 kg of water?
- b. Suppose the dam is 80% efficient at converting the water's potential energy to electrical energy. How many kilograms of water must pass through the turbines each second to generate 50 MW of electricity? This is a typical value for a small hydroelectric dam.
66. || The force required to tow a water skier at speed  $v$  is proportional to the speed. That is,  $F_{\text{tow}} = Av$ , where  $A$  is a proportionality constant. If a speed of 2.5 mph requires 2 hp, how much power is required to tow a water skier at 7.5 mph?
67. || A Porsche 944 Turbo has a rated engine power of 217 hp. 30% of the power is lost in the drive train, and 70% reaches the wheels. The total mass of the car and driver is 1480 kg, and two-thirds of the weight is over the drive wheels.
- a. What is the maximum acceleration of the Porsche on a concrete surface where  $\mu_s = 1.00$ ?
- Hint:** What force pushes the car forward?
- b. If the Porsche accelerates at  $a_{\text{max}}$ , what is its speed when it reaches maximum power output?
- c. How long does it take the Porsche to reach the maximum power output?

In Problems 68 through 71 you are given the equation(s) used to solve a problem. For each of these, you are to

- a. Write a realistic problem for which this is the correct equation(s).
- b. Draw a pictorial representation.
- c. Finish the solution of the problem.
68.  $\frac{1}{2}(2.0 \text{ kg})(4.0 \text{ m/s})^2 + 0$   
 $+ (0.15)(2.0 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) = 0 + 0 + T(2.0 \text{ m})$
69.  $\frac{1}{2}(20 \text{ kg})v_1^2 + 0$   
 $+ (0.15)(20 \text{ kg})(9.8 \text{ m/s}^2)\cos 40^\circ((2.5 \text{ m})/\sin 40^\circ)$   
 $= 0 + (20 \text{ kg})(9.8 \text{ m/s}^2)(2.5 \text{ m}) + 0$
70.  $F_{\text{push}} - (0.20)(30 \text{ kg})(9.8 \text{ m/s}^2) = 0$   
 $75 \text{ W} = F_{\text{push}} v$
71.  $T - (1500 \text{ kg})(9.8 \text{ m/s}^2) = (1500 \text{ kg})(1.0 \text{ m/s}^2)$   
 $P = T(2.0 \text{ m/s})$

### Challenge Problems

72. A 10.2 kg weather rocket generates a thrust of 200 N. The rocket, pointing upward, is clamped to the top of a vertical spring. The bottom of the spring, whose spring constant is 500 N/m, is anchored to the ground.

- a. Initially, before the engine is ignited, the rocket sits at rest on top of the spring. How much is the spring compressed?
- b. After the engine is ignited, what is the rocket's speed when the spring has stretched 40 cm? For comparison, what would be the rocket's speed after traveling this distance if it weren't attached to the spring?

73. The spring in FIGURE CP11.73 has a spring constant of 1000 N/m. It is compressed 15 cm, then launches a 200 g block. The horizontal surface is frictionless, but the block's coefficient of kinetic friction on the incline is 0.20. What distance  $d$  does the block sail through the air?

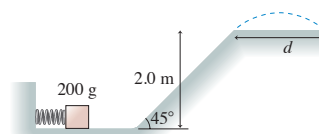


FIGURE CP11.73

74. The equation  $mgy$  for gravitational potential energy is valid only for objects near the surface of a planet. Consider two very large objects of mass  $m_1$  and  $m_2$ , such as stars or planets, whose centers are separated by the large distance  $r$ . These two large objects exert gravitational forces on each other. You'll learn in Chapter 13 that the gravitational potential energy is

$$U = -\frac{Gm_1m_2}{r}$$

where  $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$  is the *gravitational constant*.

- a. Sketch a graph of  $U$  versus  $r$ . The mathematical difficulty at  $r = 0$  is not a physically significant difficulty because the masses will collide before they get that close together.
- b. What separation  $r$  has been chosen as the point of zero potential energy? Does this make sense? Explain.
- c. Two stars are at rest  $1.0 \times 10^{14} \text{ m}$  apart. This is about 10 times the diameter of the solar system. The first star is the size of our sun, with a mass of  $2.0 \times 10^{30} \text{ kg}$  and a radius of  $7.0 \times 10^8 \text{ m}$ . The second star has mass  $8.0 \times 10^{30} \text{ kg}$  and radius of  $11.0 \times 10^8 \text{ m}$ . Gravitational forces pull the two stars together. What is the speed of each star at the moment of impact?
75. A gardener pushes a 12 kg lawnmower whose handle is tilted up  $37^\circ$  above horizontal. The lawnmower's coefficient of rolling friction is 0.15. How much power does the gardener have to supply to push the lawnmower at a constant speed of 1.2 m/s? Assume his push is parallel to the handle.

### STOP TO THINK ANSWERS

**Stop to Think 11.1:** d. Constant speed means  $\Delta K = 0$ . Gravitational potential energy is lost, and friction heats up the slide and the child's pants.

**Stop to Think 11.2:** 6.0 J.  $K_f = K_i + W$ .  $W$  is the area under the curve, which is 4.0 J.

**Stop to Think 11.3:** b. The gravitational force  $\vec{F}_G$  is in the same direction as the displacement. It does positive work. The tension force  $\vec{T}$  is opposite the displacement. It does negative work.

**Stop to Think 11.4:** c.  $W = F(\Delta r)\cos\theta$ . The 10 N force at  $90^\circ$  does no work at all.  $\cos 60^\circ = \frac{1}{2}$ , so the 8 N force does less work than the 6 N force.

**Stop to Think 11.5:** e. Force is the negative of the slope of the potential-energy diagram. At  $x = 4 \text{ m}$  the potential energy has risen by 4 J over a distance of 2 m, so the slope is 2 J/m = 2 N.

**Stop to Think 11.6:** c. Constant speed means  $\Delta K = 0$ . Gravitational potential energy is lost, and friction heats up the pole and the child's hands.

**Stop to Think 11.7:**  $P_b > P_a = P_c > P_d$ . The work done is  $mg\Delta y$ , so the power is  $mg\Delta y/\Delta t$ . Runner b does the same work as a but in less time. The ratio  $m/\Delta t$  is the same for a and c. Runner d does twice the work of a but takes more than twice as long.

# SUMMARY

The goal of Chapter 12 has been to understand the physics of rotating objects.

## General Principles

### Rotational Dynamics

Every point on a **rigid body** rotating about a fixed axis has the same angular velocity  $\omega$  and angular acceleration  $\alpha$ .

**Newton's second law** for rotational motion is

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

Use rotational kinematics to find angles and angular velocities.

### Conservation Laws

**Energy** is conserved for an isolated system.

- Pure rotation  $E = K_{\text{rot}} + U_g = \frac{1}{2}I\omega^2 + Mgy_{\text{cm}}$
- Rolling  $E = K_{\text{rot}} + K_{\text{cm}} + U_g = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2 + Mgy_{\text{cm}}$

**Angular momentum** is conserved if  $\vec{\tau}_{\text{net}} = \vec{0}$ .

- Particle  $\vec{L} = \vec{r} \times \vec{p}$
- Rotation about a symmetry axis or fixed axle  $\vec{L} = I\vec{\omega}$

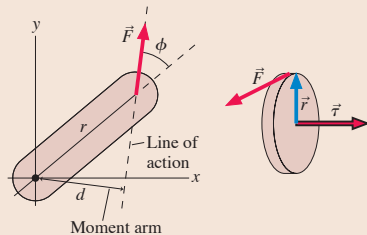
## Important Concepts

**Torque** is the rotational equivalent of force:

$$\tau = rF \sin \phi = rF_{\perp} = dF$$

The vector description of torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

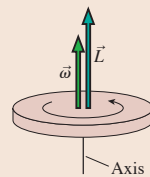


### Vector description of rotation

Angular velocity  $\vec{\omega}$  points along the rotation axis in the direction of the right-hand rule.

For a rigid body rotating about a fixed axle or an axis of symmetry, the angular momentum is  $\vec{L} = I\vec{\omega}$ .

Newton's second law is  $\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$ .



A system of particles on which there is no net force undergoes unconstrained rotation about the **center of mass**:

$$x_{\text{cm}} = \frac{1}{M} \int x \, dm \quad y_{\text{cm}} = \frac{1}{M} \int y \, dm$$

The gravitational torque on a body can be found by treating the body as a particle with all the mass  $M$  concentrated at the center of mass.

### The moment of inertia

$$I = \sum_i m_i r_i^2 = \int r^2 \, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If  $I_{\text{cm}}$  is known,  $I$  about a parallel axis distance  $d$  away is given by the **parallel-axis theorem**:  $I = I_{\text{cm}} + Md^2$ .

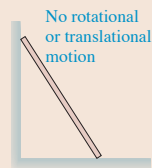
## Applications

### Rotational kinematics

$$\begin{aligned} \omega_f &= \omega_i + \alpha \Delta t \\ \theta_f &= \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2 \\ v_f &= r\omega \quad a_f = r\alpha \end{aligned}$$

### Rigid-body equilibrium

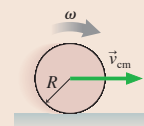
An object is in total equilibrium only if both  $\vec{F}_{\text{net}} = \vec{0}$  and  $\vec{\tau}_{\text{net}} = \vec{0}$ .



### Rolling motion

For an object that rolls without slipping

$$\begin{aligned} v_{\text{cm}} &= R\omega \\ K &= K_{\text{rot}} + K_{\text{cm}} \end{aligned}$$



## Terms and Notation

rigid body  
rigid-body model  
translational motion  
rotational motion  
combination motion

center of mass  
rotational kinetic energy,  $K_{\text{rot}}$   
moment of inertia,  $I$   
parallel-axis theorem  
torque,  $\tau$

line of action  
moment arm,  $d$   
rolling constraint  
cross product  
vector product

right-hand rule  
angular momentum,  $\vec{L}$   
law of conservation of angular momentum

# CONCEPTUAL QUESTIONS

1. Is the center of mass of the dumbbell in **FIGURE Q12.1** at point a, b, or c? Explain.

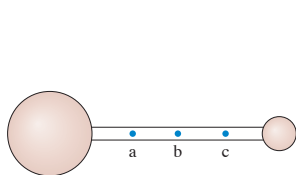


FIGURE Q12.1

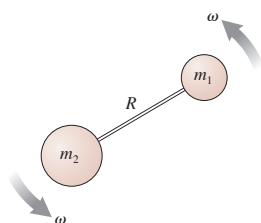


FIGURE Q12.2

2. If the angular velocity  $\omega$  is held constant, by what *factor* must  $R$  change to double the rotational kinetic energy of the dumbbell in **FIGURE Q12.2**?
3. **FIGURE Q12.3** shows three rotating disks, all of equal mass. Rank in order, from largest to smallest, their rotational kinetic energies  $K_a$  to  $K_c$ .

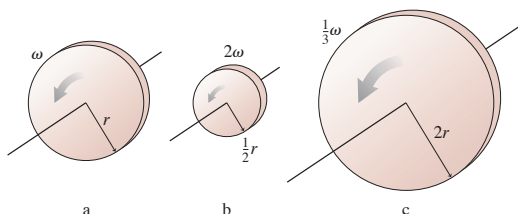


FIGURE Q12.3

4. Must an object be rotating to have a moment of inertia? Explain.
5. The moment of inertia of a uniform rod of length  $L$  about an axis through its center is  $\frac{1}{12}mL^2$ . The moment of inertia about an axis at one end is  $\frac{1}{3}mL^2$ . Explain *why* the moment of inertia is larger about the end than about the center.
6. You have two steel spheres. Sphere 2 has twice the radius of sphere 1. By what *factor* does the moment of inertia  $I_2$  of sphere 2 exceed the moment of inertia  $I_1$  of sphere 1?
7. The professor hands you two spheres. They have the same mass, the same radius, and the same exterior surface. The professor claims that one is a solid sphere and the other is hollow. Can you determine which is which without cutting them open? If so, how? If not, why not?
8. Six forces are applied to the door in **FIGURE Q12.8**. Rank in order, from largest to smallest, the six torques  $\tau_a$  to  $\tau_f$  about the hinge. Explain.

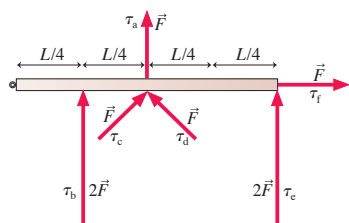


FIGURE Q12.8

9. A student gives a quick push to a ball at the end of a massless, rigid rod, as shown in **FIGURE Q12.9**, causing the ball to rotate clockwise in a *horizontal* circle. The rod's pivot is frictionless.

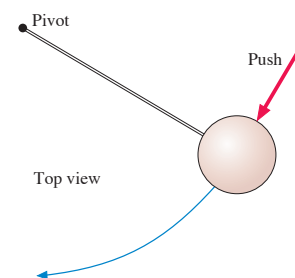


FIGURE Q12.9

- a. As the student is pushing, is the torque about the pivot positive, negative, or zero?
- b. After the push has ended, does the ball's angular velocity (i) steadily increase; (ii) increase for awhile, then hold steady; (iii) hold steady; (iv) decrease for awhile, then hold steady; or (v) steadily decrease? Explain.
- c. Right after the push has ended, is the torque positive, negative, or zero?
10. Rank in order, from largest to smallest, the angular accelerations  $\alpha_a$  to  $\alpha_d$  in **FIGURE Q12.10**. Explain.

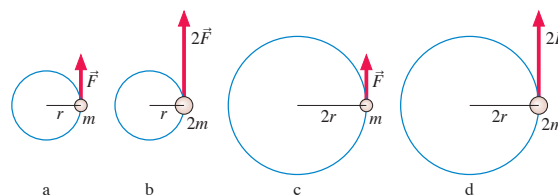


FIGURE Q12.10

11. The solid cylinder and cylindrical shell in **FIGURE Q12.11** have the same mass, same radius, and turn on frictionless, horizontal axles. (The cylindrical shell has lightweight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously. Which hits the ground first? Or is it a tie? Explain.

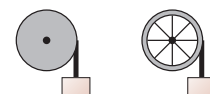


FIGURE Q12.11

12. A diver in the pike position (legs straight, hands on ankles) usually makes only one or one-and-a-half rotations. To make two or three rotations, the diver goes into a tuck position (knees bent, body curled up tight). Why?
13. Is the angular momentum of disk a in **FIGURE Q12.13** larger than, smaller than, or equal to the angular momentum of disk b? Explain.

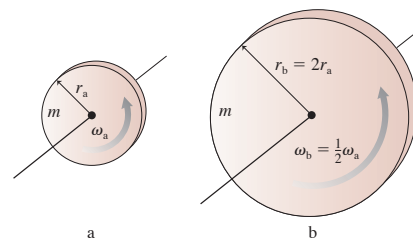






FIGURE Q12.13

# EXERCISES AND PROBLEMS



Problems labeled  integrate material from earlier chapters.

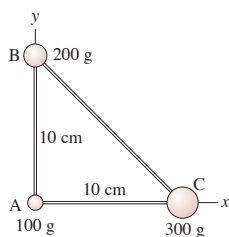
## Exercises

### Section 12.1 Rotational Motion

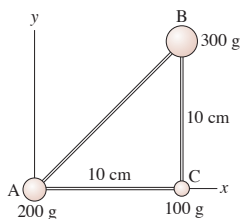
1.  A skater holds her arms outstretched as she spins at 180 rpm. What is the speed of her hands if they are 140 cm apart?
2.  A high-speed drill reaches 2000 rpm in 0.50 s.
  - a. What is the drill's angular acceleration?
  - b. Through how many revolutions does it turn during this first 0.50 s?
3.  A ceiling fan with 80-cm-diameter blades is turning at 60 rpm. Suppose the fan coasts to a stop 25 s after being turned off.
  - a. What is the speed of the tip of a blade 10 s after the fan is turned off?
  - b. Through how many revolutions does the fan turn while stopping?
4.  An 18-cm-long bicycle crank arm, with a pedal at one end, is attached to a 20-cm-diameter sprocket, the toothed disk around which the chain moves. A cyclist riding this bike increases her pedaling rate from 60 rpm to 90 rpm in 10 s.
  - a. What is the tangential acceleration of the pedal?
  - b. What length of chain passes over the top of the sprocket during this interval?

### Section 12.2 Rotation About the Center of Mass



5.  How far from the center of the earth is the center of mass of the earth + moon system? Data for the earth and moon can be found inside the back cover of the book.
6.  The three masses shown in **FIGURE EX12.6** are connected by massless, rigid rods. What are the coordinates of the center of mass?





**FIGURE EX12.6**




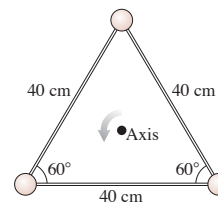
**FIGURE EX12.7**

7.  The three masses shown in **FIGURE EX12.7** are connected by massless, rigid rods. What are the coordinates of the center of mass?
8.  A 100 g ball and a 200 g ball are connected by a 30-cm-long, massless, rigid rod. The balls rotate about their center of mass at 120 rpm. What is the speed of the 100 g ball?


### Section 12.3 Rotational Energy

9.  What is the rotational kinetic energy of the earth? Assume the earth is a uniform sphere. Data for the earth can be found inside the back cover of the book.
10.  A thin, 100 g disk with a diameter of 8.0 cm rotates about an axis through its center with 0.15 J of kinetic energy. What is the speed of a point on the rim?


11.  The three 200 g masses in **FIGURE EX12.11** are connected by massless, rigid rods.

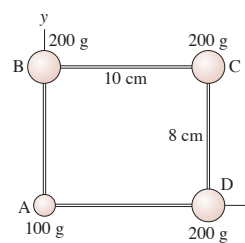


**FIGURE EX12.11**

- a. What is the triangle's moment of inertia about the axis through the center?
  - b. What is the triangle's kinetic energy if it rotates about the axis at 5.0 rev/s?
12.  A drum major twirls a 96-cm-long, 400 g baton about its center of mass at 100 rpm. What is the baton's rotational kinetic energy?


### Section 12.4 Calculating Moment of Inertia


13.  The four masses shown in **FIGURE EX12.13** are connected by massless, rigid rods.

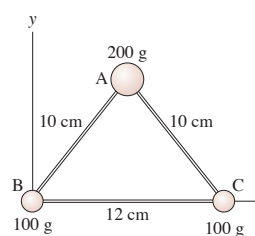


**FIGURE EX12.13**

- a. Find the coordinates of the center of mass.
  - b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.



14.  The four masses shown in **FIGURE EX12.13** are connected by massless, rigid rods.
  - a. Find the coordinates of the center of mass.
  - b. Find the moment of inertia about a diagonal axis that passes through masses B and D.

15.  The three masses shown in **FIGURE EX12.15** are connected by massless, rigid rods.




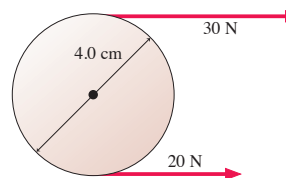
**FIGURE EX12.15**

- a. Find the coordinates of the center of mass.
  - b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.
  - c. Find the moment of inertia about an axis that passes through masses B and C.

16.  A 25 kg solid door is 220 cm tall, 91 cm wide. What is the door's moment of inertia for (a) rotation on its hinges and (b) rotation about a vertical axis inside the door, 15 cm from one edge?
17.  A 12-cm-diameter CD has a mass of 21 g. What is the CD's moment of inertia for rotation about a perpendicular axis (a) through its center and (b) through the edge of the disk?

### Section 12.5 Torque

18.  In **FIGURE EX12.18**, what is the net torque about the axle?



**FIGURE EX12.18**

19. || In **FIGURE EX12.19**, what is the net torque about the axle?

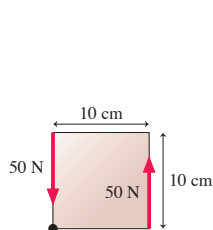


FIGURE EX12.19

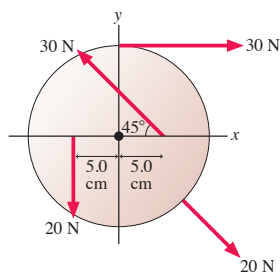


FIGURE EX12.20

20. || The 20-cm-diameter disk in **FIGURE EX12.20** can rotate on an axle through its center. What is the net torque about the axle?
21. || A 4.0-m-long, 500 kg steel beam extends horizontally from the point where it has been bolted to the framework of a new building under construction. A 70 kg construction worker stands at the far end of the beam. What is the magnitude of the torque about the point where the beam is bolted into place?
22. || An athlete at the gym holds a 3.0 kg steel ball in his hand. His arm is 70 cm long and has a mass of 4.0 kg. What is the magnitude of the torque about his shoulder if he holds his arm
- Straight out to his side, parallel to the floor?
  - Straight, but  $45^\circ$  below horizontal?

## Section 12.6 Rotational Dynamics

### Section 12.7 Rotation About a Fixed Axis

23. | An object's moment of inertia is  $2.0 \text{ kg m}^2$ . Its angular velocity is increasing at the rate of  $4.0 \text{ rad/s}$  per second. What is the torque on the object?
24. || An object whose moment of inertia is  $4.0 \text{ kg m}^2$  experiences the torque shown in **FIGURE EX12.24**. What is the object's angular velocity at  $t = 3.0 \text{ s}$ ? Assume it starts from rest.

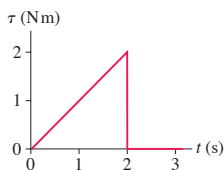


FIGURE EX12.24

25. || A 1.0 kg ball and a 2.0 kg ball are connected by a 1.0-m-long rigid, massless rod. The rod is rotating cw about its center of mass at 20 rpm. What torque will bring the balls to a halt in 5.0 s?
26. || Starting from rest, a 12-cm-diameter compact disk takes 3.0 s to reach its operating angular velocity of 2000 rpm. Assume that the angular acceleration is constant. The disk's moment of inertia is  $2.5 \times 10^{-5} \text{ kg m}^2$ .
- How much torque is applied to the disk?
  - How many revolutions does it make before reaching full speed?
27. || A 750 g, 50-cm-long metal rod is free to rotate about a frictionless axle at one end. While at rest, the rod is given a short but sharp 1000 N hammer blow at the center of the rod, aimed in a direction that causes the rod to rotate on the axle. The blow lasts a mere 2.0 ms. What is the rod's angular velocity immediately after the blow?

## Section 12.8 Static Equilibrium

28. || How much torque must the pin exert to keep the rod in **FIGURE EX12.28** from rotating?

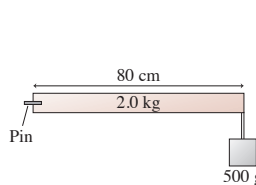


FIGURE EX12.28

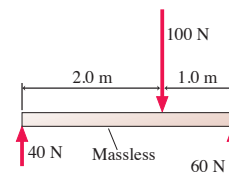


FIGURE EX12.29

29. || Is the object in **FIGURE EX12.29** in equilibrium? Explain.
30. || The two objects in **FIGURE EX12.30** are balanced on the pivot. What is distance  $d$ ?

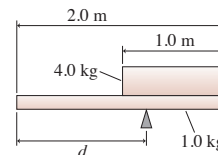


FIGURE EX12.30

31. || A 5.0 kg cat and a 2.0 kg bowl of tuna fish are at opposite ends of the 4.0-m-long seesaw of **FIGURE EX12.31**. How far to the left of the pivot must a 4.0 kg cat stand to keep the seesaw balanced?

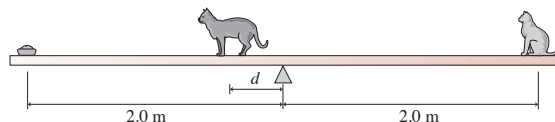


FIGURE EX12.31

## Section 12.9 Rolling Motion

32. || A car tire is 60 cm in diameter. The car is traveling at a speed of 20 m/s.
- What is the tire's angular velocity, in rpm?
  - What is the speed of a point at the top edge of the tire?
  - What is the speed of a point at the bottom edge of the tire?
33. || A 500 g, 8.0-cm-diameter can is filled with uniform, dense food. It rolls across the floor at 1.0 m/s. What is the can's kinetic energy?
34. || An 8.0-cm-diameter, 400 g solid sphere is released from rest at the top of a 2.1-m-long,  $25^\circ$  incline. It rolls, without slipping, to the bottom.
- What is the sphere's angular velocity at the bottom of the incline?
  - What fraction of its kinetic energy is rotational?
35. | A solid sphere of radius  $R$  is placed at a height of 30 cm on a  $15^\circ$  slope. It is released and rolls, without slipping, to the bottom. From what height should a circular hoop of radius  $R$  be released on the same slope in order to equal the sphere's speed at the bottom?

## Section 12.10 The Vector Description of Rotational Motion

36. | Evaluate the cross products  $\vec{A} \times \vec{B}$  and  $\vec{C} \times \vec{D}$ .

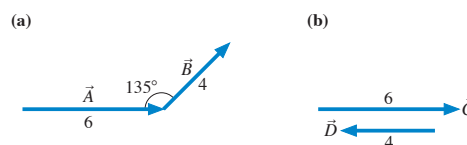


FIGURE EX12.36



37. I Evaluate the cross products  $\vec{A} \times \vec{B}$  and  $\vec{C} \times \vec{D}$ .

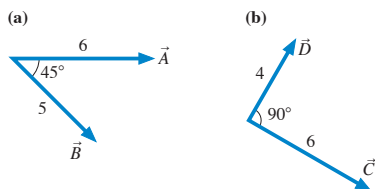


FIGURE EX12.37

38. I a. What is  $(\hat{i} \times \hat{j}) \times \hat{i}$ ?  
b. What is  $\hat{i} \times (\hat{j} \times \hat{i})$ ?  
39. I a. What is  $\hat{i} \times (\hat{i} \times \hat{j})$ ?  
b. What is  $(\hat{i} \times \hat{j}) \times \hat{k}$ ?  
40. I Vector  $\vec{A} = 3\hat{i} + \hat{j}$  and vector  $\vec{B} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ . What is the cross product  $\vec{A} \times \vec{B}$ ?  
41. I Consider the vector  $\vec{C} = 3\hat{i}$ .  
a. What is a vector  $\vec{D}$  such that  $\vec{C} \times \vec{D} = \vec{0}$ ?  
b. What is a vector  $\vec{E}$  such that  $\vec{C} \times \vec{E} = 6\hat{k}$ ?  
c. What is a vector  $\vec{F}$  such that  $\vec{C} \times \vec{F} = -3\hat{j}$ ?  
42. I Force  $\vec{F} = -10\hat{j}$  N is exerted on a particle at  $\vec{r} = (5\hat{i} + 5\hat{j})$  m. What is the torque on the particle about the origin?  
43. I What are the magnitude and direction of the angular momentum relative to the origin of the 100 g particle in FIGURE EX12.43?

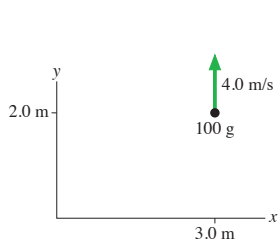


FIGURE EX12.43

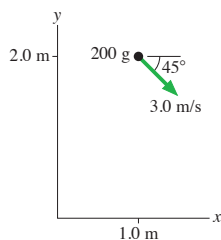


FIGURE EX12.44

44. I What are the magnitude and direction of the angular momentum relative to the origin of the 200 g particle in FIGURE EX12.44?

### Section 12.11 Angular Momentum

45. I What is the angular momentum of the 500 g rotating bar in FIGURE EX12.45?

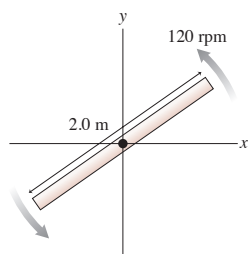


FIGURE EX12.45

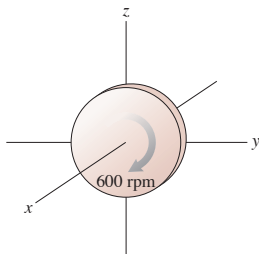


FIGURE EX12.46

46. I What is the angular momentum of the 2.0 kg, 4.0-cm-diameter rotating disk in FIGURE EX12.46?  
47. I How fast, in rpm, would a 5.0 kg, 22-cm-diameter bowling ball have to spin to have an angular momentum of  $0.23 \text{ kg m}^2/\text{s}$ ?

48. I A 2.0 kg, 20-cm-diameter turntable rotates at 100 rpm on frictionless bearings. Two 500 g blocks fall from above, hit the turntable simultaneously at opposite ends of a diameter, and stick. What is the turntable's angular velocity, in rpm, just after this event?

### Problems

49. I A 70 kg man's arm, including the hand, can be modeled as a 75-cm-long uniform rod with a mass of 3.5 kg. When the man raises both his arms, from hanging down to straight up, by how much does he raise his center of mass?  
50. I A 300 g ball and a 600 g ball are connected by a 40-cm-long massless, rigid rod. The structure rotates about its center of mass at 100 rpm. What is its rotational kinetic energy?  
51. I A 60-cm-diameter wheel is rolling along at 20 m/s. What is the speed of a point at the forward edge of the wheel?  
52. I An 800 g steel plate has the shape of the isosceles triangle shown in FIGURE P12.52. What are the x- and y-coordinates of the center of mass?

**Hint:** Divide the triangle into vertical strips of width  $dx$ , then relate the mass  $dm$  of a strip at position  $x$  to the values of  $x$  and  $dx$ .

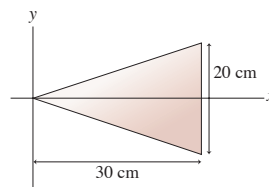


FIGURE P12.52

53. I What is the moment of inertia of a 2.0 kg, 20-cm-diameter disk for rotation about an axis (a) through the center, and (b) through the edge of the disk?  
54. I Determine the moment of inertia about the axis of the object shown in FIGURE P12.54.

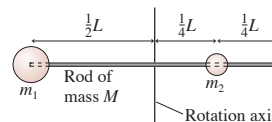


FIGURE P12.54

55. I Calculate by direct integration the moment of inertia for a thin rod of mass  $M$  and length  $L$  about an axis located distance  $d$  from one end. Confirm that your answer agrees with Table 12.2 when  $d = 0$  and when  $d = L/2$ .  
56. I a. A disk of mass  $M$  and radius  $R$  has a hole of radius  $r$  centered on the axis. Calculate the moment of inertia of the disk.  
b. Confirm that your answer agrees with Table 12.2 when  $r = 0$  and when  $r = R$ .  
c. A 4.0-cm-diameter disk with a 3.0-cm-diameter hole rolls down a 50-cm-long,  $20^\circ$  ramp. What is its speed at the bottom? What percent is this of the speed of a particle sliding down a frictionless ramp?  
57. I Calculate the moment of inertia of the rectangular plate in FIGURE P12.57 for rotation about a perpendicular axis through the center.

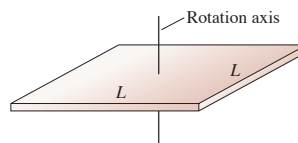
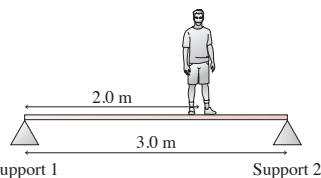


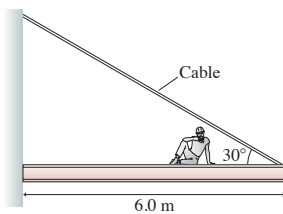
FIGURE P12.57

58. ■ Calculate the moment of inertia of the steel plate in **FIGURE P12.52** for rotation about a perpendicular axis passing through the origin.
59. ■ A person's center of mass is easily found by having the person lie on a *reaction board*. A horizontal, 2.5-m-long, 6.1 kg reaction board is supported only at the ends, with one end resting on a scale and the other on a pivot. A 60 kg woman lies on the reaction board with her feet over the pivot. The scale reads 25 kg. What is the distance from the woman's feet to her center of mass?
60. ■ A 3.0-m-long ladder, as shown in **Figure 12.37**, leans against a frictionless wall. The coefficient of static friction between the ladder and the floor is 0.40. What is the minimum angle the ladder can make with the floor without slipping?
61. ■ The 3.0-m-long, 100 kg rigid beam of **FIGURE P12.61** is supported at each end. An 80 kg student stands 2.0 m from support 1. How much upward force does each support exert on the beam?



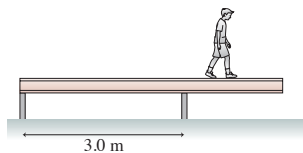
**FIGURE P12.61** Support 1 Support 2

62. ■ In **FIGURE P12.62**, an 80 kg construction worker sits down 2.0 m from the end of a 1450 kg steel beam to eat his lunch. The cable supporting the beam is rated at 15,000 N. Should the worker be worried?



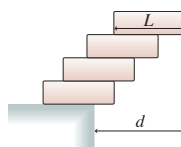
**FIGURE P12.62**

63. ■ A 40 kg, 5.0-m-long beam is supported by, but not attached to, the two posts in **FIGURE P12.63**. A 20 kg boy starts walking along the beam. How close can he get to the right end of the beam without it falling over?



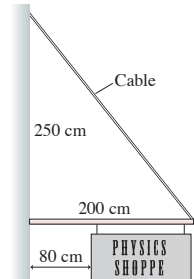
**FIGURE P12.63**

64. ■ Your task in a science contest is to stack four identical uniform bricks, each of length  $L$ , so that the top brick is as far to the right as possible without the stack falling over. Is it possible, as **FIGURE P12.64** shows, to stack the bricks such that no part of the top brick is over the table? Answer this question by determining the maximum possible value of  $d$ .



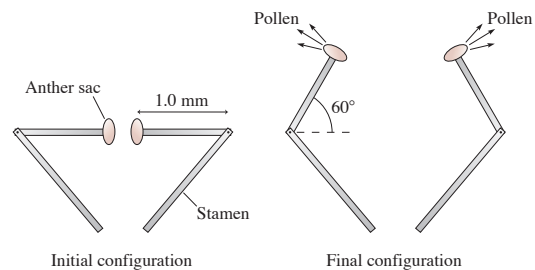
**FIGURE P12.64**

65. ■ A 120-cm-wide sign hangs from a 5.0 kg, 200-cm-long pole. A cable of negligible mass supports the end of the rod as shown in **FIGURE P12.65**. What is the maximum mass of the sign if the maximum tension in the cable without breaking is 300 N?



**FIGURE P12.65**

66. ■ The bunchberry flower has the fastest-moving parts ever observed in a plant. Initially, the stamens are held by the petals in a bent position, storing elastic energy like a coiled spring. When the petals release, the tips of the stamen act like medieval catapults, flipping through a  $60^\circ$  angle in just 0.30 ms to launch pollen from anther sacs at their ends. The human eye just sees a burst of pollen; only high-speed photography reveals the details. As **FIGURE P12.66** shows, we can model the stamen tip as a 1.0-mm-long,  $10 \mu\text{g}$  rigid rod with a  $10 \mu\text{g}$  anther sac at the end. Although oversimplifying, we'll assume a constant angular acceleration.
- How large is the "straightening torque"?
  - What is the speed of the anther sac as it releases its pollen?



**FIGURE P12.66**

67. ■ A 60-cm-long, 500 g bar rotates in a horizontal plane on an axle that passes through the center of the bar. Compressed air is fed in through the axle, passes through a small hole down the length of the bar, and escapes as air jets from holes at the ends of the bar. The jets are perpendicular to the bar's axis. Starting from rest, the bar spins up to an angular velocity of 150 rpm at the end of 10 s.
- How much force does each jet of escaping air exert on the bar?
  - If the axle is moved to one end of the bar while the air jets are unchanged, what will be the bar's angular velocity at the end of 10 seconds?
68. ■ Flywheels are large, massive wheels used to store energy. They can be spun up slowly, then the wheel's energy can be released quickly to accomplish a task that demands high power. An industrial flywheel has a 1.5 m diameter and a mass of 250 kg. Its maximum angular velocity is 1200 rpm.
- A motor spins up the flywheel with a constant torque of 50 Nm. How long does it take the flywheel to reach top speed?
  - How much energy is stored in the flywheel?
  - The flywheel is disconnected from the motor and connected to a machine to which it will deliver energy. Half the energy stored in the flywheel is delivered in 2.0 s. What is the average power delivered to the machine?
  - How much torque does the flywheel exert on the machine?

69. || The two blocks in **FIGURE P12.69** are connected by a massless rope that passes over a pulley. The pulley is 12 cm in diameter and has a mass of 2.0 kg. As the pulley turns, friction at the axle exerts a torque of magnitude 0.50 N·m. If the blocks are released from rest, how long does it take the 4.0 kg block to reach the floor?

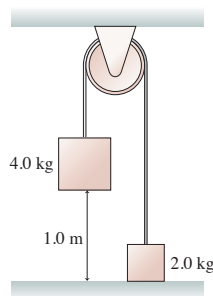


FIGURE P12.69

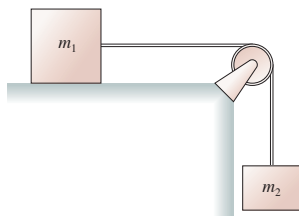


FIGURE P12.70

70. || Blocks of mass  $m_1$  and  $m_2$  are connected by a massless string that passes over the pulley in **FIGURE P12.70**. The pulley turns on frictionless bearings. Mass  $m_1$  slides on a horizontal, frictionless surface. Mass  $m_2$  is released while the blocks are at rest.
- Assume the pulley is massless. Find the acceleration of  $m_1$  and the tension in the string. This is a Chapter 7 review problem.
  - Suppose the pulley has mass  $m_p$  and radius  $R$ . Find the acceleration of  $m_1$  and the tensions in the upper and lower portions of the string. Verify that your answers agree with part a if you set  $m_p = 0$ .
71. || The 2.0 kg, 30-cm-diameter disk in **FIGURE P12.71** is spinning at 300 rpm. How much friction force must the brake apply to the rim to bring the disk to a halt in 3.0 s?

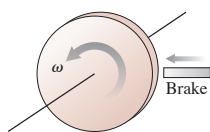


FIGURE P12.71

72. || Your engineering team has been assigned the task of measuring the properties of a new jet-engine turbine. You've previously determined that the turbine's moment of inertia is  $2.6 \text{ kg} \cdot \text{m}^2$ . The next job is to measure the frictional torque of the bearings. Your plan is to run the turbine up to a predetermined rotation speed, cut the power, and time how long it takes the turbine to reduce its rotation speed by 50%. Your data are as follows:

Rotation (rpm)	Time (s)
1500	19
1800	22
2100	25
2400	30
2700	34

Draw an appropriate graph of the data and, from the slope of the best-fit line, determine the frictional torque.

73. || A hollow sphere is rolling along a horizontal floor at 5.0 m/s when it comes to a  $30^\circ$  incline. How far up the incline does it roll before reversing direction?

74. || The 5.0 kg, 60-cm-diameter disk in **FIGURE P12.74** rotates on an axle passing through one edge. The axle is parallel to the floor. The cylinder is held with the center of mass at the same height as the axle, then released.

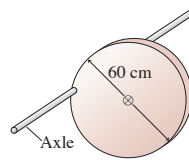


FIGURE P12.74

- What is the cylinder's initial angular acceleration?
  - What is the cylinder's angular velocity when it is directly below the axle?
75. || **FIGURE P12.75** shows a hoop of mass  $M$  and radius  $R$  rotating about an axle at the edge of the hoop. The hoop starts at its highest position and is given a very small push to start it rotating. At its lowest position, what are (a) the angular velocity and (b) the speed of the lowest point on the hoop?

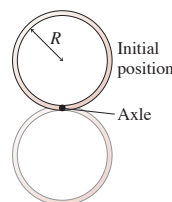


FIGURE P12.75

76. || A long, thin rod of mass  $M$  and length  $L$  is standing straight up on a table. Its lower end rotates on a frictionless pivot. A very slight push causes the rod to fall over. As it hits the table, what are (a) the angular velocity and (b) the speed of the tip of the rod?
77. || The sphere of mass  $M$  and radius  $R$  in **FIGURE P12.77** is rigidly attached to a thin rod of radius  $r$  that passes through the sphere at distance  $\frac{1}{2}R$  from the center. A string wrapped around the rod pulls with tension  $T$ . Find an expression for the sphere's angular acceleration. The rod's moment of inertia is negligible.

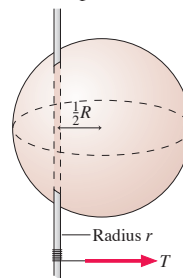


FIGURE P12.77

78. || A satellite follows the elliptical orbit shown in **FIGURE P12.78**. The only force on the satellite is the gravitational attraction of the planet. The satellite's speed at point a is 8000 m/s.
- Does the satellite experience any torque about the center of the planet? Explain.
  - What is the satellite's speed at point b?
  - What is the satellite's speed at point c?

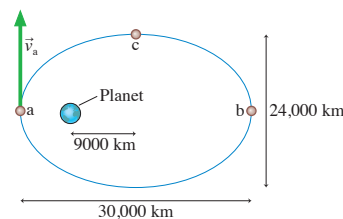


FIGURE P12.78

79. || A 10 g bullet traveling at 400 m/s strikes a 10 kg, 1.0-m-wide door at the edge opposite the hinge. The bullet embeds itself in the door, causing the door to swing open. What is the angular velocity of the door just after impact?
80. || A 200 g, 40-cm-diameter turntable rotates on frictionless bearings at 60 rpm. A 20 g block sits at the center of the turntable. A compressed spring shoots the block radially outward along a frictionless groove in the surface of the turntable. What is the turntable's rotation angular velocity when the block reaches the outer edge?

81. **||** A merry-go-round is a common piece of playground equipment. A 3.0-m-diameter merry-go-round with a mass of 250 kg is spinning at 20 rpm. John runs tangent to the merry-go-round at 5.0 m/s, in the same direction that it is turning, and jumps onto the outer edge. John's mass is 30 kg. What is the merry-go-round's angular velocity, in rpm, after John jumps on?
82. **||** A 45 kg figure skater is spinning on the toes of her skates at 1.0 rev/s. Her arms are outstretched as far as they will go. In this orientation, the skater can be modeled as a cylindrical torso (40 kg, 20 cm average diameter, 160 cm tall) plus two rod-like arms (2.5 kg each, 66 cm long) attached to the outside of the torso. The skater then raises her arms straight above her head, where she appears to be a 45 kg, 20-cm-diameter, 200-cm-tall cylinder. What is her new angular velocity, in rev/s?

### Challenge Problems

83. In **FIGURE CP12.83**, a 200 g toy car is placed on a narrow 60-cm-diameter track with wheel grooves that keep the car going in a circle. The 1.0 kg track is free to turn on a frictionless, vertical axis. The spokes have negligible mass. After the car's switch is turned on, it soon reaches a steady speed of 0.75 m/s relative to the track. What then is the track's angular velocity, in rpm?

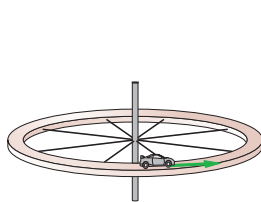


FIGURE CP12.83

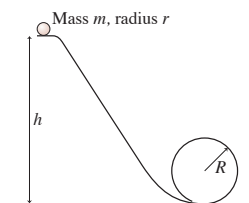


FIGURE CP12.84

84. The marble rolls down the track shown in **FIGURE CP12.84** and around a loop-the-loop of radius  $R$ . The marble has mass  $m$  and radius  $r$ . What minimum height  $h$  must the track have for the marble to make it around the loop-the-loop without falling off?
85. **FIGURE CP12.85** shows a triangular block of Swiss cheese sitting on a cheese board. You and your friends start to wonder what will happen if you slowly tilt the board, increasing angle  $\theta$ . Emily thinks the cheese will start to slide before it topples over. Fred thinks it will topple before starting to slide. Some quick Internet research on your part reveals that the coefficient of static friction of Swiss cheese on wood is 0.90. Who is right?

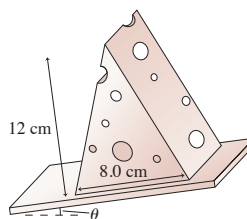


FIGURE CP12.85

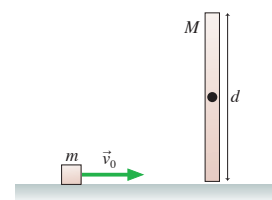


FIGURE CP12.86

86. **FIGURE CP12.86** shows a cube of mass  $m$  sliding without friction at speed  $v_0$ . It undergoes a perfectly elastic collision with the bottom tip of a rod of length  $d$  and mass  $M = 2m$ . The rod is pivoted about a frictionless axle through its center, and initially it hangs straight down and is at rest. What is the cube's velocity—both speed and direction—after the collision?
87. A 75 g, 30-cm-long rod hangs vertically on a frictionless, horizontal axle passing through its center. A 10 g ball of clay traveling horizontally at 2.5 m/s hits and sticks to the very bottom tip of the rod. To what maximum angle, measured from vertical, does the rod (with the attached ball of clay) rotate?
88. During most of its lifetime, a star maintains an equilibrium size in which the inward force of gravity on each atom is balanced by an outward pressure force due to the heat of the nuclear reactions in the core. But after all the hydrogen “fuel” is consumed by nuclear fusion, the pressure force drops and the star undergoes a *gravitational collapse* until it becomes a *neutron star*. In a neutron star, the electrons and protons of the atoms are squeezed together by gravity until they fuse into neutrons. Neutron stars spin very rapidly and emit intense pulses of radio and light waves, one pulse per rotation. These “pulsing stars” were discovered in the 1960s and are called *pulsars*.
- a. A star with the mass ( $M = 2.0 \times 10^{30}$  kg) and size ( $R = 7.0 \times 10^8$  m) of our sun rotates once every 30 days. After undergoing gravitational collapse, the star forms a pulsar that is observed by astronomers to emit radio pulses every 0.10 s. By treating the neutron star as a solid sphere, deduce its radius.
- b. What is the speed of a point on the equator of the neutron star?
- Your answers will be somewhat too large because a star cannot be accurately modeled as a solid sphere. Even so, you will be able to show that a star, whose mass is  $10^6$  larger than the earth's, can be compressed by gravitational forces to a size smaller than a typical state in the United States!

### STOP TO THINK ANSWERS

**Stop to Think 12.1:**  $I_a > I_d > I_b > I_c$ . The moment of inertia is smaller when the mass is more concentrated near the rotation axis.

**Stop to Think 12.2:**  $\tau_e > \tau_a = \tau_d > \tau_b > \tau_c$ . The tangential component in e is larger than 2 N.

**Stop to Think 12.3:**  $\alpha_b > \alpha_a > \alpha_c = \alpha_d = \alpha_e$ . Angular acceleration is proportional to torque and inversely proportional to the moment of inertia. The moment of inertia depends on the *square* of the radius. The tangential force component in e is the same as in d.

**Stop to Think 12.4:**  $c > d > a = b$ . To keep the meter stick in equilibrium, the student must supply a torque equal and opposite to the torque due to the hanging masses. Torque depends on the mass and on how far the mass is from the pivot point.

**Stop to Think 12.5:** d. There is no net torque on the bucket + rain system, so the angular momentum is conserved. The addition of mass on the outer edge of the circle increases  $I$ , so  $\omega$  must decrease. Mechanical energy is not conserved because the raindrop collisions are inelastic.

# SUMMARY

The goal of Chapter 13 has been to use Newton's theory of gravity to understand the motion of satellites and planets.

## General Principles

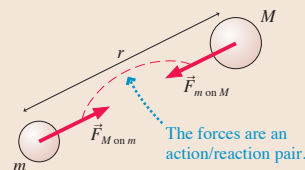
### Newton's Theory of Gravity

- Two objects with masses  $M$  and  $m$  a distance  $r$  apart exert attractive **gravitational forces** on each other of magnitude

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

where the **gravitational constant** is  $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ .

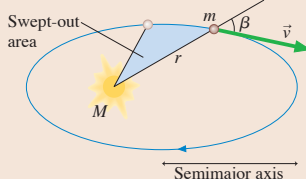
- Gravitational mass and inertial mass are equivalent.
- Newton's three laws of motion apply to all objects in the universe.



## Important Concepts

**Orbital motion** of a planet (or satellite) is described by **Kepler's laws**:

- Orbits are ellipses with the sun (or planet) at one focus.
- A line between the sun and the planet sweeps out equal areas during equal intervals of time.
- The square of the planet's period  $T$  is proportional to the cube of the orbit's semimajor axis.



**Circular orbits** are a special case of an ellipse. For a circular orbit around a mass  $M$ ,

$$v = \sqrt{\frac{GM}{r}} \quad \text{and} \quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

### Conservation of angular momentum

The angular momentum  $L = mrv \sin \beta$  remains constant throughout the orbit. Kepler's second law is a consequence of this law.

### Orbital energetics

A satellite's mechanical energy  $E_{\text{mech}} = K + U_g$  is conserved, where the gravitational potential energy is

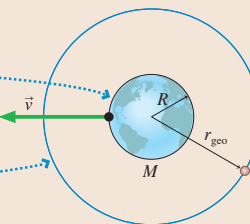
$$U_g = -\frac{GMm}{r}$$

For circular orbits,  $K = -\frac{1}{2}U_g$  and  $E_{\text{mech}} = \frac{1}{2}U_g$ . Negative total energy is characteristic of a **bound system**.

## Applications

For a planet of mass  $M$  and radius  $R$ ,

- The free-fall acceleration on the surface is  $g_{\text{surface}} = \frac{GM}{R^2}$
- The **escape speed** is  $v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$
- The radius of a **geosynchronous orbit** is  $r_{\text{geo}} = \left(\frac{GM}{4\pi^2} T^2\right)^{1/3}$



## Terms and Notation

cosmology  
Kepler's laws  
gravitational force

Newton's law of gravity  
gravitational constant,  $G$   
gravitational mass

principle of equivalence  
Newton's theory of gravity  
escape speed

satellite  
geosynchronous orbit  
bound system

## CONCEPTUAL QUESTIONS







1. Is the earth's gravitational force on the sun larger than, smaller than, or equal to the sun's gravitational force on the earth? Explain.
2. The gravitational force of a star on orbiting planet 1 is  $F_1$ . Planet 2, which is twice as massive as planet 1 and orbits at twice the distance from the star, experiences gravitational force  $F_2$ . What is the ratio  $F_1/F_2$ ?
3. A 1000 kg satellite and a 2000 kg satellite follow exactly the same orbit around the earth.
  - a. What is the ratio  $F_1/F_2$  of the force on the first satellite to that on the second satellite?
  - b. What is the ratio  $a_1/a_2$  of the acceleration of the first satellite to that of the second satellite?
4. How far away from the earth must an orbiting spacecraft be for the astronauts inside to be weightless? Explain.
5. A space shuttle astronaut is working outside the shuttle as it orbits the earth. If he drops a hammer, will it fall to earth? Explain why or why not.
6. The free-fall acceleration at the surface of planet 1 is  $20 \text{ m/s}^2$ . The radius and the mass of planet 2 are twice those of planet 1. What is  $g$  on planet 2?
7. Why is the gravitational potential energy of two masses negative? Note that saying "because that's what the equation gives" is *not* an explanation.
8. The escape speed from Planet X is  $10,000 \text{ m/s}$ . Planet Y has the same radius as Planet X but is twice as dense. What is the escape speed from Planet Y?
9. The mass of Jupiter is 300 times the mass of the earth. Jupiter orbits the sun with  $T_{\text{Jupiter}} = 11.9 \text{ yr}$  in an orbit with  $r_{\text{Jupiter}} = 5.2r_{\text{earth}}$ . Suppose the earth could be moved to the distance of Jupiter and placed in a circular orbit around the sun. Which of the following describes the earth's new period? Explain.
  - a. 1 yr
  - b. Between 1 yr and 11.9 yr
  - c. 11.9 yr
  - d. More than 11.9 yr
  - e. It would depend on the earth's speed.
  - f. It's impossible for a planet of earth's mass to orbit at the distance of Jupiter.
10. Satellites in near-earth orbit experience a very slight drag due to the extremely thin upper atmosphere. These satellites slowly but surely spiral inward, where they finally burn up as they reach the thicker lower levels of the atmosphere. The radius decreases so slowly that you can consider the satellite to have a circular orbit at all times. As a satellite spirals inward, does it speed up, slow down, or maintain the same speed? Explain.

## EXERCISES AND PROBLEMS






Problems labeled  integrate material from earlier chapters.

### Exercises



#### Section 13.3 Newton's Law of Gravity

1.  What is the ratio of the sun's gravitational force on you to the earth's gravitational force on you?
2.  The centers of a 10 kg lead ball and a 100 g lead ball are separated by 10 cm.
  - a. What gravitational force does each exert on the other?
  - b. What is the ratio of this gravitational force to the gravitational force of the earth on the 100 g ball?
3.  What is the ratio of the sun's gravitational force on the moon to the earth's gravitational force on the moon?
4.  A 1.0-m-diameter lead sphere has a mass of 5900 kg. A dust particle rests on the surface. What is the ratio of the gravitational force of the sphere on the dust particle to the gravitational force of the earth on the dust particle?
5.  Estimate the force of attraction between a 50 kg woman and a 70 kg man sitting 1.0 m apart.
6.  The space shuttle orbits 300 km above the surface of the earth. What is the gravitational force on a 1.0 kg sphere inside the space shuttle?

#### Section 13.4 Little $g$ and Big $G$

7. 
  - a. What is the free-fall acceleration at the surface of the sun?
  - b. What is the sun's free-fall acceleration at the distance of the earth?
8.  What is the free-fall acceleration at the surface of (a) the moon and (b) Jupiter?
9.  A sensitive gravimeter at a mountain observatory finds that the free-fall acceleration is  $0.0075 \text{ m/s}^2$  less than that at sea level. What is the observatory's altitude?
10.  Suppose we could shrink the earth without changing its mass. At what fraction of its current radius would the free-fall acceleration at the surface be three times its present value?
11.  Planet Z is 10,000 km in diameter. The free-fall acceleration on Planet Z is  $8.0 \text{ m/s}^2$ .
  - a. What is the mass of Planet Z?
  - b. What is the free-fall acceleration 10,000 km above Planet Z's north pole?

#### Section 13.5 Gravitational Potential Energy

12.  An astronaut on earth can throw a ball straight up to a height of 15 m. How high can he throw the ball on Mars?
13.  What is the escape speed from Jupiter?



14. || A rocket is launched straight up from the earth's surface at a speed of 15,000 m/s. What is its speed when it is very far away from the earth?
15. | A space station orbits the sun at the same distance as the earth but on the opposite side of the sun. A small probe is fired away from the station. What minimum speed does the probe need to escape the solar system?
16. || You have been visiting a distant planet. Your measurements have determined that the planet's mass is twice that of earth but the free-fall acceleration at the surface is only one-fourth as large.
  - a. What is the planet's radius?
  - b. To get back to earth, you need to escape the planet. What minimum speed does your rocket need?

### Section 13.6 Satellite Orbits and Energies

17. | The *asteroid belt* circles the sun between the orbits of Mars and Jupiter. One asteroid has a period of 5.0 earth years. What are the asteroid's orbital radius and speed?
18. | Use information about the earth and its orbit to determine the mass of the sun.
19. || Planet X orbits the star Omega with a "year" that is 200 earth days long. Planet Y circles Omega at four times the distance of Planet X. How long is a year on Planet Y?
20. | You are the science officer on a visit to a distant solar system. Prior to landing on a planet you measure its diameter to be  $1.8 \times 10^7$  m and its rotation period to be 22.3 hours. You have previously determined that the planet orbits  $2.2 \times 10^{11}$  m from its star with a period of 402 earth days. Once on the surface you find that the free-fall acceleration is  $12.2 \text{ m/s}^2$ . What is the mass of (a) the planet and (b) the star?
21. || Three satellites orbit a planet of radius  $R$ , as shown in **FIGURE EX13.21**. Satellites  $S_1$  and  $S_3$  have mass  $m$ . Satellite  $S_2$  has mass  $2m$ . Satellite  $S_1$  orbits in 250 minutes and the force on  $S_1$  is 10,000 N.
  - a. What are the periods of  $S_2$  and  $S_3$ ?
  - b. What are the forces on  $S_2$  and  $S_3$ ?
  - c. What is the kinetic-energy ratio  $K_1/K_3$  for  $S_1$  and  $S_3$ ?

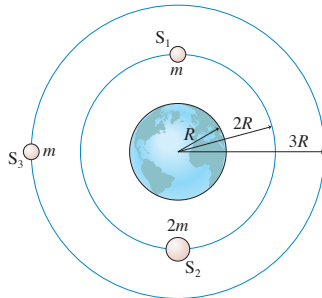


FIGURE EX13.21

22. || A satellite orbits the sun with a period of 1.0 day. What is the radius of its orbit?
23. || An earth satellite moves in a circular orbit at a speed of 5500 m/s. What is its orbital period?
24. || What are the speed and altitude of a geosynchronous satellite orbiting Mars? Mars rotates on its axis once every 24.8 hours.

### Problems

25. || Two spherical objects have a combined mass of 150 kg. The gravitational attraction between them is  $8.00 \times 10^{-6} \text{ N}$  when their centers are 20 cm apart. What is the mass of each?

26. || **FIGURE P13.26** shows three masses. What are the magnitude and the direction of the net gravitational force on (a) the 20.0 kg mass and (b) the 5.0 kg mass? Give the direction as an angle cw or ccw from the y-axis.

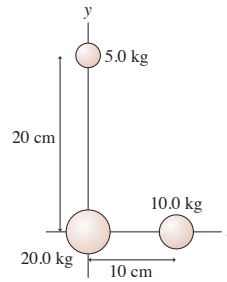


FIGURE P13.26

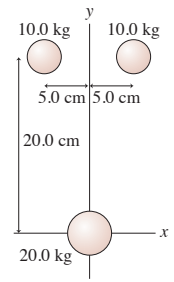


FIGURE P13.27

27. || What are the magnitude and direction of the net gravitational force on the 20.0 kg mass in **FIGURE P13.27**?
28. || What is the total gravitational potential energy of the three masses in **FIGURE P13.26**?
29. || What is the total gravitational potential energy of the three masses in **FIGURE P13.27**?
30. || Two 100 kg lead spheres are suspended from 100-m-long massless cables. The tops of the cables have been carefully anchored *exactly* 1 m apart. What is the distance between the centers of the spheres?
31. || A 20 kg sphere is at the origin and a 10 kg sphere is at  $x = 20 \text{ cm}$ . At what position on the  $x$ -axis could you place a small mass such that the net gravitational force on it due to the spheres is zero?
32. || a. At what height above the earth is the acceleration due to gravity 10% of its value at the surface?  
b. What is the speed of a satellite orbiting at that height?
33. || A 1.0 kg object is released from rest 500 km ( $\approx 300$  miles) above the earth.
  - a. What is its impact speed as it hits the ground? Ignore air resistance.
  - b. What would the impact speed be if the earth were flat?
  - c. By what percentage is the flat-earth calculation in error?
34. || An object of mass  $m$  is dropped from height  $h$  above a planet of mass  $M$  and radius  $R$ . Find an expression for the object's speed as it hits the ground.
35. || A projectile is shot straight up from the earth's surface at a speed of 10,000 km/h. How high does it go?
36. || Two meteoroids are heading for earth. Their speeds as they cross the moon's orbit are 2.0 km/s.
  - a. The first meteoroid is heading straight for earth. What is its speed of impact?
  - b. The second misses the earth by 5000 km. What is its speed at its closest point?
37. || A binary star system has two stars, each with the same mass as our sun, separated by  $1.0 \times 10^{12} \text{ m}$ . A comet is very far away and essentially at rest. Slowly but surely, gravity pulls the comet toward the stars. Suppose the comet travels along a straight line that passes through the midpoint between the two stars. What is the comet's speed at the midpoint?
38. || Suppose that on earth you can jump straight up a distance of 50 cm. Can you escape from a 4.0-km-diameter asteroid with a mass of  $1.0 \times 10^{14} \text{ kg}$ ?

39. || A projectile is fired straight away from the moon from a base on the far side of the moon, away from the earth. What is the projectile's escape speed from the earth-moon system?
40. || Two spherical asteroids have the same radius  $R$ . Asteroid 1 has mass  $M$  and asteroid 2 has mass  $2M$ . The two asteroids are released from rest with distance  $10R$  between their centers. What is the speed of each asteroid just before they collide?
- Hint:** You will need to use two conservation laws.
41. || Two Jupiter-size planets are released from rest  $1.0 \times 10^{11}$  m apart. What are their speeds as they crash together?
42. || A starship is circling a distant planet of radius  $R$ . The astronauts find that the free-fall acceleration at their altitude is half the value at the planet's surface. How far above the surface are they orbiting? Your answer will be a multiple of  $R$ .
43. || Three stars, each with the mass and radius of our sun, form an equilateral triangle  $5.0 \times 10^9$  m on a side. If all three are simultaneously released from rest, what are their speeds as they crash together in the center?
44. || The two stars in a binary star system have masses  $2.0 \times 10^{30}$  kg and  $6.0 \times 10^{30}$  kg. They are separated by  $2.0 \times 10^{12}$  m. What are
- The system's rotation period, in years?
  - The speed of each star?
45. || A 4000 kg lunar lander is in orbit 50 km above the surface of the moon. It needs to move out to a 300-km-high orbit in order to link up with the mother ship that will take the astronauts home. How much work must the thrusters do?
46. || The space shuttle is in a 250-km-high circular orbit. It needs to reach a 610-km-high circular orbit to catch the Hubble Space Telescope for repairs. The shuttle's mass is 75,000 kg. How much energy is required to boost it to the new orbit?
47. || In 2000, NASA placed a satellite in orbit around an asteroid. Consider a spherical asteroid with a mass of  $1.0 \times 10^{16}$  kg and a radius of 8.8 km.
- What is the speed of a satellite orbiting 5.0 km above the surface?
  - What is the escape speed from the asteroid?
48. || NASA would like to place a satellite in orbit around the moon such that the satellite always remains in the same position over the lunar surface. What is the satellite's altitude?
49. || A satellite orbiting the earth is directly over a point on the equator at 12:00 midnight every two days. It is not over that point at any time in between. What is the radius of the satellite's orbit?
50. || FIGURE P13.50 shows two planets of mass  $m$  orbiting a star of mass  $M$ . The planets are in the same orbit, with radius  $r$ , but are always at opposite ends of a diameter. Find an exact expression for the orbital period  $T$ .

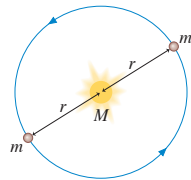


FIGURE P13.50

51. || Figure 13.17 showed a graph of  $\log T$  versus  $\log r$  for the planetary data given in Table 13.2. Such a graph is called a *log-log graph*. The scales in Figure 13.17 are logarithmic, not linear, meaning that each division along the axis corresponds to a *factor* of 10 increase in the value. Strictly speaking, the "correct" labels on the y-axis should be 7, 8, 9, and 10 because these are the logarithms of  $10^7, \dots, 10^{10}$ .
- Consider two quantities  $u$  and  $v$  that are related by the expression  $v^p = Cu^q$ , where  $C$  is a constant. The exponents  $p$  and  $q$

are not necessarily integers. Define  $x = \log u$  and  $y = \log v$ . Find an expression for  $y$  in terms of  $x$ .

- What *shape* will a graph of  $y$  versus  $x$  have? Explain.
  - What *slope* will a graph of  $y$  versus  $x$  have? Explain.
- d. Use the experimentally determined "best-fit" line in Figure 13.17 to find the mass of the sun.
52. || Large stars can explode as they finish burning their nuclear fuel, causing a *supernova*. The explosion blows away the outer layers of the star. According to Newton's third law, the forces that push the outer layers away have *reaction forces* that are inwardly directed on the core of the star. These forces compress the core and can cause the core to undergo a *gravitational collapse*. The gravitational forces keep pulling all the matter together tighter and tighter, crushing atoms out of existence. Under these extreme conditions, a proton and an electron can be squeezed together to form a neutron. If the collapse is halted when the neutrons all come into contact with each other, the result is an object called a *neutron star*, an entire star consisting of solid nuclear matter. Many neutron stars rotate about their axis with a period of  $\approx 1$  s and, as they do so, send out a pulse of electromagnetic waves once a second. These stars were discovered in the 1960s and are called *pulsars*.
- Consider a neutron star with a mass equal to the sun, a radius of 10 km, and a rotation period of 1.0 s. What is the speed of a point on the equator of the star?
  - What is  $g$  at the surface of this neutron star?
  - A stationary 1.0 kg mass has a weight on earth of 9.8 N. What would be its weight on the star?
  - How many revolutions per minute are made by a satellite orbiting 1.0 km above the surface?
  - What is the radius of a geosynchronous orbit about the neutron star?
53. || The solar system is 25,000 light years from the center of our Milky Way galaxy. One *light year* is the distance light travels in one year at a speed of  $3.0 \times 10^8$  m/s. Astronomers have determined that the solar system is orbiting the center of the galaxy at a speed of 230 km/s.
- Assuming the orbit is circular, what is the period of the solar system's orbit? Give your answer in years.
  - Our solar system was formed roughly 5 billion years ago. How many orbits has it completed?
  - The gravitational force on the solar system is the net force due to all the matter inside our orbit. Most of that matter is concentrated near the center of the galaxy. Assume that the matter has a spherical distribution, like a giant star. What is the approximate mass of the galactic center?
  - Assume that the sun is a typical star with a typical mass. If galactic matter is made up of stars, approximately how many stars are in the center of the galaxy?
- Astronomers have spent many years trying to determine how many stars there are in the Milky Way. The number of stars seems to be only about 10% of what you found in part d. In other words, about 90% of the mass of the galaxy appears to be in some form other than stars. This is called the *dark matter* of the universe. No one knows what the dark matter is. This is one of the outstanding scientific questions of our day.
54. || Three stars, each with the mass of our sun, form an equilateral triangle with sides  $1.0 \times 10^{12}$  m long. (This triangle would just about fit within the orbit of Jupiter.) The triangle has to rotate, because otherwise the stars would crash together in the center. What is the period of rotation? Give your answer in years.

55. || Pluto moves in a fairly elliptical orbit around the sun. Pluto's speed at its closest approach of  $4.43 \times 10^9$  km is 6.12 km/s. What is Pluto's speed at the most distant point in its orbit, where it is  $7.30 \times 10^9$  km from the sun?
56. || Mercury moves in a fairly elliptical orbit around the sun. Mercury's speed is 38.8 km/s when it is at its most distant point,  $6.99 \times 10^{10}$  m from the sun. How far is Mercury from the sun at its closest point, where its speed is 59.0 km/s?
57. || Comets move around the sun in very elliptical orbits. At its closest approach, in 1986, Comet Halley was  $8.79 \times 10^7$  km from the sun and moving with a speed of 54.6 km/s. What was the comet's speed when it crossed Neptune's orbit in 2006?
58. || A spaceship is in a circular orbit of radius  $r_0$  about a planet of mass  $M$ . A brief but intense firing of its engine in the forward direction decreases the spaceship's speed by 50%. This causes the spaceship to move into an elliptical orbit.
- What is the spaceship's new speed, just after the rocket burn is complete, in terms of  $M$ ,  $G$ , and  $r_0$ ?
  - In terms of  $r_0$ , what are the spaceship's maximum and minimum distances from the planet in its new orbit?

In Problems 59 through 61 you are given the equation(s) used to solve a problem. For each of these, you are to

- Write a realistic problem for which this is the correct equation(s).
- Draw a pictorial representation.
- Finish the solution of the problem.

$$59. \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.68 \times 10^{26} \text{ kg})}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m})^2}$$

$$60. \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(1000 \text{ kg})}{r^2} = \frac{(1000 \text{ kg})(1997 \text{ m/s})^2}{r}$$

$$61. \frac{1}{2}(100 \text{ kg})v_2^2 - \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})(100 \text{ kg})}{1.74 \times 10^6 \text{ m}} = 0 - \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})(100 \text{ kg})}{3.48 \times 10^6 \text{ m}}$$

### Challenge Problems

62. A satellite in a circular orbit of radius  $r$  has period  $T$ . A satellite in a nearby orbit with radius  $r + \Delta r$ , where  $\Delta r \ll r$ , has the very slightly different period  $T + \Delta T$ .
- Show that

$$\frac{\Delta T}{T} = \frac{3}{2} \frac{\Delta r}{r}$$

- Two earth satellites are in parallel orbits with radii 6700 km and 6701 km. One day they pass each other, 1 km apart, along a line radially outward from the earth. How long will it be until they are again 1 km apart?
63. In 1996, the Solar and Heliospheric Observatory (SOHO) was "parked" in an orbit slightly inside the earth's orbit, as shown in FIGURE CP13.63. The satellite's period in this orbit is exactly one year, so it remains fixed relative to the earth. At this point, called

a *Lagrange point*, the light from the sun is never blocked by the earth, yet the satellite remains "nearby" so that data are easily transmitted to earth. What is SOHO's distance from the earth?

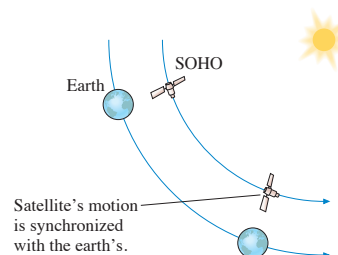


FIGURE CP13.63

**Hint:** Use the binomial approximation. SOHO's distance from the earth is much less than the earth's distance from the sun.

64. A projectile is fired from the earth in the direction of the earth's motion around the sun. What minimum speed must the projectile have relative to the earth to escape the solar system? Ignore the earth's rotation.

**Hint:** This is a three-part problem. First find the speed a projectile at the earth's distance needs to escape the sun. Transform that speed into the earth's reference frame, then determine how fast the projectile must be launched to have this speed when far from the earth.

65. Your job with NASA is to monitor satellite orbits. One day, during a routine survey, you find that a 400 kg satellite in a 1000-km-high circular orbit is going to collide with a smaller 100 kg satellite traveling in the same orbit but in the opposite direction. Knowing the construction of the two satellites, you expect they will become enmeshed into a single piece of space debris. When you notify your boss of this impending collision, he asks you to quickly determine whether the space debris will continue to orbit or crash into the earth. What will the outcome be?
66. While visiting Planet Physics, you toss a rock straight up at 11 m/s and catch it 2.5 s later. While you visit the surface, your cruise ship orbits at an altitude equal to the planet's radius every 230 min. What are the (a) mass and (b) radius of Planet Physics?
67. A moon lander is orbiting the moon at an altitude of 1000 km. By what percentage must it decrease its speed so as to just graze the moon's surface one-half period later?
68. Let's look in more detail at how a satellite is moved from one circular orbit to another. FIGURE CP13.68 shows two circular orbits, of radii  $r_1$  and  $r_2$ , and an elliptical orbit that connects them. Points 1 and 2 are at the ends of the semimajor axis of the ellipse.

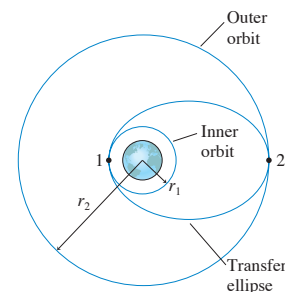


FIGURE CP13.68

- A satellite moving along the elliptical orbit has to satisfy two conservation laws. Use these two laws to prove that the velocities at points 1 and 2 are

$$v_1' = \sqrt{\frac{2GM(r_2/r_1)}{r_1 + r_2}} \quad \text{and} \quad v_2' = \sqrt{\frac{2GM(r_1/r_2)}{r_1 + r_2}}$$

The prime indicates that these are the velocities on the elliptical orbit. Both reduce to Equation 13.22 if  $r_1 = r_2 = r$ .

- b. Consider a 1000 kg communications satellite that needs to be boosted from an orbit 300 km above the earth to a geosynchronous orbit 35,900 km above the earth. Find the velocity  $v_1$  on the inner circular orbit and the velocity  $v'_1$  at the low point on the elliptical orbit that spans the two circular orbits.
  - c. How much work must the rocket motor do to transfer the satellite from the circular orbit to the elliptical orbit?
  - d. Now find the velocity  $v'_2$  at the high point of the elliptical orbit and the velocity  $v_2$  of the outer circular orbit.
  - e. How much work must the rocket motor do to transfer the satellite from the elliptical orbit to the outer circular orbit?
  - f. Compute the total work done and compare your answer to the result of Example 13.6.
69. FIGURE CP13.69 shows a particle of mass  $m$  at distance  $x$  from the center of a very thin cylinder of mass  $M$  and length  $L$ . The particle is outside the cylinder, so  $x > L/2$ .

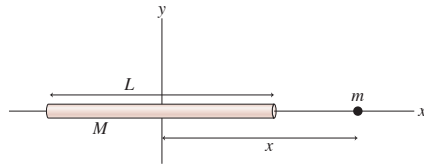


FIGURE CP13.69

- a. Calculate the gravitational potential energy of these two masses.
  - b. Use what you know about the relationship between force and potential energy to find the magnitude of the gravitational force on  $m$  when it is at position  $x$ .
70. FIGURE CP13.70 shows a particle of mass  $m$  at distance  $x$  along the axis of a very thin ring of mass  $M$  and radius  $R$ .
- a. Calculate the gravitational potential energy of these two masses.
  - b. Use what you know about the relationship between force and potential energy to find the magnitude of the gravitational force on  $m$  when it is at position  $x$ .

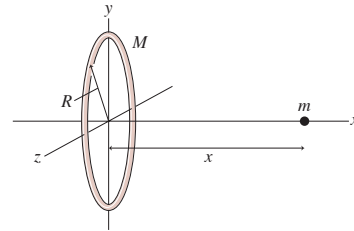


FIGURE CP13.70

## STOP TO THINK ANSWERS

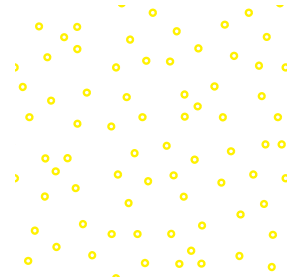
**Stop to Think 13.1: e.** The acceleration decreases inversely with the square of the distance. At height  $R_e$ , the distance from the center of the earth is  $2R_e$ .

**Stop to Think 13.2: c.** Newton's third law requires  $F_{1 \text{ on } 2} = F_{2 \text{ on } 1}$ .

**Stop to Think 13.3: b.**  $g_{\text{surface}} = GM/R^2$ . Because of the square, a radius twice as large balances a mass four times as large.

**Stop to Think 13.4:** In absolute value,  $U_e > U_a = U_b = U_d > U_c$ .  $|U_g|$  is proportional to  $m_1 m_2 / r$ .

**Stop to Think 13.5: a.**  $T^2$  is proportional to  $r^3$ , or  $T$  is proportional to  $r^{3/2}$ .  $4^{3/2} = 8$ .



# SUMMARY

The goal of Chapter 14 has been to understand systems that oscillate with simple harmonic motion.

## General Principles

### Dynamics

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position.

#### Horizontal spring

$$(F_{\text{net}})_x = -kx$$

#### Vertical spring

The origin is at the equilibrium position  $\Delta L = mg/k$ .

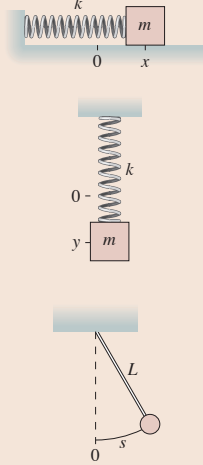
$$(F_{\text{net}})_y = -ky$$

Both:  $\omega = \sqrt{\frac{k}{m}} \quad T = 2\pi \sqrt{\frac{m}{k}}$

#### Pendulum

$$(F_{\text{net}})_t = -\left(\frac{mg}{L}\right)s$$

$$\omega = \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}}$$



### Energy

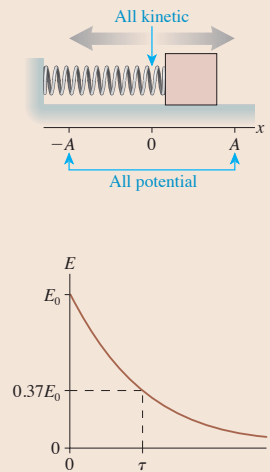
If there is **no friction** or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy  $E = K + U$  is conserved.

$$\begin{aligned} E &= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}m(v_{\text{max}})^2 \\ &= \frac{1}{2}kA^2 \end{aligned}$$

In a **damped system**, the energy decays exponentially

$$E = E_0 e^{-t/\tau}$$

where  $\tau$  is the **time constant**.



## Important Concepts

**Simple harmonic motion (SHM)** is a sinusoidal oscillation with period  $T$  and amplitude  $A$ .

**Frequency**  $f = \frac{1}{T}$

#### Angular frequency

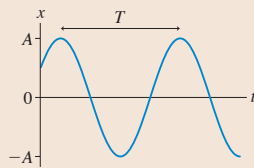
$$\omega = 2\pi f = \frac{2\pi}{T}$$

**Position**  $x(t) = A \cos(\omega t + \phi_0)$

$$= A \cos\left(\frac{2\pi t}{T} + \phi_0\right)$$

**Velocity**  $v_x(t) = -v_{\text{max}} \sin(\omega t + \phi_0)$  with maximum speed  $v_{\text{max}} = \omega A$

**Acceleration**  $a_x(t) = -\omega^2 x(t) = -\omega^2 A \cos(\omega t + \phi_0)$



SHM is the projection onto the  $x$ -axis of **uniform circular motion**.

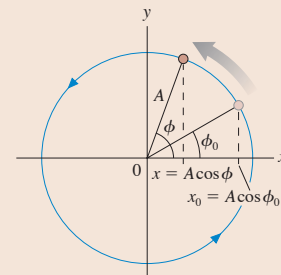
$\phi = \omega t + \phi_0$  is the **phase**

The position at time  $t$  is

$$\begin{aligned} x(t) &= A \cos \phi \\ &= A \cos(\omega t + \phi_0) \end{aligned}$$

The **phase constant**  $\phi_0$  determines the initial conditions:

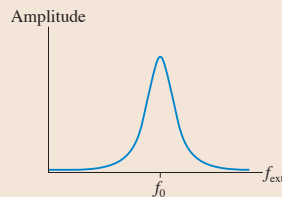
$$x_0 = A \cos \phi_0 \quad v_{0x} = -\omega A \sin \phi_0$$



## Applications

### Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if  $f_{\text{ext}} \approx f_0$ , where  $f_0$  is the system's natural oscillation frequency, or **resonant frequency**.

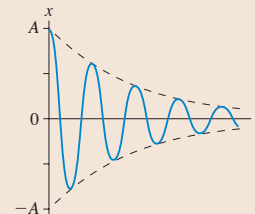


### Damping

If there is a drag force  $\vec{D} = -b\vec{v}$ , where  $b$  is the damping constant, then (for lightly damped systems)

$$x(t) = A e^{-bt/2m} \cos(\omega t + \phi_0)$$

The time constant for energy loss is  $\tau = m/b$ .



## Terms and Notation

oscillatory motion  
oscillator  
period,  $T$   
frequency,  $f$   
hertz, Hz  
simple harmonic motion,  
SHM

amplitude,  $A$   
angular frequency,  $\omega$   
phase,  $\phi$   
phase constant,  $\phi_0$   
restoring force  
equation of motion  
small-angle approximation

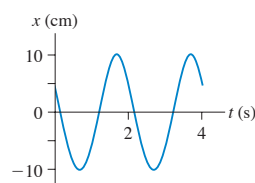
linear restoring force  
damped oscillation  
damping constant,  $b$   
envelope  
time constant,  $\tau$   
half-life,  $t_{1/2}$   
driven oscillation

natural frequency,  $f_0$   
driving frequency,  $f_{\text{ext}}$   
response curve  
resonance  
resonance frequency,  $f_0$

## CONCEPTUAL QUESTIONS

- A block oscillating on a spring has period  $T = 2$  s. What is the period if:
  - The block's mass is doubled? Explain. Note that you do not know the value of either  $m$  or  $k$ , so do *not* assume any particular values for them. The required analysis involves thinking about ratios.
  - The value of the spring constant is quadrupled?
  - The oscillation amplitude is doubled while  $m$  and  $k$  are unchanged?
- A pendulum on Planet X, where the value of  $g$  is unknown, oscillates with a period  $T = 2$  s. What is the period of this pendulum if:
  - Its mass is doubled? Explain. Note that you do not know the value of  $m$ ,  $L$ , or  $g$ , so do not assume any specific values. The required analysis involves thinking about ratios.
  - Its length is doubled?
  - Its oscillation amplitude is doubled?
- FIGURE Q14.3 shows a position-versus-time graph for a particle in SHM. What are (a) the amplitude  $A$ , (b) the angular frequency  $\omega$ , and (c) the phase constant  $\phi_0$ ? Explain.

FIGURE Q14.3



- Equation 14.25 states that  $\frac{1}{2}kA^2 = \frac{1}{2}m(v_{\text{max}})^2$ . What does this mean? Write a couple of sentences explaining how to interpret this equation.
- A block oscillating on a spring has an amplitude of 20 cm. What will the amplitude be if the total energy is doubled? Explain.
- A block oscillating on a spring has a maximum speed of 20 cm/s. What will the block's maximum speed be if the total energy is doubled? Explain.
- FIGURE Q14.7 shows a position-versus-time graph for a particle in SHM.
  - What is the phase constant  $\phi_0$ ? Explain.
  - What is the phase of the particle at each of the three numbered points on the graph?

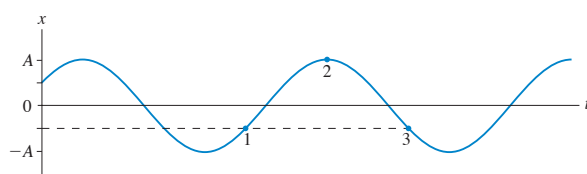


FIGURE Q14.7

- FIGURE Q14.8 shows a velocity-versus-time graph for a particle in SHM.
  - What is the phase constant  $\phi_0$ ? Explain.
  - What is the phase of the particle at each of the three numbered points on the graph?

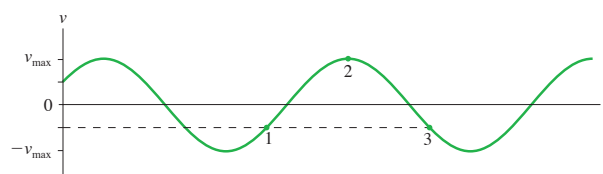


FIGURE Q14.8

- FIGURE Q14.9 shows the potential-energy diagram and the total energy line of a particle oscillating on a spring.
  - What is the spring's equilibrium length?
  - Where are the turning points of the motion? Explain.
  - What is the particle's maximum kinetic energy?
  - What will be the turning points if the particle's total energy is doubled?

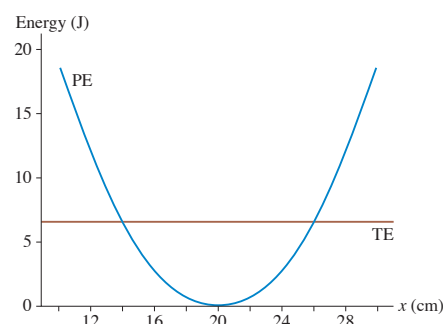



FIGURE Q14.9

- Suppose the damping constant  $b$  of an oscillator increases.
  - Is the medium more resistive or less resistive?
  - Do the oscillations damp out more quickly or less quickly?
  - Is the time constant  $\tau$  increased or decreased?
- Describe the difference between  $\tau$  and  $T$ . Don't just *name* them; say what is different about the physical concepts they represent.
  - Describe the difference between  $\tau$  and  $t_{1/2}$ .
- What is the difference between the driving frequency and the natural frequency of an oscillator?



# EXERCISES AND PROBLEMS

Problems labeled  integrate material from earlier chapters.

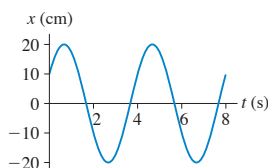
## Exercises

### Section 14.1 Simple Harmonic Motion

1. | When a guitar string plays the note “A,” the string vibrates at 440 Hz. What is the period of the vibration?
2. | An air-track glider attached to a spring oscillates between the 10 cm mark and the 60 cm mark on the track. The glider completes 10 oscillations in 33 s. What are the (a) period, (b) frequency, (c) angular frequency, (d) amplitude, and (e) maximum speed of the glider?
3. || An air-track glider is attached to a spring. The glider is pulled to the right and released from rest at  $t = 0$  s. It then oscillates with a period of 2.0 s and a maximum speed of 40 cm/s.
  - a. What is the amplitude of the oscillation?
  - b. What is the glider’s position at  $t = 0.25$  s?

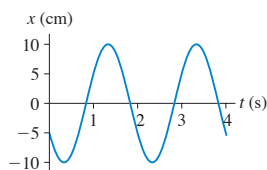
### Section 14.2 Simple Harmonic Motion and Circular Motion

4. | What are the (a) amplitude, (b) frequency, and (c) phase constant of the oscillation shown in **FIGURE EX14.4**?



**FIGURE EX14.4**

5. || What are the (a) amplitude, (b) frequency, and (c) phase constant of the oscillation shown in **FIGURE EX14.5**?



**FIGURE EX14.5**

6. || An object in simple harmonic motion has an amplitude of 4.0 cm, a frequency of 2.0 Hz, and a phase constant of  $2\pi/3$  rad. Draw a position graph showing two cycles of the motion.
7. || An object in simple harmonic motion has an amplitude of 8.0 cm, a frequency of 0.25 Hz, and a phase constant of  $-\pi/2$  rad. Draw a position graph showing two cycles of the motion.
8. | An object in simple harmonic motion has amplitude 4.0 cm and frequency 4.0 Hz, and at  $t = 0$  s it passes through the equilibrium point moving to the right. Write the function  $x(t)$  that describes the object’s position.
9. | An object in simple harmonic motion has amplitude 8.0 cm and frequency 0.50 Hz. At  $t = 0$  s it has its most negative position. Write the function  $x(t)$  that describes the object’s position.
10. || An air-track glider attached to a spring oscillates with a period of 1.5 s. At  $t = 0$  s the glider is 5.00 cm left of the equilibrium position and moving to the right at 36.3 cm/s.
  - a. What is the phase constant?
  - b. What is the phase at  $t = 0$  s, 0.5 s, 1.0 s, and 1.5 s?

### Section 14.3 Energy in Simple Harmonic Motion

### Section 14.4 The Dynamics of Simple Harmonic Motion

11. | A block attached to a spring with unknown spring constant oscillates with a period of 2.0 s. What is the period if
  - a. The mass is doubled?
  - b. The mass is halved?
  - c. The amplitude is doubled?
  - d. The spring constant is doubled?
 Parts a to d are independent questions, each referring to the initial situation.
12. || A 200 g air-track glider is attached to a spring. The glider is pushed in 10 cm and released. A student with a stopwatch finds that 10 oscillations take 12.0 s. What is the spring constant?
13. || A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At  $t = 0$  s, the mass is at  $x = 5.0$  cm and has  $v_x = -30$  cm/s. Determine:
  - a. The period.
  - b. The angular frequency.
  - c. The amplitude.
  - d. The phase constant.
  - e. The maximum speed.
  - f. The maximum acceleration.
  - g. The total energy.
  - h. The position at  $t = 0.40$  s.
14. | The position of a 50 g oscillating mass is given by  $x(t) = (2.0 \text{ cm}) \cos(10t - \pi/4)$ , where  $t$  is in s. Determine:
  - a. The amplitude.
  - b. The period.
  - c. The spring constant.
  - d. The phase constant.
  - e. The initial conditions.
  - f. The maximum speed.
  - g. The total energy.
  - h. The velocity at  $t = 0.40$  s.
15. || A 1.0 kg block is attached to a spring with spring constant 16 N/m. While the block is sitting at rest, a student hits it with a hammer and almost instantaneously gives it a speed of 40 cm/s. What are
  - a. The amplitude of the subsequent oscillations?
  - b. The block’s speed at the point where  $x = \frac{1}{2}A$ ?

### Section 14.5 Vertical Oscillations

16. | A spring is hanging from the ceiling. Attaching a 500 g physics book to the spring causes it to stretch 20 cm in order to come to equilibrium.
  - a. What is the spring constant?
  - b. From equilibrium, the book is pulled down 10 cm and released. What is the period of oscillation?
  - c. What is the book’s maximum speed?
17. || A spring with spring constant 15 N/m hangs from the ceiling. A ball is attached to the spring and allowed to come to rest. It is then pulled down 6.0 cm and released. If the ball makes 30 oscillations in 20 s, what are its (a) mass and (b) maximum speed?
18. || A spring is hung from the ceiling. When a block is attached to its end, it stretches 2.0 cm before reaching its new equilibrium length. The block is then pulled down slightly and released. What is the frequency of oscillation?

### Section 14.6 The Pendulum

19. | A mass on a string of unknown length oscillates as a pendulum with a period of 4.0 s. What is the period if
  - a. The mass is doubled?

- b. The string length is doubled?
- c. The string length is halved?
- d. The amplitude is doubled?

Parts a to d are independent questions, each referring to the initial situation.

- 20. **|** A 200 g ball is tied to a string. It is pulled to an angle of  $8.0^\circ$  and released to swing as a pendulum. A student with a stopwatch finds that 10 oscillations take 12 s. How long is the string?
- 21. **|** What is the period of a 1.0-m-long pendulum on (a) the earth and (b) Venus?
- 22. **|** What is the length of a pendulum whose period on the moon matches the period of a 2.0-m-long pendulum on the earth?
- 23. **|** Astronauts on the first trip to Mars take along a pendulum that has a period on earth of 1.50 s. The period on Mars turns out to be 2.45 s. What is the free-fall acceleration on Mars?
- 24. **|** A uniform steel bar swings from a pivot at one end with a period of 1.2 s. How long is the bar?

### Section 14.7 Damped Oscillations

#### Section 14.8 Driven Oscillations and Resonance

- 25. **|** A 2.0 g spider is dangling at the end of a silk thread. You can make the spider bounce up and down on the thread by tapping lightly on his feet with a pencil. You soon discover that you can give the spider the largest amplitude on his little bungee cord if you tap exactly once every second. What is the spring constant of the silk thread?
- 26. **|** The amplitude of an oscillator decreases to 36.8% of its initial value in 10.0 s. What is the value of the time constant?
- 27. **|** Sketch a position graph from  $t = 0$  s to  $t = 10$  s of a damped oscillator having a frequency of 1.0 Hz and a time constant of 4.0 s.
- 28. **|** In a science museum, a 110 kg brass pendulum bob swings at the end of a 15.0-m-long wire. The pendulum is started at exactly 8:00 A.M. every morning by pulling it 1.5 m to the side and releasing it. Because of its compact shape and smooth surface, the pendulum's damping constant is only 0.010 kg/s. At exactly 12:00 noon, how many oscillations will the pendulum have completed and what is its amplitude?
- 29. **|** Vision is blurred if the head is vibrated at 29 Hz because the vibrations are resonant with the natural frequency of the eyeball in its socket. If the mass of the eyeball is 7.5 g, a typical value, what is the effective spring constant of the musculature that holds the eyeball in the socket?

### Problems

- 30. **|** **FIGURE P14.30** is the velocity-versus-time graph of a particle in simple harmonic motion.
  - a. What is the amplitude of the oscillation?
  - b. What is the phase constant?
  - c. What is the position at  $t = 0$  s?

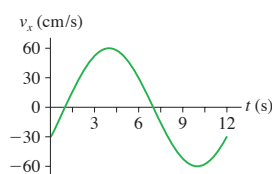


FIGURE P14.30

- 31. **|** **FIGURE P14.31** is the position-versus-time graph of a particle in simple harmonic motion.
  - a. What is the phase constant?
  - b. What is the velocity at  $t = 0$  s?
  - c. What is  $v_{\max}$ ?

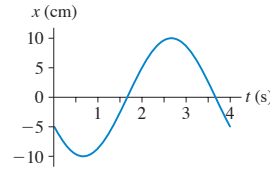


FIGURE P14.31

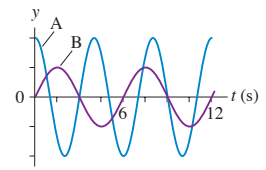


FIGURE P14.32

- 32. **|** The two graphs in **FIGURE P14.32** are for two different vertical mass-spring systems. If both systems have the same mass, what is the ratio  $k_A/k_B$  of their spring constants?
- 33. **|** An object in SHM oscillates with a period of 4.0 s and an amplitude of 10 cm. How long does the object take to move from  $x = 0.0$  cm to  $x = 6.0$  cm?
- 34. **|** A 1.0 kg block oscillates on a spring with spring constant 20 N/m. At  $t = 0$  s the block is 20 cm to the right of the equilibrium position and moving to the left at a speed of 100 cm/s. Determine (a) the period and (b) the amplitude.
- 35. **|** Astronauts in space cannot weigh themselves by standing on a bathroom scale. Instead, they determine their mass by oscillating on a large spring. Suppose an astronaut attaches one end of a large spring to her belt and the other end to a hook on the wall of the space capsule. A fellow astronaut then pulls her away from the wall and releases her. The spring's length as a function of time is shown in **FIGURE P14.35**.
  - a. What is her mass if the spring constant is 240 N/m?
  - b. What is her speed when the spring's length is 1.2 m?

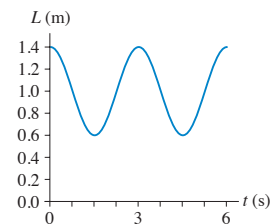


FIGURE P14.35

- 36. **|** The motion of a particle is given by  $x(t) = (25 \text{ cm})\cos(10t)$ , where  $t$  is in s. At what time is the kinetic energy twice the potential energy?
- 37. **|** a. When the displacement of a mass on a spring is  $\frac{1}{2}A$ , what fraction of the energy is kinetic energy and what fraction is potential energy?  
b. At what displacement, as a fraction of  $A$ , is the energy half kinetic and half potential?
- 38. **|** For a particle in simple harmonic motion, show that  $v_{\max} = (\pi/2)v_{\text{avg}}$  where  $v_{\text{avg}}$  is the average speed during one cycle of the motion.
- 39. **|** A 100 g block attached to a spring with spring constant 2.5 N/m oscillates horizontally on a frictionless table. Its velocity is 20 cm/s when  $x = -5.0$  cm.
  - a. What is the amplitude of oscillation?
  - b. What is the block's maximum acceleration?
  - c. What is the block's position when the acceleration is maximum?
  - d. What is the speed of the block when  $x = 3.0$  cm?

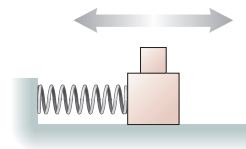
40. || A block on a spring is pulled to the right and released at  $t = 0$  s. It passes  $x = 3.00$  cm at  $t = 0.685$  s, and it passes  $x = -3.00$  cm at  $t = 0.886$  s.
- What is the angular frequency?
  - What is the amplitude?
- Hint:**  $\cos(\pi - \theta) = -\cos \theta$ .
41. || A 300 g oscillator has a speed of 95.4 cm/s when its displacement is 3.0 cm and 71.4 cm/s when its displacement is 6.0 cm. What is the oscillator's maximum speed?
42. || BIO An ultrasonic transducer, of the type used in medical ultrasound imaging, is a very thin disk ( $m = 0.10$  g) driven back and forth in SHM at 1.0 MHz by an electromagnetic coil.
- The maximum restoring force that can be applied to the disk without breaking it is 40,000 N. What is the maximum oscillation amplitude that won't rupture the disk?
  - What is the disk's maximum speed at this amplitude?
43. || A 5.0 kg block hangs from a spring with spring constant 2000 N/m. The block is pulled down 5.0 cm from the equilibrium position and given an initial velocity of 1.0 m/s back toward equilibrium. What are the (a) frequency, (b) amplitude, and (c) total mechanical energy of the motion?
44. || Your lab instructor has asked you to measure a spring constant using a dynamic method—letting it oscillate—rather than a static method of stretching it. You and your lab partner suspend the spring from a hook, hang different masses on the lower end, and start them oscillating. One of you uses a meter stick to measure the amplitude, the other uses a stopwatch to time 10 oscillations. Your data are as follows:

Mass (g)	Amplitude (cm)	Time (s)
100	6.5	7.8
150	5.5	9.8
200	6.0	10.9
250	3.5	12.4

Use the best-fit line of an appropriate graph to determine the spring constant.

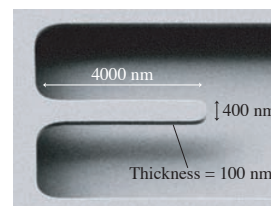
45. || A 200 g block hangs from a spring with spring constant 10 N/m. At  $t = 0$  s the block is 20 cm below the equilibrium point and moving upward with a speed of 100 cm/s. What are the block's
- Oscillation frequency?
  - Distance from equilibrium when the speed is 50 cm/s?
  - Distance from equilibrium at  $t = 1.0$  s?
46. || A spring with spring constant  $k$  is suspended vertically from a support and a mass  $m$  is attached. The mass is held at the point where the spring is not stretched. Then the mass is released and begins to oscillate. The lowest point in the oscillation is 20 cm below the point where the mass was released. What is the oscillation frequency?
47. || While grocery shopping, you put several apples in the spring scale in the produce department. The scale reads 20 N, and you use your ruler (which you always carry with you) to discover that the pan goes down 9.0 cm when the apples are added. If you tap the bottom of the apple-filled pan to make it bounce up and down a little, what is its oscillation frequency? Ignore the mass of the pan.
48. || A compact car has a mass of 1200 kg. Assume that the car has one spring on each wheel, that the springs are identical, and that the mass is equally distributed over the four springs.

- What is the spring constant of each spring if the empty car bounces up and down 2.0 times each second?
  - What will be the car's oscillation frequency while carrying four 70 kg passengers?
49. || The two blocks in **FIGURE P14.49** oscillate on a frictionless surface with a period of 1.5 s. The upper block just begins to slip when the amplitude is increased to 40 cm. What is the coefficient of static friction between the two blocks?



**FIGURE P14.49**

50. || BIO It has recently become possible to “weigh” DNA molecules by measuring the influence of their mass on a nano-oscillator. **FIGURE P14.50** shows a thin rectangular cantilever etched out of silicon (density  $2300 \text{ kg/m}^3$ ) with a small gold dot at the end. If pulled down and released, the end of the cantilever vibrates with simple harmonic motion, moving up and down like a diving board after a jump. When bathed with DNA molecules whose ends have been modified to bind with gold, one or more molecules may attach to the gold dot. The addition of their mass causes a very slight—but measurable—decrease in the oscillation frequency.



**FIGURE P14.50**

- A vibrating cantilever of mass  $M$  can be modeled as a block of mass  $\frac{1}{3}M$  attached to a spring. (The factor of  $\frac{1}{3}$  arises from the moment of inertia of a bar pivoted at one end.) Neither the mass nor the spring constant can be determined very accurately—perhaps to only two significant figures—but the oscillation frequency can be measured with very high precision simply by counting the oscillations. In one experiment, the cantilever was initially vibrating at exactly 12 MHz. Attachment of a DNA molecule caused the frequency to decrease by 50 Hz. What was the mass of the DNA?
51. || It is said that Galileo discovered a basic principle of the pendulum—that the period is independent of the amplitude—by using his pulse to time the period of swinging lamps in the cathedral as they swayed in the breeze. Suppose that one oscillation of a swinging lamp takes 5.5 s.
- How long is the lamp chain?
  - What maximum speed does the lamp have if its maximum angle from vertical is  $3.0^\circ$ ?
52. || A 100 g mass on a 1.0-m-long string is pulled  $8.0^\circ$  to one side and released. How long does it take for the pendulum to reach  $4.0^\circ$  on the opposite side?
53. || BIO Orangutans can move by brachiation, swinging like a pendulum beneath successive handholds. If an orangutan has arms that are 0.90 m long and repeatedly swings to a  $20^\circ$  angle, taking one swing after another, estimate its speed of forward motion in m/s. While this is somewhat beyond the range of validity of the small-angle approximation, the standard results for a pendulum are adequate for making an estimate.

54. | Show that Equation 14.51 for the angular frequency of a physical pendulum gives Equation 14.48 when applied to a simple pendulum of a mass on a string.
55. || A 15-cm-long, 200 g rod is pivoted at one end. A 20 g ball of clay is stuck on the other end. What is the period if the rod and clay swing as a pendulum?
56. || A uniform rod of mass  $M$  and length  $L$  swings as a pendulum on a pivot at distance  $L/4$  from one end of the rod. Find an expression for the frequency  $f$  of small-angle oscillations.
57. || A solid sphere of mass  $M$  and radius  $R$  is suspended from a thin rod, as shown in **FIGURE P14.57**. The sphere can swing back and forth at the bottom of the rod. Find an expression for the frequency  $f$  of small-angle oscillations.

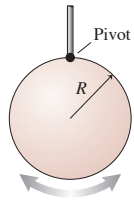


FIGURE P14.57

58. || A geologist needs to determine the local value of  $g$ . Unfortunately, his only tools are a meter stick, a saw, and a stopwatch. He starts by hanging the meter stick from one end and measuring its frequency as it swings. He then saws off 20 cm—using the centimeter markings—and measures the frequency again. After two more cuts, these are his data:

Length (cm)	Frequency (Hz)
100	0.61
80	0.67
60	0.79
40	0.96

Use the best-fit line of an appropriate graph to determine the local value of  $g$ .

59. || Interestingly, there have been several studies using cadavers to determine the moments of inertia of human body parts, information that is important in biomechanics. In one study, the center of mass of a 5.0 kg lower leg was found to be 18 cm from the knee. When the leg was allowed to pivot at the knee and swing freely as a pendulum, the oscillation frequency was 1.6 Hz. What was the moment of inertia of the lower leg about the knee joint?
60. || A 500 g air-track glider attached to a spring with spring constant 10 N/m is sitting at rest on a frictionless air track. A 250 g glider is pushed toward it from the far end of the track at a speed of 120 cm/s. It collides with and sticks to the 500 g glider. What are the amplitude and period of the subsequent oscillations?
61. || A 200 g block attached to a horizontal spring is oscillating with an amplitude of 2.0 cm and a frequency of 2.0 Hz. Just as it passes through the equilibrium point, moving to the right, a sharp blow directed to the left exerts a 20 N force for 1.0 ms. What are the new (a) frequency and (b) amplitude?
62. || **FIGURE P14.62** is a top view of an object of mass  $m$  connected between two stretched rubber bands of length  $L$ . The object rests on a frictionless surface. At equilibrium, the tension in each rubber band is  $T$ . Find an expression for the frequency of oscillations *perpendicular* to the rubber bands. Assume the amplitude is sufficiently small that the magnitude of the tension in the rubber bands is essentially unchanged as the mass oscillates.
63. || A molecular bond can be modeled as a spring between two atoms that vibrate with simple harmonic motion. **FIGURE P14.63** shows an SHM approximation for the potential energy of an HCl molecule. For  $E < 4 \times 10^{-19}$  J it is a good approximation to the more accurate HCl potential-energy curve that was shown in Figure 10.31. Because the chlorine atom is so much more massive than the hydrogen atom, it is reasonable to assume that the hydrogen atom ( $m = 1.67 \times 10^{-27}$  kg) vibrates back and forth while the chlorine atom remains at rest. Use the graph to estimate the vibrational frequency of the HCl molecule.
64. || An ice cube can slide around the inside of a vertical circular hoop of radius  $R$ . It undergoes small-amplitude oscillations if displaced slightly from the equilibrium position at the lowest point. Find an expression for the period of these small-amplitude oscillations.
65. || A penny rides on top of a piston as it undergoes vertical simple harmonic motion with an amplitude of 4.0 cm. If the frequency is low, the penny rides up and down without difficulty. If the frequency is steadily increased, there comes a point at which the penny leaves the surface.
- At what point in the cycle does the penny first lose contact with the piston?
  - What is the maximum frequency for which the penny just barely remains in place for the full cycle?
66. || On your first trip to Planet X you happen to take along a 200 g mass, a 40-cm-long spring, a meter stick, and a stopwatch. You're curious about the free-fall acceleration on Planet X, where ordinary tasks seem easier than on earth, but you can't find this information in your Visitor's Guide. One night you suspend the spring from the ceiling in your room and hang the mass from it. You find that the mass stretches the spring by 31.2 cm. You then pull the mass down 10.0 cm and release it. With the stopwatch you find that 10 oscillations take 14.5 s. Based on this information, what is  $g$ ?
67. || The 15 g head of a bobble-head doll oscillates in SHM at a frequency of 4.0 Hz.
- What is the spring constant of the spring on which the head is mounted?
  - The amplitude of the head's oscillations decreases to 0.5 cm in 4.0 s. What is the head's damping constant?
68. || An oscillator with a mass of 500 g and a period of 0.50 s has an amplitude that decreases by 2.0% during each complete oscillation. If the initial amplitude is 10 cm, what will be the amplitude after 25 oscillations?
69. || A spring with spring constant 15.0 N/m hangs from the ceiling. A 500 g ball is attached to the spring and allowed to come to rest. It is then pulled down 6.0 cm and released. What is the time constant if the ball's amplitude has decreased to 3.0 cm after 30 oscillations?

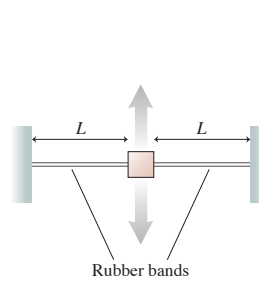


FIGURE P14.62

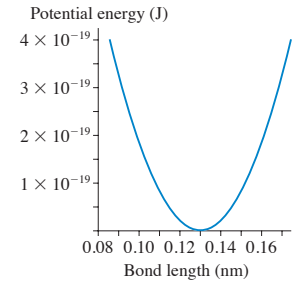


FIGURE P14.63

70. || A 250 g air-track glider is attached to a spring with spring constant 4.0 N/m. The damping constant due to air resistance is 0.015 kg/s. The glider is pulled out 20 cm from equilibrium and released. How many oscillations will it make during the time in which the amplitude decays to  $e^{-1}$  of its initial value?
71. || A 200 g oscillator in a vacuum chamber has a frequency of 2.0 Hz. When air is admitted, the oscillation decreases to 60% of its initial amplitude in 50 s. How many oscillations will have been completed when the amplitude is 30% of its initial value?
72. || Prove that the expression for  $x(t)$  in Equation 14.55 is a solution to the equation of motion for a damped oscillator, Equation 14.54, if and only if the angular frequency  $\omega$  is given by the expression in Equation 14.56.
73. || A block on a frictionless table is connected as shown in **FIGURE P14.73** to two springs having spring constants  $k_1$  and  $k_2$ . Show that the block's oscillation frequency is given by

$$f = \sqrt{f_1^2 + f_2^2}$$

where  $f_1$  and  $f_2$  are the frequencies at which it would oscillate if attached to spring 1 or spring 2 alone.

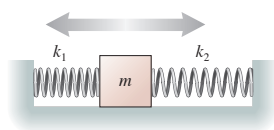


FIGURE P14.73

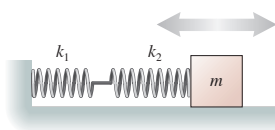


FIGURE P14.74

74. || A block on a frictionless table is connected as shown in **FIGURE P14.74** to two springs having spring constants  $k_1$  and  $k_2$ . Find an expression for the block's oscillation frequency  $f$  in terms of the frequencies  $f_1$  and  $f_2$  at which it would oscillate if attached to spring 1 or spring 2 alone.

### Challenge Problems

75. A block hangs in equilibrium from a vertical spring. When a second identical block is added, the original block sags by 5.0 cm. What is the oscillation frequency of the two-block system?

76. A 1.00 kg block is attached to a horizontal spring with spring constant 2500 N/m. The block is at rest on a frictionless surface. A 10 g bullet is fired into the block, in the face opposite the spring, and sticks. What was the bullet's speed if the subsequent oscillations have an amplitude of 10.0 cm?
77. A spring is standing upright on a table with its bottom end fastened to the table. A block is dropped from a height 3.0 cm above the top of the spring. The block sticks to the top end of the spring and then oscillates with an amplitude of 10 cm. What is the oscillation frequency?
78. The analysis of a simple pendulum assumed that the mass was a particle, with no size. A realistic pendulum is a small, uniform sphere of mass  $M$  and radius  $R$  at the end of a massless string, with  $L$  being the distance from the pivot to the center of the sphere.
- Find an expression for the period of this pendulum.
  - Suppose  $M = 25$  g,  $R = 1.0$  cm, and  $L = 1.0$  m, typical values for a real pendulum. What is the ratio  $T_{\text{real}}/T_{\text{simple}}$ , where  $T_{\text{real}}$  is your expression from part a and  $T_{\text{simple}}$  is the expression derived in this chapter?
79. a. A mass  $m$  oscillating on a spring has period  $T$ . Suppose the mass changes very slightly from  $m$  to  $m + \Delta m$ , where  $\Delta m \ll m$ . Find an expression for  $\Delta T$ , the small change in the period. Your expression should involve  $T$ ,  $m$ , and  $\Delta m$  but not the spring constant.
- b. Suppose the period is 2.000 s and the mass increases by 0.1%. What is the new period?
80. **FIGURE CP14.80** shows a 200 g uniform rod pivoted at one end. The other end is attached to a horizontal spring. The spring is neither stretched nor compressed when the rod hangs straight down. What is the rod's oscillation period? You can assume that the rod's angle from vertical is always small.

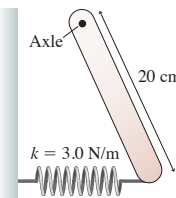


FIGURE CP14.80

### STOP TO THINK ANSWERS

**Stop to Think 14.1:** c.  $v_{\text{max}} = 2\pi A/T$ . Doubling  $A$  and  $T$  leaves  $v_{\text{max}}$  unchanged.

**Stop to Think 14.2:** d. Think of circular motion. At  $45^\circ$ , the particle is in the first quadrant (positive  $x$ ) and moving to the left (negative  $v_x$ ).

**Stop to Think 14.3:** c > b > a = d. Energy conservation  $\frac{1}{2}kA^2 = \frac{1}{2}m(v_{\text{max}})^2$  gives  $v_{\text{max}} = \sqrt{k/m}A$ .  $k$  or  $m$  has to be increased or decreased by a factor of 4 to have the same effect as increasing or decreasing  $A$  by a factor of 2.

**Stop to Think 14.4:** c.  $v_x = 0$  because the slope of the position graph is zero. The negative value of  $x$  shows that the particle is left of the equilibrium position, so the restoring force is to the right.

**Stop to Think 14.5:** c. The period of a pendulum does not depend on its mass.

**Stop to Think 14.6:**  $\tau_d > \tau_b = \tau_c > \tau_a$ . The time constant is the time to decay to 37% of the initial height. The time constant is independent of the initial height.

