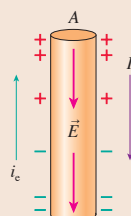


SUMMARY

The goal of Chapter 30 has been to learn how and why charge moves through a conductor as what we call a current.

General Principles

Current is a nonequilibrium motion of charges sustained by an electric field. Nonuniform surface charge density creates an electric field in a wire. The electric field pushes the electron current i_e in a direction opposite to \vec{E} . The conventional current I is in the direction in which positive charge *seems* to move.

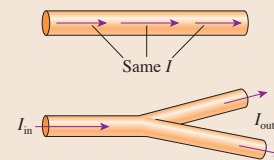


Conservation of Current

The current is the same at any two points in a wire. At a junction,

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

This is **Kirchhoff's junction law**.



Electron current

$$i_e = \text{rate of electron flow}$$

$$N_e = i_e \Delta t$$

Conventional current

$$I = \text{rate of charge flow} = e i_e$$

$$Q = I \Delta t$$

Current density

$$J = I/A$$

Important Concepts

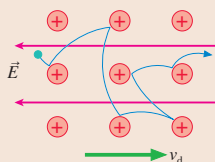
Sea of electrons

Conduction electrons move freely around the positive ions that form the atomic lattice.

Conduction

An electric field causes a slow drift at speed v_d to be superimposed on the rapid but random thermal motions of the electrons.

Collisions of electrons with the ions transfer energy to the atoms. This makes the wire warm and lightbulbs glow. More collisions mean a higher resistivity ρ and a lower conductivity σ .



The **drift speed** is $v_d = \frac{e\tau}{m}E$, where τ is the mean time between collisions.

The electron current is related to the drift speed by

$$i_e = n_e A v_d$$

where n_e is the electron density.

An electric field E in a conductor causes a current density $J = n_e e v_d = \sigma E$, where the **conductivity** is

$$\sigma = \frac{n_e e^2 \tau}{m}$$

The **resistivity** is $\rho = 1/\sigma$.

Applications

Resistors

A potential difference ΔV_{wire} between the ends of a wire creates an electric field inside the wire:

$$E_{\text{wire}} = \frac{\Delta V_{\text{wire}}}{L}$$

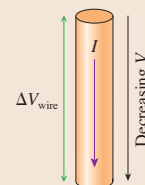
The electric field causes a current in the direction of decreasing potential.

The size of the current is

$$I = \frac{\Delta V_{\text{wire}}}{R}$$

where $R = \frac{\rho L}{A}$ is the wire's **resistance**.

This is **Ohm's law**.



Terms and Notation

current, I	ampere, A	conductivity, σ	Ohm's law
drift speed, v_d	current density, J	resistivity, ρ	resistor
electron current, i_e	law of conservation of current	superconductivity	ideal wire
mean time between collisions, τ	junction	resistance, R	ideal insulator
	Kirchhoff's junction law	ohm, Ω	

CONCEPTUAL QUESTIONS

1. Suppose a time machine has just brought you forward from 1750 (post-Newton but pre-electricity) and you've been shown the lightbulb demonstration of **FIGURE Q30.1**. Do observations or *simple* measurements you might make—measurements that must make sense to you with your 1700s knowledge—prove that something is *flowing* through the wires? Or might you advance an alternative hypothesis for why the bulb is glowing? If your answer to the first question is yes, state what observations and/or measurements are relevant and the reasoning from which you can infer that something must be flowing. If not, can you offer an alternative hypothesis about why the bulb glows that could be tested?

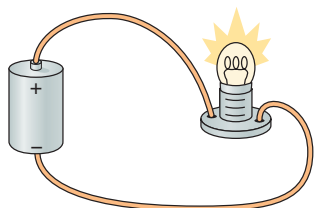


FIGURE Q30.1

2. Consider a lightbulb circuit such as the one in **FIGURE Q30.1**.
- From the simple observations and measurements you can make on this circuit, can you distinguish a current composed of positive charge carriers from a current composed of negative charge carriers? If so, describe how you can tell which it is. If not, why not?
 - One model of current is the motion of discrete charged particles. Another model is that current is the flow of a continuous charged fluid. Do simple observations and measurements on this circuit provide evidence in favor of either one of these models? If so, describe how.
3. The electron drift speed in a wire is exceedingly slow—typically only a fraction of a millimeter per second. Yet when you turn on a flashlight switch, the light comes on almost instantly. Resolve this apparent paradox.
4. Is **FIGURE Q30.4** a possible surface charge distribution for a current-carrying wire? If so, in which direction is the current? If not, why not?



FIGURE Q30.4

5. What is the difference between current and current density?
6. All the wires in **FIGURE Q30.6** are made of the same material and have the same diameter. Rank in order, from largest to smallest, the currents I_a to I_d . Explain.

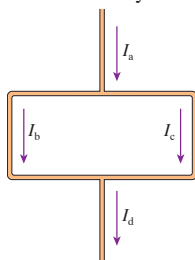


FIGURE Q30.6

7. Both batteries in **FIGURE Q30.7** are identical and all lightbulbs are the same. Rank in order, from brightest to least bright, the brightness of bulbs a to c. Explain.

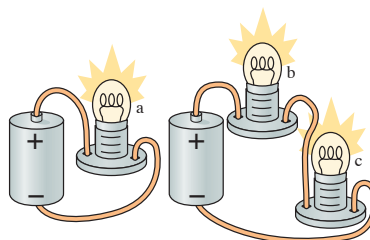


FIGURE Q30.7

8. Both batteries in **FIGURE Q30.8** are identical and all lightbulbs are the same. Rank in order, from brightest to least bright, the brightness of bulbs a to c. Explain.

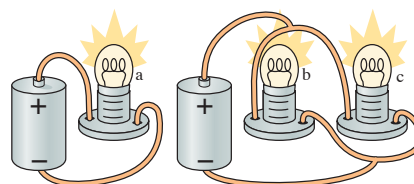


FIGURE Q30.8

9. The wire in **FIGURE Q30.9** consists of two segments of different diameters but made from the same metal. The current in segment 1 is I_1 .
- Compare the currents in the two segments. That is, is I_2 greater than, less than, or equal to I_1 ? Explain.
 - Compare the current densities J_1 and J_2 in the two segments.
 - Compare the electric field strengths E_1 and E_2 in the two segments.
 - Compare the drift speeds $(v_d)_1$ and $(v_d)_2$ in the two segments.
10. The current in a wire is doubled. What happens to (a) the current density, (b) the conduction-electron density, (c) the mean time between collisions, and (d) the electron drift speed? Are each of these doubled, halved, or unchanged? Explain.
11. The wires in **FIGURE Q30.11** are all made of the same material. Rank in order, from largest to smallest, the resistances R_a to R_e of these wires. Explain.

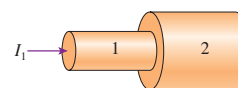


FIGURE Q30.9

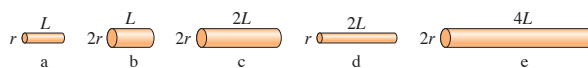


FIGURE Q30.11





12. Which, if any, of these statements are true? (More than one may be true.) Explain.
- A battery supplies the energy to a circuit.
 - A battery is a source of potential difference; the potential difference between the terminals of the battery is always the same.
 - A battery is a source of current; the current leaving the battery is always the same.

EXERCISES AND PROBLEMS





Problems labeled  integrate material from earlier chapters.

Exercises







Section 30.1 The Electron Current




1.  The electron drift speed in a 1.0-mm-diameter gold wire is 5.0×10^{-5} m/s. How long does it take 1 mole of electrons to flow through a cross section of the wire?
2.  1.0×10^{20} electrons flow through a cross section of a 2.0-mm-diameter iron wire in 5.0 s. What is the electron drift speed?
3.  Electrons flow through a 1.6-mm-diameter aluminum wire at 2.0×10^{-4} m/s. How many electrons move through a cross section of the wire each day?
4.  1.0×10^{16} electrons flow through a cross section of silver wire in 320 μ s with a drift speed of 8.0×10^{-4} m/s. What is the diameter of the wire?

Section 30.2 Creating a Current








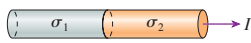

5.  The electron drift speed is 2.0×10^{-4} m/s in a metal with a mean time between collisions of 5.0×10^{-14} s. What is the electric field strength?
6.  a. How many conduction electrons are there in a 1.0-mm-diameter gold wire that is 10 cm long?
b. How far must the sea of electrons in the wire move to deliver -32 nC of charge to an electrode?
7.  The mean time between collisions in iron is 4.2×10^{-15} s. What electron current is driven through a 1.8-mm-diameter iron wire by a 0.065 V/m electric field?
8.  A 2.0×10^{-3} V/m electric field creates a 3.5×10^{17} electrons/s current in a 1.0-mm-diameter aluminum wire. What are (a) the drift speed and (b) the mean time between collisions for electrons in this wire?

Section 30.3 Current and Current Density





9.  The wires leading to and from a 0.12-mm-diameter lightbulb filament are 1.5 mm in diameter. The wire to the filament carries a current with a current density of 4.5×10^5 A/m². What are (a) the current and (b) the current density in the filament?
10.  The current in a 100 watt lightbulb is 0.85 A. The filament inside the bulb is 0.25 mm in diameter.
a. What is the current density in the filament?
b. What is the electron current in the filament?
11.  In an integrated circuit, the current density in a $2.5\text{-}\mu\text{m}$ -thick \times $75\text{-}\mu\text{m}$ -wide gold film is 7.5×10^5 A/m². How much charge flows through the film in 15 min?
12.  When a nerve cell fires, charge is transferred across the cell membrane to change the cell's potential from negative to positive. For a typical nerve cell, 9.0 pC of charge flows in a time of 0.50 ms. What is the average current through the cell membrane?
13.  The current in an electric hair dryer is 10.0 A. How many electrons flow through the hair dryer in 5.0 min?
14.  2.0×10^{13} electrons flow through a transistor in 1.0 ms. What is the current through the transistor?

15.  In an ionic solution, 5.0×10^{15} positive ions with charge $+2e$ pass to the right each second while 6.0×10^{15} negative ions with charge $-e$ pass to the left. What is the current in the solution?
16.  A hollow copper wire with an inner diameter of 1.0 mm and an outer diameter of 2.0 mm carries a current of 10 A. What is the current density in the wire?
17.  The current in a $2.0\text{ mm} \times 2.0\text{ mm}$ square aluminum wire is 2.5 A. What are (a) the current density and (b) the electron drift speed?

Section 30.4 Conductivity and Resistivity

18.  What is the mean time between collisions for electrons in an aluminum wire and in an iron wire?
19.  The electric field in a $2.0\text{ mm} \times 2.0\text{ mm}$ square aluminum wire is 0.012 V/m. What is the current in the wire?
20.  A 15-cm-long nichrome wire is connected across the terminals of a 1.5 V battery.
a. What is the electric field inside the wire?
b. What is the current density inside the wire?
c. If the current in the wire is 2.0 A, what is the wire's diameter?
21.  A 3.0-mm-diameter wire carries a 12 A current when the electric field is 0.085 V/m. What is the wire's resistivity?
22.  A 0.0075 V/m electric field creates a 3.9 mA current in a 1.0-mm-diameter wire. What material is the wire made of?
23.  A 0.50-mm-diameter silver wire carries a 20 mA current. What are (a) the electric field and (b) the electron drift speed in the wire?
24.  The two segments of the wire in  **FIGURE EX30.24** have equal diameters but different conductivities σ_1 and σ_2 . Current I passes through this wire. If the conductivities have the ratio $\sigma_2/\sigma_1 = 2$, what is the ratio E_2/E_1 of the electric field strengths in the two segments of the wire?
25.  A metal cube 1.0 cm on each side is sandwiched between two electrodes. The electrodes create a 0.0050 V/m electric field in the metal. A current of 9.0 A passes through the cube, from the positive electrode to the negative electrode. Identify the metal.

Section 30.5 Resistance and Ohm's Law

26.  A 1.5 V battery provides 0.50 A of current.
a. At what rate (C/s) is charge lifted by the charge escalator?
b. How much work does the charge escalator do to lift 1.0 C of charge?
c. What is the power output of the charge escalator?
27.  Wires 1 and 2 are made of the same metal. Wire 2 has twice the length and twice the diameter of wire 1. What are the ratios (a) ρ_2/ρ_1 of the resistivities and (b) R_2/R_1 of the resistances of the two wires?
28.  What is the resistance of
a. A 2.0-m-long gold wire that is 0.20 mm in diameter?
b. A 10-cm-long piece of carbon with a $1.0\text{ mm} \times 1.0\text{ mm}$ square cross section?
29.  A 10-m-long wire with a diameter of 0.80 mm has a resistance of 1.1 Ω . Of what material is the wire made?

30. | The electric field inside a 30-cm-long copper wire is 5.0 mV/m. What is the potential difference between the ends of the wire?
31. | a. How long must a 0.60-mm-diameter aluminum wire be to have a 0.50 A current when connected to the terminals of a 1.5 V flashlight battery?
b. What is the current if the wire is half this length?
32. | The terminals of a 0.70 V watch battery are connected by a 100-m-long gold wire with a diameter of 0.10 mm. What is the current in the wire?
33. | BIO The femoral artery is the large artery that carries blood to the leg. What is the resistance of a 20-cm-long column of blood in a 1.0-cm-diameter femoral artery? The conductivity of blood is $0.63 \Omega^{-1} \text{ m}^{-1}$.
34. | Pencil “lead” is actually carbon. What is the current if a 9.0 V potential difference is applied between the ends of a 0.70-mm-diameter, 6.0-cm-long lead from a mechanical pencil?
35. | The resistance of a very fine aluminum wire with a $10 \mu\text{m} \times 10 \mu\text{m}$ square cross section is 1000Ω . A 1000Ω resistor is made by wrapping this wire in a spiral around a 3.0-mm-diameter glass core. How many turns of wire are needed?

36. | FIGURE EX30.36 is a current-versus-potential-difference graph for a material. What is the material's resistance?

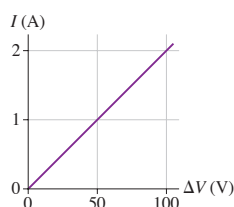


FIGURE EX30.36

37. | A circuit calls for a 0.50-mm-diameter copper wire to be stretched between two points. You don't have any copper wire, but you do have aluminum wire in a wide variety of diameters. What diameter aluminum wire will provide the same resistance?

Problems

38. | For what electric field strength would the current in a 2.0-mm-diameter nichrome wire be the same as the current in a 1.0-mm-diameter aluminum wire in which the electric field strength is 0.0080 V/m?
39. | You've been asked to determine whether a new material your company has made is ohmic and, if so, to measure its electrical conductivity. Taking a $0.50 \text{ mm} \times 1.0 \text{ mm} \times 45 \text{ mm}$ sample, you wire the ends of the long axis to a power supply and then measure the current for several different potential differences. Your data are as follows:

Voltage (V)	Current (A)
0.200	0.47
0.400	1.06
0.600	1.53
0.800	1.97

Use an appropriate graph of the data to determine whether the material is ohmic and, if so, its conductivity.

40. | The electron beam inside a television picture tube is 0.40 mm in diameter and carries a current of $50 \mu\text{A}$. This electron beam impinges on the inside of the picture tube screen.
a. How many electrons strike the screen each second?
b. What is the current density in the electron beam?

- c. The electrons move with a velocity of $4.0 \times 10^7 \text{ m/s}$. What electric field strength is needed to accelerate electrons from rest to this velocity in a distance of 5.0 mm?
- d. Each electron transfers its kinetic energy to the picture tube screen upon impact. What is the *power* delivered to the screen by the electron beam?

41. | FIGURE P30.41 shows a 4.0-cm-wide plastic film being wrapped onto a 2.0-cm-diameter roller that turns at 90 rpm. The plastic has a uniform surface charge density of -2.0 nC/cm^2 .

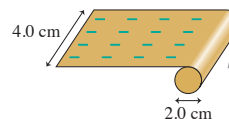


FIGURE P30.41

- a. What is the current of the moving film?
b. How long does it take the roller to accumulate a charge of $-10 \mu\text{C}$?
42. | A sculptor has asked you to help electroplate gold onto a brass statue. You know that the charge carriers in the ionic solution are gold ions, and you've calculated that you must deposit 0.50 g of gold to reach the necessary thickness. How much current do you need, in mA, to plate the statue in 3.0 hours?
43. | In a classic model of the hydrogen atom, the electron moves around the proton in a circular orbit of radius 0.053 nm.
a. What is the electron's orbital frequency?
b. What is the effective current of the electron?
44. | BIO The biochemistry that takes place inside cells depends on various elements, such as sodium, potassium, and calcium, that are dissolved in water as ions. These ions enter cells through narrow pores in the cell membrane known as *ion channels*. Each ion channel, which is formed from a specialized protein molecule, is selective for one type of ion. Measurements with microelectrodes have shown that a 0.30-nm-diameter potassium ion (K^+) channel carries a current of 1.8 pA.
a. How many potassium ions pass through if the ion channel opens for 1.0 ms?
b. What is the current density in the ion channel?
45. | The starter motor of a car engine draws a current of 150 A from the battery. The copper wire to the motor is 5.0 mm in diameter and 1.2 m long. The starter motor runs for 0.80 s until the car engine starts.
a. How much charge passes through the starter motor?
b. How far does an electron travel along the wire while the starter motor is on?
46. | A car battery is rated at 90 Ah, meaning that it can supply a 90 A current for 1 h before being completely discharged. If you leave your headlights on until the battery is completely dead, how much charge leaves the battery?
47. | BIO Variations in the resistivity of blood can give valuable clues about changes in various properties of the blood. Suppose a medical device attaches two electrodes into a 1.5-mm-diameter vein at positions 5.0 cm apart. What is the blood resistivity if a 9.0 V potential difference causes a $230 \mu\text{A}$ current through the blood in the vein?
48. | BIO The conducting path between the right hand and the left hand can be modeled as a 10-cm-diameter, 160-cm-long cylinder. The average resistivity of the interior of the human body is $5.0 \Omega \cdot \text{m}$. Dry skin has a much higher resistivity, but skin resistance can be made negligible by soaking the hands in salt water. If skin resistance is neglected, what potential difference between the hands is needed for a lethal shock of 100 mA across the chest? Your result shows that even small potential differences can produce dangerous currents when the skin is wet.

49. || You need to design a 1.0 A fuse that “blows” if the current exceeds 1.0 A. The fuse material in your stockroom melts at a current density of 500 A/cm². What diameter wire of this material will do the job?
50. || A hollow metal cylinder has inner radius a , outer radius b , length L , and conductivity σ . The current I is *radially* outward from the inner surface to the outer surface.
- Find an expression for the electric field strength inside the metal as a function of the radius r from the cylinder’s axis.
 - Evaluate the electric field strength at the inner and outer surfaces of an iron cylinder if $a = 1.0$ cm, $b = 2.5$ cm, $L = 10$ cm, and $I = 25$ A.
51. || A hollow metal sphere has inner radius a , outer radius b , and conductivity σ . The current I is *radially* outward from the inner surface to the outer surface.
- Find an expression for the electric field strength inside the metal as a function of the radius r from the center.
 - Evaluate the electric field strength at the inner and outer surfaces of a copper sphere if $a = 1.0$ cm, $b = 2.5$ cm, and $I = 25$ A.
52. || The total amount of charge in coulombs that has entered a wire at time t is given by the expression $Q = 4t - t^2$, where t is in seconds and $t \geq 0$.
- Find an expression for the current in the wire at time t .
 - Graph I versus t for the interval $0 \leq t \leq 4$ s.
53. || The total amount of charge that has entered a wire at time t is given by the expression $Q = (20 \text{ C})(1 - e^{-t/(2.0 \text{ s})})$, where t is in seconds and $t \geq 0$.
- Find an expression for the current in the wire at time t .
 - What is the maximum value of the current?
 - Graph I versus t for the interval $0 \leq t \leq 10$ s.
54. || The current in a wire at time t is given by the expression $I = (2.0 \text{ A})e^{-t/(2.0 \mu\text{s})}$, where t is in microseconds and $t \geq 0$.
- Find an expression for the total amount of charge (in coulombs) that has entered the wire at time t . The initial conditions are $Q = 0$ C at $t = 0$ μs .
 - Graph Q versus t for the interval $0 \leq t \leq 10 \mu\text{s}$.
55. || The two wires in [FIGURE P30.55](#) are made of the same material. What are the current and the electron drift speed in the 2.0-mm-diameter segment of the wire?

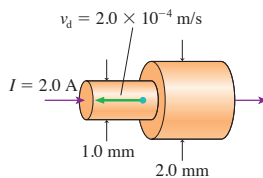


FIGURE P30.55

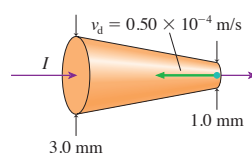


FIGURE P30.56

56. || What is the electron drift speed at the 3.0-mm-diameter end (the left end) of the wire in [FIGURE P30.56](#)?
57. | What diameter should the nichrome wire in [FIGURE P30.57](#) be in order for the electric field strength to be the same in both wires?

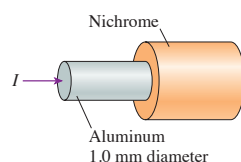


FIGURE P30.57

58. || An aluminum wire consists of the three segments shown in [FIGURE P30.58](#). The current in the top segment is 10 A. For each of these three segments, find the
- Current I .
 - Current density J .
 - Electric field E .
 - Drift velocity v_d .
 - Electron current i .

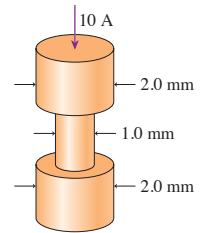


FIGURE P30.58

- Place your results in a table for easy viewing.
59. || What electric field strength is needed to create a 5.0 A current in a 2.0-mm-diameter iron wire?
60. || A 20-cm-long hollow nichrome tube of inner diameter 2.8 mm, outer diameter 3.0 mm is connected to a 3.0 V battery. What is the current in the tube?
61. || The batteries in [FIGURE P30.61](#) are identical. Both resistors have equal currents. What is the resistance of the resistor on the right?

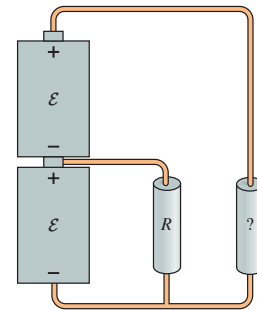


FIGURE P30.61

62. || A 1.5 V flashlight battery is connected to a wire with a resistance of 3.0 Ω . [FIGURE P30.62](#) shows the battery’s potential difference as a function of time. What is the total charge lifted by the charge escalator?

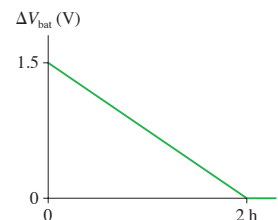


FIGURE P30.62

63. || Two 10-cm-diameter metal plates 1.0 cm apart are charged to ± 12.5 nC. They are suddenly connected together by a 0.224-mm-diameter copper wire stretched taut from the center of one plate to the center of the other.
- What is the maximum current in the wire?
 - Does the current increase with time, decrease with time, or remain steady? Explain.
 - What is the total amount of energy dissipated in the wire?
64. || A long, round wire has resistance R . What will the wire’s resistance be if you stretch it to twice its initial length?

65. || **FIGURE P30.65** shows the potential along a tungsten wire. What is the current density in the wire?

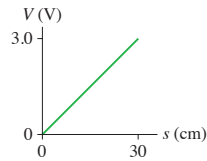


FIGURE P30.65

66. || Household wiring often uses 2.0-mm-diameter copper wires. The wires can get rather long as they snake through the walls from the fuse box to the farthest corners of your house. What is the potential difference across a 20-m-long, 2.0-mm-diameter copper wire carrying an 8.0 A current?
67. || You've decided to protect your house by placing a 5.0-m-tall iron lightning rod next to the house. The top is sharpened to a point and the bottom is in good contact with the ground. From your research, you've learned that lightning bolts can carry up to 50 kA of current and last up to 50 μ s.
- How much charge is delivered by a lightning bolt with these parameters?
 - You don't want the potential difference between the top and bottom of the lightning rod to exceed 100 V. What minimum diameter must the rod have?

Challenge Problems

68. The conductive tissues of the upper leg can be modeled as a **BIO** 40-cm-long, 12-cm-diameter cylinder of muscle and fat. The resistivities of muscle and fat are 13 Ω m and 25 Ω m, respectively. One person's upper leg is 82% muscle, 18% fat. What current is measured if a 1.5 V potential difference is applied between the person's hip and knee?
69. The current supplied by a battery slowly decreases as the battery runs down. Suppose that the current as a function of time is $I = (0.75 \text{ A})e^{-t/(6 \text{ h})}$. What is the total number of electrons transported from the positive electrode to the negative electrode by the charge escalator from the time the battery is first used until it is completely dead?

70. The electric field in a current-carrying wire can be modeled as the electric field at the midpoint between two charged rings. Model a 3.0-mm-diameter aluminum wire as two 3.0-mm-diameter rings 2.0 mm apart. What is the current in the wire after 20 electrons are transferred from one ring to the other?
71. A 5.0-mm-diameter proton beam carries a total current of 1.5 mA. The current density in the proton beam, which increases with distance from the center, is given by $J = J_{\text{edge}}(r/R)$, where R is the radius of the beam and J_{edge} is the current density at the edge.
- How many protons per second are delivered by this proton beam?
 - Determine the value of J_{edge} .
72. A metal wire connecting the terminals of a battery with potential difference ΔV_{bat} gets warm as it draws a current I .
- What is ΔU , the change in potential energy of charge Q as it passes through the wire?
 - Where does this energy go?
 - Power is the *rate* of transfer of energy. Based on your answer to part a, find an expression for the power supplied by the battery to warm the wire.
 - What power does a 1.5 V battery supply to a wire drawing a 1.2 A current?

73. **FIGURE CP30.73** shows a wire that is made of two equal-diameter segments with conductivities σ_1 and σ_2 . When current I passes through the wire, a thin layer of charge appears at the boundary between the segments.

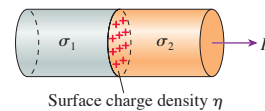


FIGURE CP30.73

- Find an expression for the surface charge density η on the boundary. Give your result in terms of I , σ_1 , σ_2 , and the wire's cross-section area A .
- A 1.0-mm-diameter wire made of copper and iron segments carries a 5.0 A current. How much charge accumulates at the boundary between the segments?

STOP TO THINK ANSWERS

Stop to Think 30.1: $i_c > i_b > i_a > i_d$. The electron current is proportional to $r^2 v_d$. Changing r by a factor of 2 has more influence than changing v_d by a factor of 2.

Stop to Think 30.2: The electrons don't have to move from the switch to the bulb, which could take hours. Because the wire between the switch and the bulb is already full of electrons, a flow of electrons from the switch into the wire immediately causes electrons to flow from the other end of the wire into the lightbulb.

Stop to Think 30.3: $E_d > E_b > E_c > E_a = E_e$. The electric field strength depends on the *difference* in the charge on the two wires. The electric fields of the rings in a and c are opposed to each other, so the net field is zero. The rings in d have the largest charge *difference*.

Stop to Think 30.4: 1 A into the junction. The total current entering the junction must equal the total current leaving the junction.

Stop to Think 30.5: $J_b > J_a = J_d > J_c$. The current density $J = I/\pi r^2$ is independent of the conductivity σ , so a and d are the same. Changing r by a factor of 2 has more influence than changing I by a factor of 2.

Stop to Think 30.6: $I_a = I_b = I_c = I_d$. Conservation of current requires $I_a = I_b$. The current in each wire is $I = \Delta V_{\text{wire}}/R$. All the wires have the same resistance because they are identical, and they all have the same potential difference because each is connected directly to the battery, which is a *source of potential*.

SUMMARY

The goal of Chapter 31 has been to understand the fundamental physical principles that govern electric circuits.

General Strategy

MODEL Assume that wires and, where appropriate, batteries are ideal.

VISUALIZE Draw a circuit diagram. Label all known and unknown quantities.

SOLVE Base the solution on Kirchhoff's laws.

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Write one loop equation for each independent loop.
- Find the current and the potential difference.
- Rebuild the circuit to find I and ΔV for each resistor.

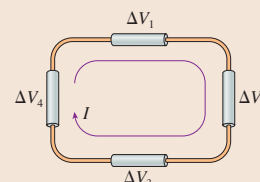
ASSESS Verify that

- The sum of potential differences across series resistors matches ΔV for the equivalent resistor.
- The sum of the currents through parallel resistors matches I for the equivalent resistor.

Kirchhoff's loop law

For a closed loop:

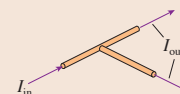
- Assign a direction to the current I .
- $\sum(\Delta V)_i = 0$



Kirchhoff's junction law

For a junction:

- $\sum I_{\text{in}} = \sum I_{\text{out}}$



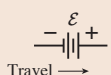
Important Concepts

Ohm's Law

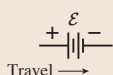
A potential difference ΔV between the ends of a conductor with resistance R creates a current

$$I = \frac{\Delta V}{R}$$

Signs of ΔV for Kirchhoff's loop law



$$\Delta V_{\text{bat}} = +\mathcal{E}$$



$$\Delta V_{\text{bat}} = -\mathcal{E}$$



$$\Delta V_{\text{res}} = -IR$$

The **energy used by a circuit** is supplied by the emf \mathcal{E} of the battery through the energy transformations

$$E_{\text{chem}} \rightarrow U \rightarrow K \rightarrow E_{\text{th}}$$

The battery *supplies* energy at the rate

$$P_{\text{bat}} = I\mathcal{E}$$

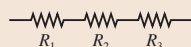
The resistors *dissipate* energy at the rate

$$P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R}$$

Applications

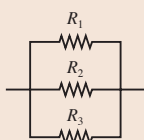
Series resistors

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$$



Parallel resistors

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \right)^{-1}$$



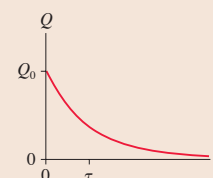
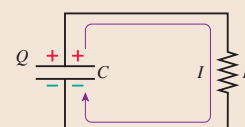
RC circuits

The discharge of a capacitor through a resistor satisfies:

$$Q = Q_0 e^{-t/\tau}$$

$$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau} = I_0 e^{-t/\tau}$$

where $\tau = RC$ is the **time constant**.



Terms and Notation

circuit diagram
Kirchhoff's junction law
Kirchhoff's loop law
complete circuit
load

source
kilowatt hour, kWh
series resistors
equivalent resistance, R_{eq}
ammeter

internal resistance, r
terminal voltage, ΔV_{bat}
short circuit
parallel resistors
voltmeter

grounded
RC circuit
time constant, τ

CONCEPTUAL QUESTIONS

1. Rank in order, from largest to smallest, the currents I_a to I_d through the four resistors in **FIGURE Q31.1**.

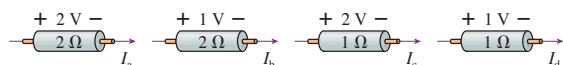


FIGURE Q31.1

2. The tip of a flashlight bulb is touching the top of the 3 V battery in **FIGURE Q31.2**. Does the bulb light? Why or why not?



FIGURE Q31.2

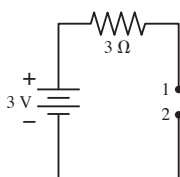


FIGURE Q31.3

3. The wire is broken on the right side of the circuit in **FIGURE Q31.3**. What is the potential difference ΔV_{12} between points 1 and 2? Explain.
4. The circuit of **FIGURE Q31.4** has two resistors, with $R_1 > R_2$. Which of the two resistors dissipates the larger amount of power? Explain.

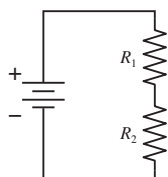


FIGURE Q31.4

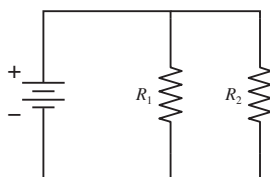


FIGURE Q31.5

5. The circuit of **FIGURE Q31.5** has two resistors, with $R_1 > R_2$. Which of the two resistors dissipates the larger amount of power? Explain.
6. Rank in order, from largest to smallest, the powers P_a to P_d dissipated by the four resistors in **FIGURE Q31.6**.

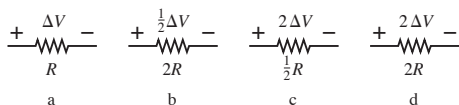


FIGURE Q31.6

7. Are the two resistors in **FIGURE Q31.7** in series or in parallel? Explain.



FIGURE Q31.7

8. A battery with internal resistance r is connected to a load resistance R . If R is increased, does the terminal voltage of the battery increase, decrease, or stay the same? Explain.
9. Initially bulbs A and B in **FIGURE Q31.9** are glowing. What happens to each bulb if the switch is closed? Does it get brighter, stay the same, get dimmer, or go out? Explain.

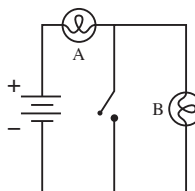


FIGURE Q31.9

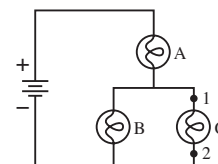


FIGURE Q31.10

10. Bulbs A, B, and C in **FIGURE Q31.10** are identical, and all are glowing.
- Rank in order, from most to least, the brightnesses of the three bulbs. Explain.
 - Suppose a wire is connected between points 1 and 2. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain.
11. Bulbs A and B in **FIGURE Q31.11** are identical, and both are glowing. Bulb B is removed from its socket. Does the potential difference ΔV_{12} between points 1 and 2 increase, stay the same, decrease, or become zero? Explain.

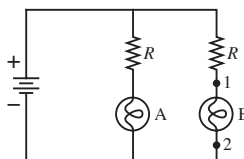


FIGURE Q31.11

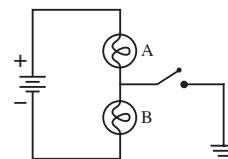


FIGURE Q31.12

12. Bulbs A and B in **FIGURE Q31.12** are identical, and both are glowing. What happens to each bulb when the switch is closed? Does its brightness increase, stay the same, decrease, or go out? Explain.
13. **FIGURE Q31.13** shows the voltage as a function of time of a capacitor as it is discharged (separately) through three different resistors. Rank in order, from largest to smallest, the values of the resistances R_1 to R_3 .

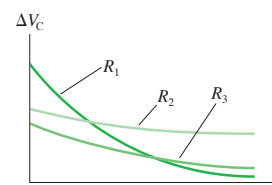


FIGURE Q31.13

EXERCISES AND PROBLEMS

Problems labeled  integrate material from earlier chapters.

Exercises

Section 31.1 Circuit Elements and Diagrams

1.  Draw a circuit diagram for the circuit of **FIGURE EX31.1**.

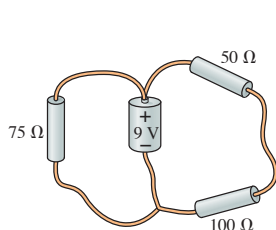


FIGURE EX31.1

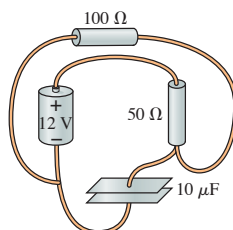



FIGURE EX31.2

2.  Draw a circuit diagram for the circuit of **FIGURE EX31.2**.

Section 31.2 Kirchhoff's Laws and the Basic Circuit

3.  In **FIGURE EX31.3**, what is the current in the wire to the right of the junction? Does the charge in this wire flow to the right or to the left?

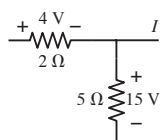


FIGURE EX31.3

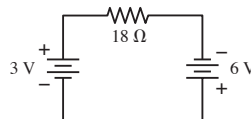




FIGURE EX31.4

4.  a. What are the magnitude and direction of the current in the $18\ \Omega$ resistor in **FIGURE EX31.4**?
 b. Draw a graph of the potential as a function of the distance traveled through the circuit, traveling cw from $V = 0\text{ V}$ at the lower left corner.
5.  a. What are the magnitude and direction of the current in the $10\ \Omega$ resistor in **FIGURE EX31.5**?
 b. Draw a graph of the potential as a function of the distance traveled through the circuit, traveling cw from $V = 0\text{ V}$ at the lower left corner.

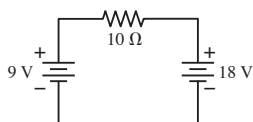


FIGURE EX31.5

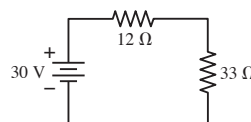




FIGURE EX31.6

6.  What is the potential difference across each resistor in **FIGURE EX31.6**?

Section 31.3 Energy and Power

7.  What is the resistance of a 1500 W (120 V) hair dryer? What is the current in the hair dryer when it is used?

8.  How much power is dissipated by each resistor in **FIGURE EX31.8**?

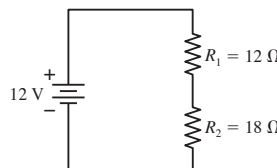


FIGURE EX31.8

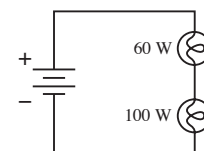







FIGURE EX31.9

9.  A 60 W lightbulb and a 100 W lightbulb are placed one after the other in the circuit of **FIGURE EX31.9**. The battery's emf is large enough that both bulbs are glowing. Which is the true statement?
 A. The 60 W bulb is brighter.
 B. Both bulbs are equally bright.
 C. The 100 W bulb is brighter.
 D. There's not enough information to tell which bulb is brighter.
10.  A standard 100 W (120 V) lightbulb contains a 7.0-cm-long tungsten filament. The high-temperature resistivity of tungsten is $9.0 \times 10^{-7}\ \Omega\text{ m}$. What is the diameter of the filament?
11.  A typical American family uses 1000 kWh of electricity a month.
 a. What is the average current in the 120 V power line to the house?
 b. On average, what is the resistance of a household?
12.  A waterbed heater uses 450 W of power. It is on 25% of the time, off 75%. What is the annual cost of electricity at a billing rate of \$0.12/kWh?

Section 31.4 Series Resistors

Section 31.5 Real Batteries

13.  Two of the three resistors in **FIGURE EX31.13** are unknown but equal. The total resistance between points a and b is $200\ \Omega$. What is the value of R ?

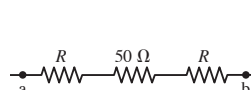


FIGURE EX31.13

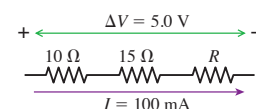




FIGURE EX31.14

14.  What is the value of resistor R in **FIGURE EX31.14**?
15.  The battery in **FIGURE EX31.15** is short-circuited by an ideal ammeter having zero resistance.
 a. What is the battery's internal resistance?
 b. How much power is dissipated inside the battery?

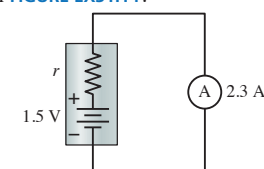



FIGURE EX31.15

16.  The voltage across the terminals of a 9.0 V battery is 8.5 V when the battery is connected to a $20\ \Omega$ load. What is the battery's internal resistance?

17. || Compared to an ideal battery, by what percentage does the battery's internal resistance reduce the potential difference across the $20\ \Omega$ resistor in **FIGURE EX31.17**?

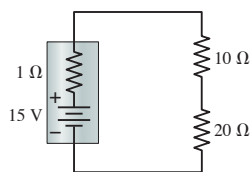


FIGURE EX31.17

Section 31.6 Parallel Resistors

18. || A metal wire of resistance R is cut into two pieces of equal length. The two pieces are connected together side by side. What is the resistance of the two connected wires?
19. | Two of the three resistors in **FIGURE EX31.19** are unknown but equal. The total resistance between points a and b is $75\ \Omega$. What is the value of R ?

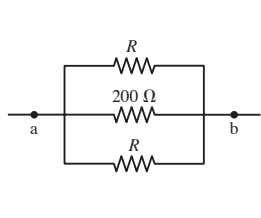


FIGURE EX31.19

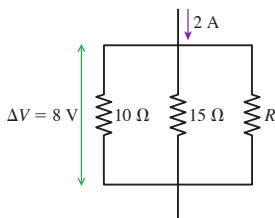


FIGURE EX31.20

20. | What is the value of resistor R in **FIGURE EX31.20**?
21. | What is the equivalent resistance between points a and b in **FIGURE EX31.21**?

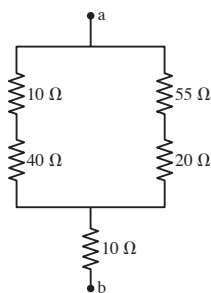


FIGURE EX31.21

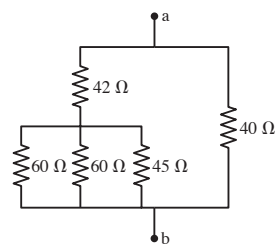


FIGURE EX31.22

22. | What is the equivalent resistance between points a and b in **FIGURE EX31.22**?
23. | What is the equivalent resistance between points a and b in **FIGURE EX31.23**?

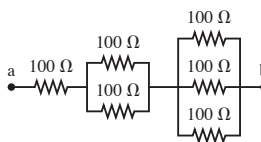


FIGURE EX31.23

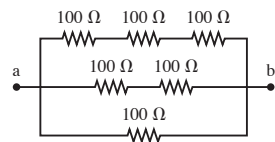


FIGURE EX31.24

24. | What is the equivalent resistance between points a and b in **FIGURE EX31.24**?

Section 31.8 Getting Grounded

25. || In **FIGURE EX31.25**, what is the value of the potential at points a and b?

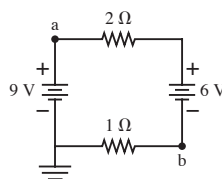


FIGURE EX31.25

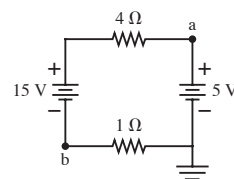


FIGURE EX31.26

26. || In **FIGURE EX31.26**, what is the value of the potential at points a and b?

Section 31.9 RC Circuits

27. | Show that the product RC has units of s.
28. | What is the time constant for the discharge of the capacitors in **FIGURE EX31.28**?

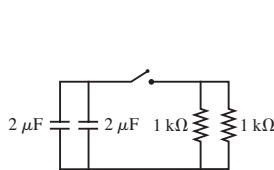


FIGURE EX31.28

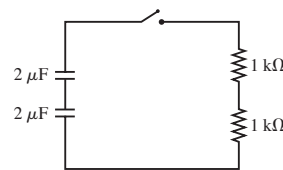


FIGURE EX31.29

29. || What is the time constant for the discharge of the capacitors in **FIGURE EX31.29**?
30. || A $10\ \mu\text{F}$ capacitor initially charged to $20\ \mu\text{C}$ is discharged through a $1.0\ \text{k}\Omega$ resistor. How long does it take to reduce the capacitor's charge to $10\ \mu\text{C}$?
31. | The switch in **FIGURE EX31.31** has been in position a for a long time. It is changed to position b at $t = 0\ \text{s}$. What are the charge Q on the capacitor and the current I through the resistor (a) immediately after the switch is closed? (b) at $t = 50\ \mu\text{s}$? (c) at $t = 200\ \mu\text{s}$?
32. || What value resistor will discharge a $1.0\ \mu\text{F}$ capacitor to 10% of its initial charge in $2.0\ \text{ms}$?
33. || A capacitor is discharged through a $100\ \Omega$ resistor. The discharge current decreases to 25% of its initial value in $2.5\ \text{ms}$. What is the value of the capacitor?

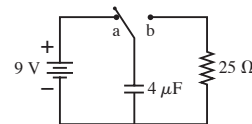


FIGURE EX31.31

Problems

34. || The five identical bulbs in **FIGURE P31.34** are all glowing. The battery is ideal. What is the order of brightness of the bulbs, from brightest to dimmest? Some may be equal.
- A. $P = S > Q = R = T$
 B. $P = S = T > Q = R$
 C. $P > S = T > Q = R$
 D. $P > Q = R > S = T$

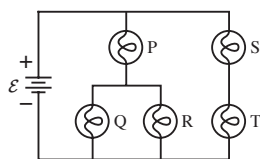


FIGURE P31.34

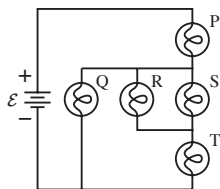


FIGURE P31.35

35. || The five identical bulbs in FIGURE P31.35 are all glowing. The battery is ideal. What is the order of brightness of the bulbs, from brightest to dimmest? Some may be equal.
- $P = T > Q = R = S$
 - $P > Q = R = S > T$
 - $P = T > Q > R = S$
 - $P > Q > T > R = S$
36. || Two 75 W (120 V) lightbulbs are wired in series, then the combination is connected to a 120 V supply. How much power is dissipated by each bulb?
37. || The corroded contacts in a lightbulb socket have $5.0\ \Omega$ resistance. How much actual power is dissipated by a 100 W (120 V) lightbulb screwed into this socket?
38. || An electric eel develops a 450 V potential difference between its head and tail. The eel can stun a fish or other prey by using this potential difference to drive a 0.80 A current pulse for 1.0 ms. What are (a) the energy delivered by this pulse and (b) the total charge that flows?
39. || You have a $2.0\ \Omega$ resistor, a $3.0\ \Omega$ resistor, a $6.0\ \Omega$ resistor, and a 6.0 V battery. Draw a diagram of a circuit in which all three resistors are used and the battery delivers 9.0 W of power.
40. | You have three $12\ \Omega$ resistors. Draw diagrams showing how you could arrange all three so that their equivalent resistance is (a) $4.0\ \Omega$, (b) $8.0\ \Omega$, (c) $18\ \Omega$, and (d) $36\ \Omega$.
41. || What is the equivalent resistance between points a and b in FIGURE P31.41?

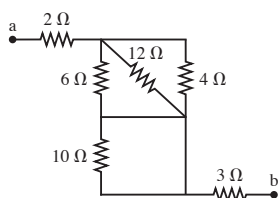


FIGURE P31.41

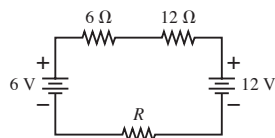


FIGURE P31.42

42. | There is a current of 0.25 A in the circuit of FIGURE P31.42. What is the power dissipated by R ?
43. || A variable resistor R is connected across the terminals of a battery. FIGURE P31.43 shows the current in the circuit as R is varied. What are the emf and internal resistance of the battery?

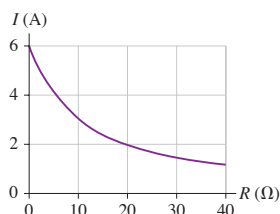


FIGURE P31.43

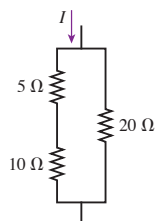


FIGURE P31.44

44. | The $10\ \Omega$ resistor in FIGURE P31.44 is dissipating 40 W of power. How much power are the other two resistors dissipating?
45. || What are the emf and internal resistance of the battery in FIGURE P31.45?

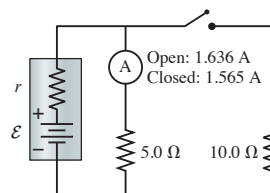


FIGURE P31.45

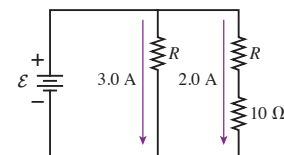


FIGURE P31.46

46. | What is the emf of the battery in FIGURE P31.46?
47. || A 2.5 V battery with $0.70\ \Omega$ internal resistance is connected in parallel with a 1.5 V battery having $0.30\ \Omega$ internal resistance. That is, their positive terminals are connected by a wire and their negative terminals are connected by a wire. What is the terminal voltage of the 2.5 V battery?
48. || a. Load resistor R is attached to a battery of emf \mathcal{E} and internal resistance r . For what value of the resistance R , in terms of \mathcal{E} and r , will the power dissipated by the load resistor be a maximum?
- b. What is the maximum power that the load can dissipate if the battery has $\mathcal{E} = 9.0\ \text{V}$ and $r = 1.0\ \Omega$?
- c. Why should the power dissipated by the load have a maximum value? Explain.
- Hint:** What happens to the power dissipation when R is either very small or very large?
49. | The ammeter in FIGURE P31.49 reads 3.0 A. Find I_1 , I_2 , and \mathcal{E} .

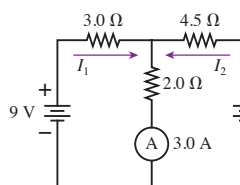


FIGURE P31.49

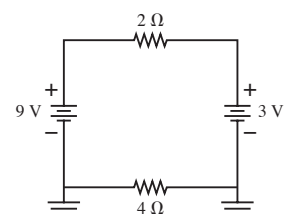


FIGURE P31.50

50. || What is the current in the $2\ \Omega$ resistor in FIGURE P31.50?
51. | It seems hard to justify spending \$5 for a compact fluorescent lightbulb when an ordinary incandescent bulb costs 50¢. To see if this makes sense, compare a 60 W incandescent bulb lasting 1000 hours to a 15 W compact fluorescent bulb having a lifetime of 10,000 hours. Both bulbs produce the same amount of visible light and are interchangeable. If electricity costs \$0.10/kWh, what is the total cost—purchase plus energy—to obtain 10,000 hours of light from each type of bulb? This is called the *life-cycle cost*.
52. || A refrigerator has a 1000 W compressor, but the compressor runs only 20% of the time.
- If electricity costs \$0.10/kWh, what is the monthly (30 day) cost of running the refrigerator?
 - A more energy-efficient refrigerator with an 800 W compressor costs \$100 more. If you buy the more expensive refrigerator, how many months will it take to recover your additional cost?

53. I For an ideal battery ($r = 0\ \Omega$), closing the switch in **FIGURE P31.53** does not affect the brightness of bulb A. In practice, bulb A dims *just a little* when the switch closes. To see why, assume that the 1.50 V battery has an internal resistance $r = 0.50\ \Omega$ and that the resistance of a glowing bulb is $R = 6.00\ \Omega$.
- What is the current through bulb A when the switch is open?
 - What is the current through bulb A after the switch has closed?
 - By what percentage does the current through A change when the switch is closed?

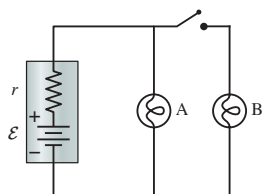


FIGURE P31.53

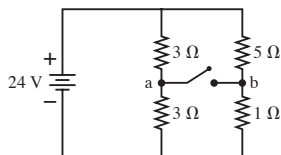


FIGURE P31.54

54. I What are the battery current I_{bat} and the potential difference ΔV_{ab} between points a and b when the switch in **FIGURE P31.54** is (a) open and (b) closed?
55. I The circuit in **FIGURE P31.55** is called a *voltage divider*. What value of R will make $V_{\text{out}} = V_{\text{in}}/10$?

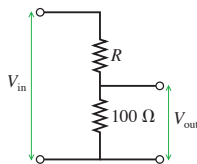


FIGURE P31.55

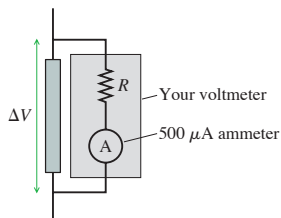


FIGURE P31.56

56. I A circuit you're building needs a voltmeter that goes from 0 V to a full-scale reading of 5.0 V. Unfortunately, the only meter in the storeroom is an *ammeter* that goes from 0 μA to a full-scale reading of 500 μA . Fortunately, you've just finished a physics class, and you realize that you can convert this meter to a *voltmeter* by putting a resistor in series with it, as shown in **FIGURE P31.56**. You've measured that the resistance of the ammeter is 50.0 Ω , not the 0 Ω of an ideal ammeter. What value of R must you use so that the meter will go to full scale when the potential difference across the object being measured is 5.0 V?
57. I A circuit you're building needs an ammeter that goes from 0 mA to a full-scale reading of 50 mA. Unfortunately, the only ammeter in the storeroom goes from 0 μA to a full-scale reading of only 500 μA . Fortunately, you've just finished a physics class, and you realize that you can make this ammeter work by putting a resistor in parallel with it, as shown in **FIGURE P31.57**. You've measured that the resistance of the ammeter is 50.0 Ω , not the 0 Ω of an ideal ammeter.
- What value of R must you use so that the meter will go to full scale when the current I is 50 mA?
 - What is the effective resistance of your ammeter?

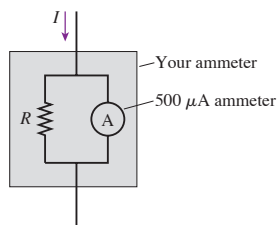


FIGURE P31.57

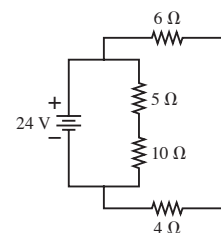


FIGURE P31.58

58. I For the circuit shown in **FIGURE P31.58**, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.
59. I For the circuit shown in **FIGURE P31.59**, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.

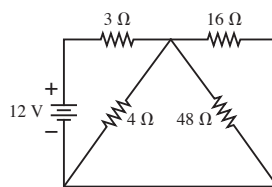


FIGURE P31.59

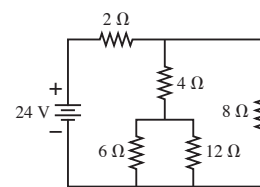


FIGURE P31.60

60. I For the circuit shown in **FIGURE P31.60**, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.
61. I For the circuit shown in **FIGURE P31.61**, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.

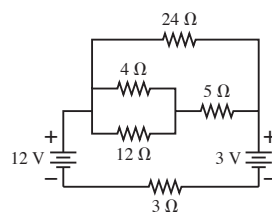


FIGURE P31.61

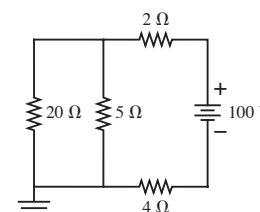


FIGURE P31.62

62. I What is the current through the 20 Ω resistor in **FIGURE P31.62**?
63. I What is the current through the 10 Ω resistor in **FIGURE P31.63**? Is the current from left to right or right to left?

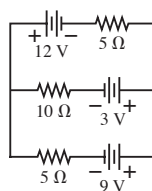


FIGURE P31.63

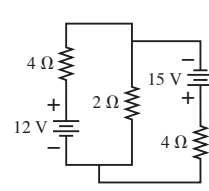


FIGURE P31.64

64. I What power is dissipated by the 2 Ω resistor in **FIGURE P31.64**?

65. || For what emf \mathcal{E} does the $200\ \Omega$ resistor in FIGURE P31.65 dissipate no power? Should the emf be oriented with its positive terminal at the top or at the bottom?

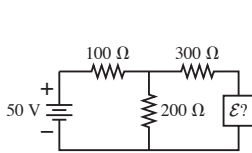


FIGURE P31.65

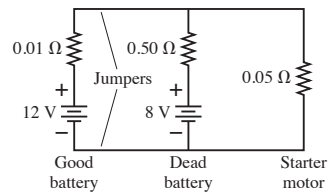


FIGURE P31.66

66. || A 12 V car battery dies not so much because its voltage drops but because chemical reactions increase its internal resistance. A good battery connected with jumper cables can both start the engine and recharge the dead battery. Consider the automotive circuit of FIGURE P31.66.
- How much current could the good battery alone drive through the starter motor?
 - How much current is the dead battery alone able to drive through the starter motor?
 - With the jumper cables attached, how much current passes through the starter motor?
 - With the jumper cables attached, how much current passes through the dead battery, and in which direction?
67. || How much current flows through the bottom wire in FIGURE P31.67, and in which direction?

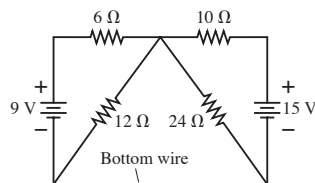


FIGURE P31.67

68. || The capacitor in an RC circuit is discharged with a time constant of 10 ms. At what time after the discharge begins are (a) the charge on the capacitor reduced to half its initial value and (b) the energy stored in the capacitor reduced to half its initial value?
69. || A circuit you're using discharges a $20\ \mu\text{F}$ capacitor through an unknown resistor. After charging the capacitor, you close a switch at $t = 0$ s and then monitor the resistor current with an ammeter. Your data are as follows:

Time (s)	Current (μA)
0.5	890
1.0	640
1.5	440
2.0	270
2.5	200

Use an appropriate graph of the data to determine (a) the resistance and (b) the initial capacitor voltage.

70. || A $150\ \mu\text{F}$ defibrillator capacitor is charged to 1500 V. When fired through a patient's chest, it loses 95% of its charge in 40 ms. What is the resistance of the patient's chest?

71. || A $50\ \mu\text{F}$ capacitor that had been charged to 30 V is discharged through a resistor. FIGURE P31.71 shows the capacitor voltage as a function of time. What is the value of the resistance?

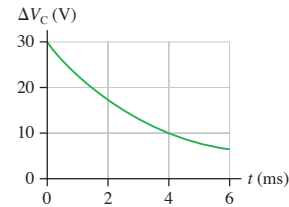


FIGURE P31.71

72. || A $0.25\ \mu\text{F}$ capacitor is charged to 50 V. It is then connected in series with a $25\ \Omega$ resistor and a $100\ \Omega$ resistor and allowed to discharge completely. How much energy is dissipated by the $25\ \Omega$ resistor?
73. || The capacitor in FIGURE P31.73 begins to charge after the switch closes at $t = 0$ s.
- What is ΔV_C a very long time after the switch has closed?
 - What is Q_{max} in terms of \mathcal{E} , R , and C ?
 - In this circuit, does $I = +dQ/dt$ or $-dQ/dt$? Explain.
 - Find an expression for the current I at time t . Graph I from $t = 0$ to $t = 5\tau$.
74. || The capacitors in FIGURE P31.74 are charged and the switch closes at $t = 0$ s. At what time has the current in the $8\ \Omega$ resistor decayed to half the value it had immediately after the switch was closed?

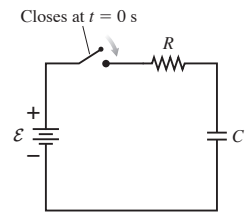


FIGURE P31.73

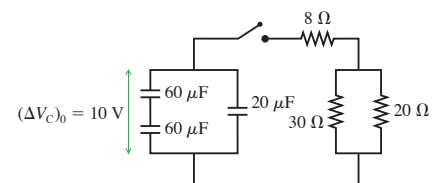


FIGURE P31.74

Challenge Problems

75. You've made the finals of the Science Olympics! As one of your tasks, you're given 1.0 g of aluminum and asked to make a wire, using all the aluminum, that will dissipate 7.5 W when connected to a 1.5 V battery. What length and diameter will you choose for your wire?
76. The switch in FIGURE CP31.76 has been closed for a very long time.
- What is the charge on the capacitor?
 - The switch is opened at $t = 0$ s. At what time has the charge on the capacitor decreased to 10% of its initial value?

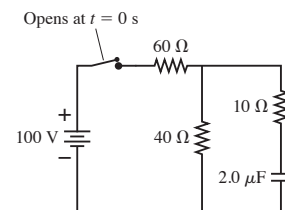


FIGURE CP31.76

77. A capacitor-charging circuit has a time constant of 40 ms. When the switch is closed, the initial current to the $50\ \mu\text{F}$ capacitor is 65 mA. What is the capacitor's voltage after 20 ms? Assume the capacitor was completely uncharged when the switch closed.
78. The capacitor in Figure 31.38a begins to charge after the switch closes at $t = 0$ s. Analyze this circuit and show that $Q = Q_{\text{max}}(1 - e^{-t/\tau})$, where $Q_{\text{max}} = C\mathcal{E}$.
79. The switch in Figure 31.38a closes at $t = 0$ s and, after a very long time, the capacitor is fully charged. Find expressions for (a) the total energy supplied by the battery as the capacitor is being charged, (b) total energy dissipated by the resistor as the capacitor is being charged, and (c) the energy stored in the capacitor when it is fully charged. Your expressions will be in terms of \mathcal{E} , R , and C . (d) Do your results for parts a to c show that energy is conserved? Explain.
80. An *oscillator circuit* is important to many applications. A simple oscillator circuit can be built by adding a neon gas tube to an RC circuit, as shown in **FIGURE CP31.80**. Gas is normally a good insulator, and the resistance of the gas tube is essentially infinite when the light is off. This allows the capacitor to charge. When the capacitor voltage reaches a value V_{on} , the electric field inside the tube becomes strong enough to ionize the neon gas. Visually, the tube lights with an orange glow. Electrically, the ionization of the gas provides a very-low-resistance path through the tube. The capacitor very rapidly (we can think of it as instantaneously) discharges through the tube and the capacitor voltage drops. When the capacitor voltage has dropped to a value V_{off} ,

the electric field inside the tube becomes too weak to sustain the ionization and the neon light turns off. The capacitor then starts to charge again. The capacitor voltage oscillates between V_{off} , when it starts charging, and V_{on} , when the light comes on to discharge it.

- a. Show that the oscillation period is

$$T = RC \ln \left(\frac{\mathcal{E} - V_{\text{off}}}{\mathcal{E} - V_{\text{on}}} \right)$$

- b. A neon gas tube has $V_{\text{on}} = 80\ \text{V}$ and $V_{\text{off}} = 20\ \text{V}$. What resistor value should you choose to go with a $10\ \mu\text{F}$ capacitor and a 90 V battery to make a 10 Hz oscillator?

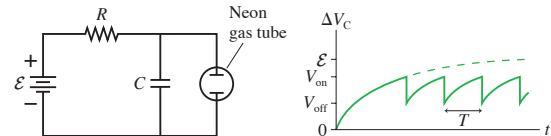


FIGURE CP31.80

81. A 2.0-m-long, 1.0-mm-diameter wire has a variable resistivity given by

$$\rho(x) = (2.5 \times 10^{-6}) \left[1 + \left(\frac{x}{1.0\ \text{m}} \right)^2 \right] \Omega\text{m}$$

where x is measured from one end of the wire. What is the current if this wire is connected to the terminals of a 9.0 V battery?

STOP TO THINK ANSWERS

Stop to Think 31.1: a, b, and d. These three are the same circuit because the logic of the connections is the same. In c, the functioning of the circuit is changed by the extra wire connecting the two sides of the capacitor.

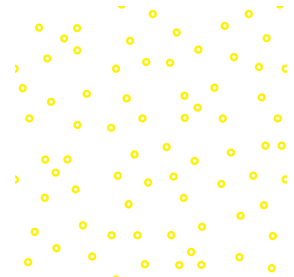
Stop to Think 31.2: ΔV increases by 2 V in the direction of I . Kirchhoff's loop law, starting on the left side of the battery, is then $+12\ \text{V} + 2\ \text{V} - 8\ \text{V} - 6\ \text{V} = 0\ \text{V}$.

Stop to Think 31.3: $P_b > P_d > P_a > P_c$. The power dissipated by a resistor is $P_R = (\Delta V_R)^2/R$. Increasing R decreases P_R ; increasing ΔV_R increases P_R . But the potential has a larger effect because P_R depends on the square of ΔV_R .

Stop to Think 31.4: $I = 2\ \text{A}$ for all. $V_a = 20\ \text{V}$, $V_b = 16\ \text{V}$, $V_c = 10\ \text{V}$, $V_d = 8\ \text{V}$, $V_e = 0\ \text{V}$. Current is conserved. The potential is 0 V on the right and increases by IR for each resistor going to the left.

Stop to Think 31.5: $A > B > C = D$. All the current from the battery goes through A, so it is brightest. The current divides at the junction, but not equally. Because B is in parallel with C + D but has half the resistance, twice as much current travels through B as through C + D. So B is dimmer than A but brighter than C and D. C and D are equal because of conservation of current.

Stop to Think 31.6: b. The two $2\ \Omega$ resistors are in series and equivalent to a $4\ \Omega$ resistor. Thus $\tau = RC = 4\ \text{s}$.

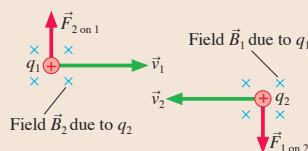


SUMMARY

The goal of Chapter 32 has been to learn how to calculate and use the magnetic field.

General Principles

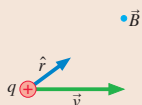
At its most fundamental level, **magnetism** is an interaction between moving charges. The magnetic field of one moving charge exerts a force on another moving charge.



Magnetic Fields

The Biot-Savart law

- A point charge, $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$
- A short current element, $\vec{B} = \frac{\mu_0}{4\pi} \frac{I\Delta\vec{s} \times \hat{r}}{r^2}$



To find the magnetic field of a current:

- Divide the wire into many short segments.
- Find the field of each segment $\Delta\vec{s}$.
- Find \vec{B} by summing the fields of all $\Delta\vec{s}$, usually as an integral.

An alternative method for fields with a high degree of symmetry is **Ampère's law**:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

where I_{through} is the current through the area bounded by the integration path.

Magnetic Forces

The magnetic force on a moving charge is

$$\vec{F} = q\vec{v} \times \vec{B}$$

The force is perpendicular to \vec{v} and \vec{B} .

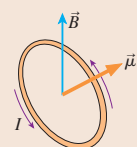
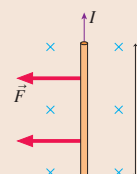
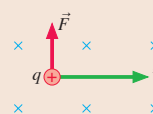
The magnetic force on a current-carrying wire is

$$\vec{F} = I\vec{l} \times \vec{B}$$

$\vec{F} = \vec{0}$ for a charge or current moving parallel to \vec{B} .

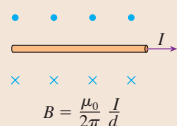
The magnetic torque on a magnetic dipole is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

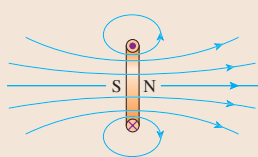


Applications

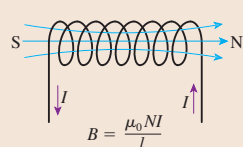
Wire



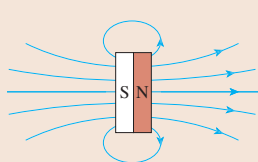
Loop



Solenoid



Flat magnet



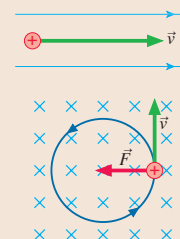
Right-hand rule

Point your right thumb in the direction of I . Your fingers curl in the direction of \vec{B} . For a dipole, \vec{B} emerges from the side that is the north pole.

Charged-particle motion

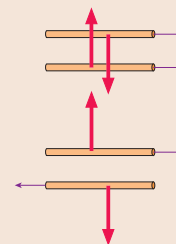
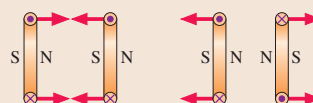
No force if \vec{v} is parallel to \vec{B}

Circular motion at the cyclotron frequency $f_{\text{cyc}} = qB/2\pi m$ if \vec{v} is perpendicular to \vec{B}



Parallel wires and current loops

Parallel currents attract. Opposite currents repel.



Terms and Notation

north pole
south pole
magnetic dipole
magnetic material
right-hand rule
magnetic field, \vec{B}
magnetic field lines

Biot-Savart law
tesla, T
permeability constant, μ_0
cross product
current loop
electromagnet
magnetic dipole moment, $\vec{\mu}$

line integral
Ampère's law
uniform magnetic field
solenoid
cyclotron motion
cyclotron frequency, f_{cyc}
cyclotron

Hall effect
Hall voltage, ΔV_H
ferromagnetic
magnetic domain
induced magnetic dipole
permanent magnet

CONCEPTUAL QUESTIONS

- The lightweight glass sphere in **FIGURE Q32.1** hangs by a thread. The north pole of a bar magnet is brought near the sphere.
 - Suppose the sphere is electrically neutral. Is it attracted to, repelled by, or not affected by the magnet? Explain.
 - Answer the same question if the sphere is positively charged.

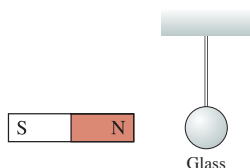


FIGURE Q32.1

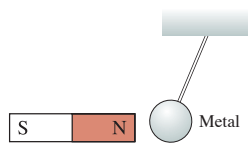


FIGURE Q32.2

- The metal sphere in **FIGURE Q32.2** hangs by a thread. When the north pole of a magnet is brought near, the sphere is strongly attracted to the magnet. Then the magnet is reversed and its south pole is brought near the sphere. How does the sphere respond? Explain.
- You have two electrically neutral metal cylinders that exert strong attractive forces on each other. You have no other metal objects. Can you determine if *both* of the cylinders are magnets, or if one is a magnet and the other is just a piece of iron? If so, how? If not, why not?
- What is the current direction in the wire of **FIGURE Q32.4**? Explain.

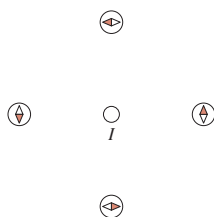


FIGURE Q32.4

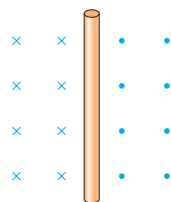


FIGURE Q32.5

- What is the current direction in the wire of **FIGURE Q32.5**? Explain.
- What is the *initial* direction of deflection for the charged particles entering the magnetic fields shown in **FIGURE Q32.6**?

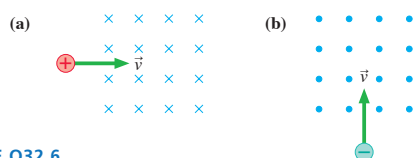


FIGURE Q32.6

- What is the *initial* direction of deflection for the charged particles entering the magnetic fields shown in **FIGURE Q32.7**?

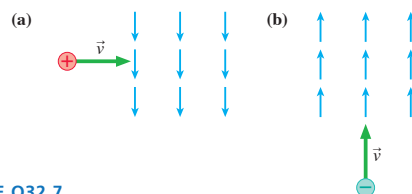


FIGURE Q32.7

- Determine the magnetic field direction that causes the charged particles shown in **FIGURE Q32.8** to experience the indicated magnetic force.

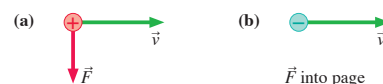


FIGURE Q32.8

- Determine the magnetic field direction that causes the charged particles shown in **FIGURE Q32.9** to experience the indicated magnetic force.




FIGURE Q32.9

- You have a horizontal cathode-ray tube (CRT) for which the controls have been adjusted such that the electron beam *should* make a single spot of light exactly in the center of the screen. You observe, however, that the spot is deflected to the right. It is possible that the CRT is broken. But as a clever scientist, you realize that your laboratory might be in either an electric or a magnetic field. Assuming that you do not have a compass, any magnets, or any charged rods, how can you use the CRT itself to determine whether the CRT is broken, is in an electric field, or is in a magnetic field? You cannot remove the CRT from the room.
- The south pole of a bar magnet is brought toward the current loop of **FIGURE Q32.11**. Does the bar magnet attract, repel, or have no effect on the loop? Explain.
- Give a step-by-step explanation, using both words and pictures, of how a permanent magnet can pick up a piece of nonmagnetized iron.




FIGURE Q32.11

EXERCISES AND PROBLEMS

Problems labeled  integrate material from earlier chapters.

Exercises

Section 32.3 The Source of the Magnetic Field: Moving Charges

1.  Points 1 and 2 in **FIGURE EX32.1** are the same distance from the wires as the point where $B = 2.0$ mT. What are the strength and direction of \vec{B} at points 1 and 2?

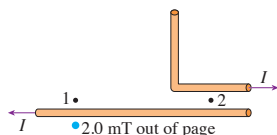


FIGURE EX32.1

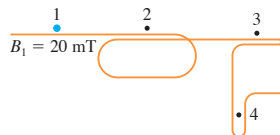






FIGURE EX32.2

2.  What is the magnetic field strength at points 2 to 4 in **FIGURE EX32.2**? Assume that the wires overlap closely and that points 1 to 4 are equally distant from the wires.
3.  A proton moves along the x -axis with $v_x = 1.0 \times 10^7$ m/s. As it passes the origin, what are the strength and direction of the magnetic field at the (x, y, z) positions (a) (1 cm, 0 cm, 0 cm), (b) (0 cm, 1 cm, 0 cm), and (c) (0 cm, -2 cm, 0 cm)?
4.  An electron moves along the z -axis with $v_z = 2.0 \times 10^7$ m/s. As it passes the origin, what are the strength and direction of the magnetic field at the (x, y, z) positions (a) (1 cm, 0 cm, 0 cm), (b) (0 cm, 0 cm, 1 cm), and (c) (0 cm, 1 cm, 1 cm)?
5.  What is the magnetic field at the position of the dot in **FIGURE EX32.5**? Give your answer as a vector.

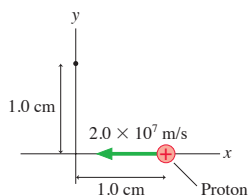


FIGURE EX32.5

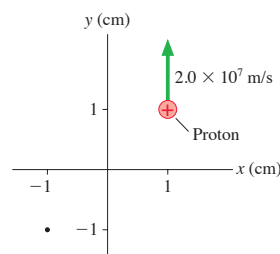









FIGURE EX32.6

6.  What is the magnetic field at the position of the dot in **FIGURE EX32.6**? Give your answer as a vector.
7.  A proton is passing the origin. The magnetic field at the (x, y, z) position (1 mm, 0 mm, 0 mm) is $1.0 \times 10^{-13} \hat{j}$ T. The field at (0 mm, 1 mm, 0 mm) is $-1.0 \times 10^{-13} \hat{i}$ T. What are the speed and direction of the proton?

Section 32.4 The Magnetic Field of a Current

8.  What currents are needed to generate the magnetic field strengths of Table 32.1 at a point 1.0 cm from a long, straight wire?
9.  At what distances from a very thin, straight wire carrying a 10 A current would the magnetic field strengths of Table 32.1 be generated?

10.  The element niobium, which is a metal, is a superconductor (i.e., no electrical resistance) at temperatures below 9 K. However, the superconductivity is destroyed if the magnetic field at the surface of the metal reaches or exceeds 0.10 T. What is the maximum current in a straight, 3.0-mm-diameter superconducting niobium wire?
11.  The magnetic field at the center of a 1.0-cm-diameter loop is 2.5 mT.
- a. What is the current in the loop?
- b. A long straight wire carries the same current you found in part a. At what distance from the wire is the magnetic field 2.5 mT?
12.  A wire carries current I into the junction shown in **FIGURE EX32.12**. What is the magnetic field at the dot?

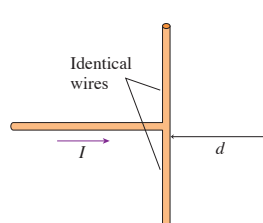


FIGURE EX32.12

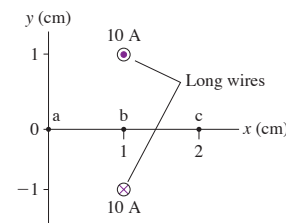




FIGURE EX32.13

13.  What are the magnetic fields at points a to c in **FIGURE EX32.13**? Give your answers as vectors.
14.  What are the magnetic field strength and direction at points a to c in **FIGURE EX32.14**?

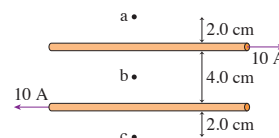






FIGURE EX32.14

Section 32.5 Magnetic Dipoles

15.  The on-axis magnetic field strength 10 cm from a small bar magnet is $5.0 \mu\text{T}$.
- a. What is the bar magnet's magnetic dipole moment?
- b. What is the on-axis field strength 15 cm from the magnet?
16.  A 100 A current circulates around a 2.0-mm-diameter superconducting ring.
- a. What is the ring's magnetic dipole moment?
- b. What is the on-axis magnetic field strength 5.0 cm from the ring?
17.  A small, square loop carries a 25 A current. The on-axis magnetic field strength 50 cm from the loop is 7.5 nT. What is the edge length of the square?
18.  The earth's magnetic dipole moment is 8.0×10^{22} A m².
- a. What is the magnetic field strength on the surface of the earth at the earth's north magnetic pole? How does this compare to the value in Table 32.1? You can assume that the current loop is deep inside the earth.
- b. Astronauts discover an earth-size planet without a magnetic field. To create a magnetic field with the same strength as earth's, they propose running a current through a wire around the equator. What size current would be needed?

Section 32.6 Ampère's Law and Solenoids

19. **|** What is the line integral of \vec{B} between points i and f in **FIGURE EX32.19**?

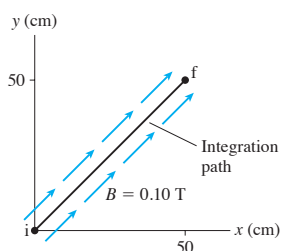


FIGURE EX32.19

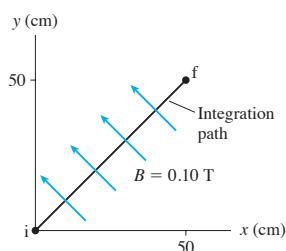


FIGURE EX32.20

20. **|** What is the line integral of \vec{B} between points i and f in **FIGURE EX32.20**?
21. **||** The value of the line integral of \vec{B} around the closed path in **FIGURE EX32.21** is 3.77×10^{-6} T·m. What is I_3 ?

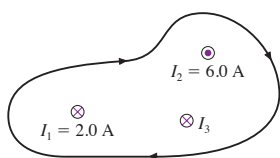


FIGURE EX32.21

22. **||** The value of the line integral of \vec{B} around the closed path in **FIGURE EX32.22** is 1.38×10^{-5} T·m. What are the direction (in or out of the page) and magnitude of I_3 ?

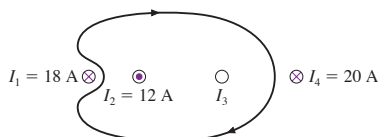


FIGURE EX32.22

23. **||** What is the line integral of \vec{B} between points i and f in **FIGURE EX32.23**?

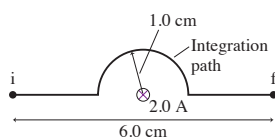


FIGURE EX32.23

24. **||** Magnetic resonance imaging needs a magnetic field strength of 1.5 T. The solenoid is 1.8 m long and 75 cm in diameter. It is tightly wound with a single layer of 2.0-mm-diameter superconducting wire. What size current is needed?
25. **||** A 2.0-cm-diameter, 15-cm-long solenoid is tightly wound with one layer of wire. A 2.5 A current through the wire generates a 3.0 mT magnetic field inside the solenoid. What is the diameter of the wire, in mm?

Section 32.7 The Magnetic Force on a Moving Charge

26. **|** A proton moves in the magnetic field $\vec{B} = 0.50 \hat{i}$ T with a speed of 1.0×10^7 m/s in the directions shown in **FIGURE EX32.26**. For each, what is magnetic force \vec{F} on the proton? Give your answers in component form.

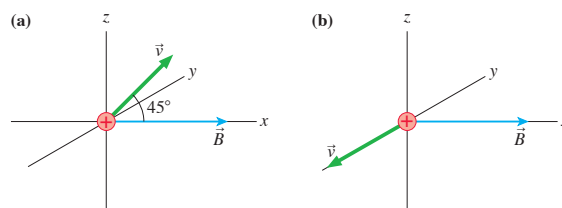


FIGURE EX32.26

27. **||** An electron moves in the magnetic field $\vec{B} = 0.50 \hat{i}$ T with a speed of 1.0×10^7 m/s in the directions shown in **FIGURE EX32.27**. For each, what is magnetic force \vec{F} on the electron? Give your answers in component form.

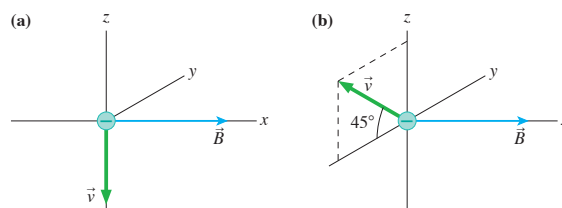


FIGURE EX32.27

28. **||** To five significant figures, what are the cyclotron frequencies in a 3.0000 T magnetic field of the ions (a) O_2^+ , (b) N_2^+ , and (c) CO^+ ? The atomic masses are shown in the table; the mass of the missing electron is less than 0.001 u and is not relevant at this level of precision. Although N_2^+ and CO^+ both have a nominal molecular mass of 28, they are easily distinguished by virtue of their slightly different cyclotron frequencies. Use the following constants: $1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$, $e = 1.6022 \times 10^{-19} \text{ C}$.

Atomic masses

^{12}C	12.000
^{14}N	14.003
^{16}O	15.995

29. **|** Radio astronomers detect electromagnetic radiation at 45 MHz from an interstellar gas cloud. They suspect this radiation is emitted by electrons spiraling in a magnetic field. What is the magnetic field strength inside the gas cloud?
30. **|** For your senior project, you would like to build a cyclotron that will accelerate protons to 10% of the speed of light. The largest vacuum chamber you can find is 50 cm in diameter. What magnetic field strength will you need?
31. **|** The Hall voltage across a conductor in a 55 mT magnetic field is $1.9 \mu\text{V}$. When used with the same current in a different magnetic field, the voltage across the conductor is $2.8 \mu\text{V}$. What is the strength of the second field?
32. **|** Test instruments to measure magnetic field strengths are often based on the Hall effect. In one instrument, the "probe" is a 1.0-mm-thick, 6.0-mm-wide semiconductor with a charge-carrier density of $2.1 \times 10^{21} \text{ m}^{-3}$, much less than the charge-carrier density in a conductor. Passing a 60 mA current through the probe generates a Hall voltage of 120 mV. What is the magnetic field strength?

Section 32.8 Magnetic Forces on Current-Carrying Wires

33. | What magnetic field strength and direction will levitate the 2.0 g wire in **FIGURE EX32.33**?

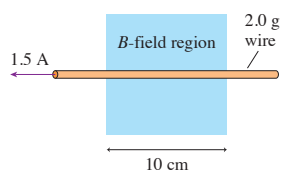
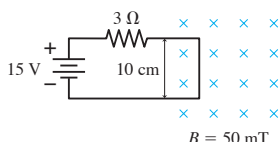


FIGURE EX32.33

34. | The right edge of the circuit in **FIGURE EX32.34** extends into a 50 mT uniform magnetic field. What are the magnitude and direction of the net force on the circuit?

FIGURE EX32.34



35. | The two 10-cm-long parallel wires in **FIGURE EX32.35** are separated by 5.0 mm. For what value of the resistor R will the force between the two wires be 5.4×10^{-5} N?

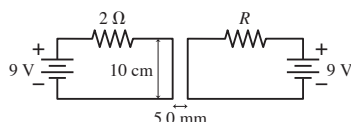
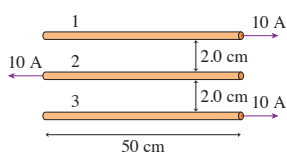


FIGURE EX32.35

36. | What is the net force (magnitude and direction) on each wire in **FIGURE EX32.36**?

FIGURE EX32.36



Section 32.9 Forces and Torques on Current Loops

37. | A square current loop 5.0 cm on each side carries a 500 mA current. The loop is in a 1.2 T uniform magnetic field. The axis of the loop, perpendicular to the plane of the loop, is 30° away from the field direction. What is the magnitude of the torque on the current loop?
38. | A small bar magnet experiences a 0.020 N·m torque when the axis of the magnet is at 45° to a 0.10 T magnetic field. What is the magnitude of its magnetic dipole moment?
39. | a. What is the magnitude of the torque on the current loop in **FIGURE EX32.39**?
b. What is the loop's equilibrium orientation?

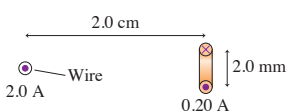


FIGURE EX32.39

Problems

40. | Although the evidence is weak, there has been concern in recent years over possible health effects from the magnetic fields generated by electric transmission lines. A typical high-voltage transmission line is 20 m above the ground and carries a 200 A current at a potential of 110 kV.
- a. What is the magnetic field strength on the ground directly under such a transmission line?
- b. What percentage is this of the earth's magnetic field of $50 \mu\text{T}$?

41. | A biophysics experiment uses a very sensitive magnetic field probe to determine the current associated with a nerve impulse traveling along an axon. If the peak field strength 1.0 mm from an axon is 8.0 pT, what is the peak current carried by the axon?
42. | A long wire carrying a 5.0 A current perpendicular to the xy -plane intersects the x -axis at $x = -2.0$ cm. A second, parallel wire carrying a 3.0 A current intersects the x -axis at $x = +2.0$ cm. At what point or points on the x -axis is the magnetic field zero if (a) the two currents are in the same direction and (b) the two currents are in opposite directions?
43. | The two insulated wires in **FIGURE P32.43** cross at a 30° angle but do not make electrical contact. Each wire carries a 5.0 A current. Points 1 and 2 are each 4.0 cm from the intersection and equally distant from both wires. What are the magnitude and direction of the magnetic fields at points 1 and 2?

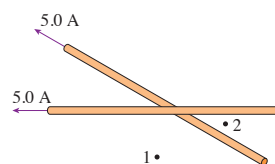


FIGURE P32.43

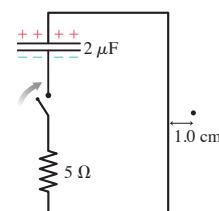


FIGURE P32.44

44. | The capacitor in **FIGURE P32.44** is charged to 50 V. The switch closes at $t = 0$ s. Draw a graph showing the magnetic field strength as a function of time at the position of the dot. On your graph indicate the maximum field strength, and provide an appropriate numerical scale on the horizontal axis.
45. | At what distance on the axis of a current loop is the magnetic field half the strength of the field at the center of the loop? Give your answer as a multiple of R .
46. | Find an expression for the magnetic field strength at the center (point P) of the circular arc in **FIGURE P32.46**.

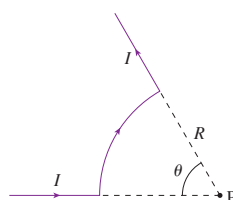


FIGURE P32.46

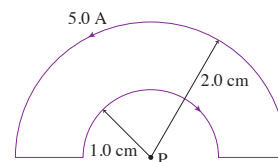


FIGURE P32.47

47. | What are the strength and direction of the magnetic field at point P in **FIGURE P32.47**?
48. | What are the strength and direction of the magnetic field at the center of the loop in **FIGURE P32.48**?

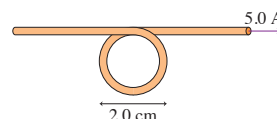


FIGURE P32.48

49. || Your employer asks you to build a 20-cm-long solenoid with an interior field of 5.0 mT. The specifications call for a single layer of wire, wound with the coils as close together as possible. You have two spools of wire available. Wire with a #18 gauge has a diameter of 1.02 mm and has a maximum current rating of 6 A. Wire with a #26 gauge is 0.41 mm in diameter and can carry up to 1 A. Which wire should you use, and what current will you need?
50. || The magnetic field strength at the north pole of a 2.0-cm-diameter, 8-cm-long Alnico magnet is 0.10 T. To produce the same field with a solenoid of the same size, carrying a current of 2.0 A, how many turns of wire would you need? Does this seem feasible?
51. || The earth's magnetic field, with a magnetic dipole moment of $8.0 \times 10^{22} \text{ A m}^2$, is generated by currents within the molten iron of the earth's outer core. Suppose we model the core current as a 3000-km-diameter current loop made from a 1000-km-diameter "wire." The loop diameter is measured from the centers of this very fat wire.
- What is the current in the current loop?
 - What is the current density J in the current loop?
 - To decide whether this is a large or a small current density, compare it to the current density of a 1.0 A current in a 1.0-mm-diameter wire.
52. || Weak magnetic fields can be measured at the surface of the brain. Although the currents causing these fields are quite complicated, we can estimate their size by modeling them as a current loop around the equator of a 16-cm-diameter (the width of a typical head) sphere. What current is needed to produce a 3.0 pT field—the strength measured for one subject—at the pole of this sphere?
53. || The heart produces a weak magnetic field that can be used to diagnose certain heart problems. It is a dipole field produced by a current loop in the outer layers of the heart.
- It is estimated that the field at the center of the heart is 90 pT. What current must circulate around an 8.0-cm-diameter loop, about the size of a human heart, to produce this field?
 - What is the magnitude of the heart's magnetic dipole moment?
54. || Two identical coils are parallel to each other on the same axis. They are separated by a distance equal to their radius. They each have N turns and carry equal currents I in the same direction.
- Find an expression for the magnetic field strength at the midpoint between the loops.
 - Calculate the field strength if the loops are 10 cm in diameter, have 10 turns, and carry a 1.0 A current.
55. || Use the Biot-Savart law to find the magnetic field strength at the center of the semicircle in **FIGURE P32.55**.
56. || The *toroid* of **FIGURE P32.56** is a coil of wire wrapped around a doughnut-shaped ring (a *torus*) made of nonconducting material. Toroidal magnetic fields are used to confine fusion plasmas.
- From symmetry, what must be the *shape* of the magnetic field in this toroid? Explain.
 - Consider a toroid with N closely spaced turns carrying current I . Use Ampère's law to find an expression for the magnetic field strength at a point inside the torus at distance r from the axis.
 - Is a toroidal magnetic field a uniform field? Explain.



FIGURE P32.55

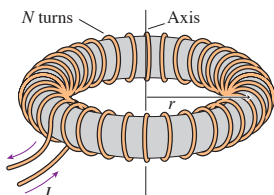


FIGURE P32.56

57. || A long, hollow wire has inner radius R_1 and outer radius R_2 . The wire carries current I uniformly distributed across the area of the wire. Use Ampère's law to find an expression for the magnetic field strength in the three regions $0 < r < R_1$, $R_1 < r < R_2$, and $R_2 < r$.
58. || An electron orbits in a 5.0 mT field with angular momentum $8.0 \times 10^{-26} \text{ kg m}^2/\text{s}$. What is the diameter of the orbit?
59. || A proton moving in a uniform magnetic field with $\vec{v}_1 = 1.00 \times 10^6 \hat{i} \text{ m/s}$ experiences force $\vec{F}_1 = 1.20 \times 10^{-16} \hat{k} \text{ N}$. A second proton with $\vec{v}_2 = 2.00 \times 10^6 \hat{j} \text{ m/s}$ experiences $\vec{F}_2 = -4.16 \times 10^{-16} \hat{k} \text{ N}$ in the same field. What is \vec{B} ? Give your answer as a magnitude and an angle measured ccw from the $+x$ -axis.
60. || An electron travels with speed $1.0 \times 10^7 \text{ m/s}$ between the two parallel charged plates shown in **FIGURE P32.60**. The plates are separated by 1.0 cm and are charged by a 200 V battery. What magnetic field strength and direction will allow the electron to pass between the plates without being deflected?

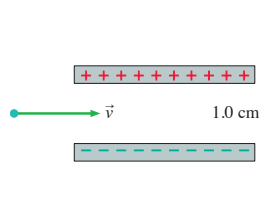


FIGURE P32.60

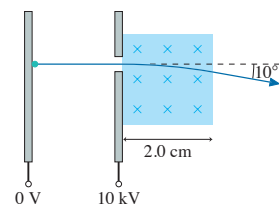


FIGURE P32.61

61. || An electron in a cathode-ray tube is accelerated through a potential difference of 10 kV, then passes through the 2.0-cm-wide region of uniform magnetic field in **FIGURE P32.61**. What field strength will deflect the electron by 10° ?
62. || The microwaves in a microwave oven are produced in a special tube called a *magnetron*. The electrons orbit the magnetic field at 2.4 GHz, and as they do so they emit 2.4 GHz electromagnetic waves.
- What is the magnetic field strength?
 - If the maximum diameter of the electron orbit before the electron hits the wall of the tube is 2.5 cm, what is the maximum electron kinetic energy?
63. || An antiproton (same properties as a proton except that $q = -e$) is moving in the combined electric and magnetic fields of **FIGURE P32.63**. What are the magnitude and direction of the antiproton's acceleration at this instant?

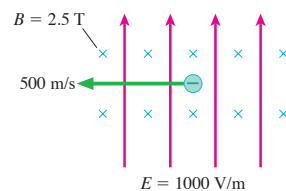


FIGURE P32.63

64. || a. A 65-cm-diameter cyclotron uses a 500 V oscillating potential difference between the dees. What is the maximum kinetic energy of a proton if the magnetic field strength is 0.75 T?
- b. How many revolutions does the proton make before leaving the cyclotron?

65. || **FIGURE P32.65** shows a *mass spectrometer*, an analytical instrument used to identify the various molecules in a sample by measuring their charge-to-mass ratio q/m . The sample is ionized, the positive ions are accelerated (starting from rest) through a potential difference ΔV , and they then enter a region of uniform magnetic field. The field bends the ions into circular trajectories, but after just half a circle they either strike the wall or pass through a small opening to a detector. As the accelerating voltage is slowly increased, different ions reach the detector and are measured. Consider a mass spectrometer with a 200.00 mT magnetic field and an 8.0000 cm spacing between the entrance and exit holes. To five significant figures, what accelerating potential differences ΔV are required to detect the ions (a) O_2^+ , (b) N_2^+ , and (c) CO^+ ? See Exercise 28 for atomic masses; the mass of the missing electron is less than 0.001 u and is not relevant at this level of precision. Although N_2^+ and CO^+ both have a nominal molecular mass of 28, they are easily distinguished by virtue of their slightly different accelerating voltages. Use the following constants: $1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$, $e = 1.6022 \times 10^{-19} \text{ C}$.

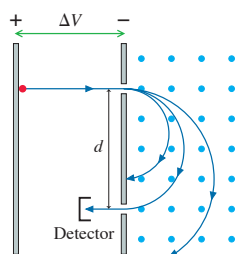


FIGURE P32.65

66. || A Hall-effect probe to measure magnetic field strengths needs to be calibrated in a known magnetic field. Although it is not easy to do, magnetic fields can be precisely measured by measuring the cyclotron frequency of protons. A testing laboratory adjusts a magnetic field until the proton's cyclotron frequency is 10.0 MHz. At this field strength, the Hall voltage on the probe is 0.543 mV when the current through the probe is 0.150 mA. Later, when an unknown magnetic field is measured, the Hall voltage at the same current is 1.735 mV. What is the strength of this magnetic field?
67. || The 10-turn loop of wire shown in **FIGURE P32.67** lies in a horizontal plane, parallel to a uniform horizontal magnetic field, and carries a 2.0 A current. The loop is free to rotate about a nonmagnetic axle through the center. A 50 g mass hangs from one edge of the loop. What magnetic field strength will prevent the loop from rotating about the axle?

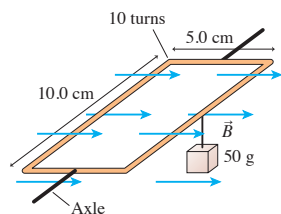


FIGURE P32.67

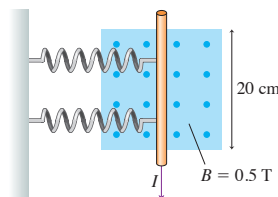


FIGURE P32.68

68. || The two springs in **FIGURE P32.68** each have a spring constant of 10 N/m. They are compressed by 1.0 cm when a current passes through the wire. How big is the current?

69. || Magnetic fields are sometimes measured by balancing magnetic forces against known mechanical forces. Your task is to measure the strength of a horizontal magnetic field using a 12-cm-long rigid metal rod that hangs from two nonmagnetic springs, one at each end, with spring constants 1.3 N/m. You first position the rod to be level and perpendicular to the field, whose direction you determined with a compass. You then connect the ends of the rod to wires that run parallel to the field and thus experience no forces. Finally, you measure the downward deflection of the rod, stretching the springs, as you pass current through it. Your data are as follows:

Current (A)	Deflection (mm)
1.0	4
2.0	9
3.0	12
4.0	15
5.0	21

Use an appropriate graph of the data to determine the magnetic field strength.

70. || A conducting bar of length l and mass m rests at the left end of the two frictionless rails of length d in **FIGURE P32.70**. A uniform magnetic field of strength B points upward.
- In which direction, into or out of the page, will a current through the conducting bar cause the bar to experience a force to the right?
 - Find an expression for the bar's speed as it leaves the rails at the right end.

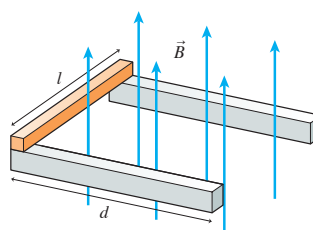


FIGURE P32.70

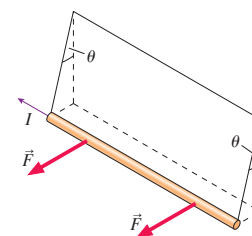


FIGURE P32.71

71. || a. In **FIGURE P32.71**, a long, straight, current-carrying wire of linear mass density μ is suspended by threads. A magnetic field perpendicular to the wire exerts a horizontal force that deflects the wire to an equilibrium angle θ . Find an expression for the strength and direction of the magnetic field \vec{B} .
b. What \vec{B} deflects a 55 g/m wire to a 12° angle when the current is 10 A?
72. || **FIGURE P32.72** is a cross section through three long wires with linear mass density 50 g/m. They each carry equal currents in the directions shown. The lower two wires are 4.0 cm apart and are attached to a table. What current I will allow the upper wire to "float" so as to form an equilateral triangle with the lower wires?

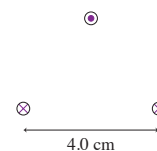


FIGURE P32.72

73. || In the semiclassical Bohr model of the hydrogen atom, the electron moves in a circular orbit of radius $5.3 \times 10^{-11} \text{ m}$ with speed $2.2 \times 10^6 \text{ m/s}$. According to this model, what is the magnetic field at the center of a hydrogen atom?

Hint: Determine the *average* current of the orbiting electron.

74. || A wire along the x -axis carries current I in the negative x -direction through the magnetic field

$$\vec{B} = \begin{cases} B_0 \frac{x}{l} \hat{k} & 0 \leq x \leq l \\ 0 & \text{elsewhere} \end{cases}$$

- Draw a graph of B versus x over the interval $-\frac{3}{2}l < x < \frac{3}{2}l$.
 - Find an expression for the net force \vec{F}_{net} on the wire.
 - Find an expression for the net torque on the wire about the point $x = 0$.
75. || A nonuniform magnetic field exerts a net force on a current loop of radius R . FIGURE P32.75 shows a magnetic field that is diverging from the end of a bar magnet. The magnetic field at the position of the current loop makes an angle θ with respect to the vertical.
- Find an expression for the net magnetic force on the current.
 - Calculate the force if $R = 2.0$ cm, $I = 0.50$ A, $B = 200$ mT, and $\theta = 20^\circ$.

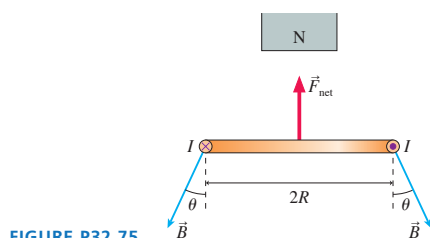


FIGURE P32.75

Challenge Problems

- You have a 1.0-m-long copper wire. You want to make an N -turn current loop that generates a 1.0 mT magnetic field at the center when the current is 1.0 A. You must use the entire wire. What will be the diameter of your coil?
 - Derive an expression for the magnetic field strength at distance d from the center of a straight wire of finite length l that carries current I .
 - Determine the field strength at the center of a current-carrying square loop having sides of length $2R$.
 - Compare your answer to part b to the field at the center of a circular loop of diameter $2R$. Do so by computing the ratio $B_{\text{square}}/B_{\text{circle}}$.
78. A flat, circular disk of radius R is uniformly charged with total charge Q . The disk spins at angular velocity ω about an axis through its center. What is the magnetic field strength at the center of the disk?

79. A long, straight conducting wire of radius R has a nonuniform current density $J = J_0 r/R$, where J_0 is a constant. The wire carries total current I .

- Find an expression for J_0 in terms of I and R .
- Find an expression for the magnetic field strength inside the wire at radius r .
- At the boundary, $r = R$, does your solution match the known field outside a long, straight current-carrying wire?

80. The coaxial cable shown in FIGURE CP32.80 consists of a solid inner conductor of radius R_1 surrounded by a hollow, very thin outer conductor of radius R_2 . The two carry equal currents I , but in opposite directions. The current density is uniformly distributed over each conductor.

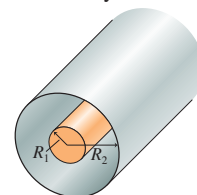


FIGURE CP32.80

- Find expressions for three magnetic fields: within the inner conductor, in the space between the conductors, and outside the outer conductor.
- Draw a graph of B versus r from $r = 0$ to $r = 2R_2$ if $R_1 = \frac{1}{3}R_2$.

81. An infinitely wide flat sheet of charge flows out of the page in FIGURE CP32.81. The current per unit width along the sheet (amps per meter) is given by the linear current density J_s .

- What is the shape of the magnetic field? To answer this question, you may find it helpful to approximate the current sheet as many parallel, closely spaced current-carrying wires. Give your answer as a picture showing magnetic field vectors.
- Find the magnetic field strength at distance d above or below the current sheet.

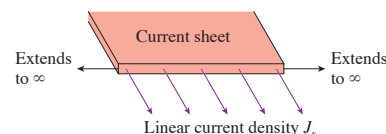


FIGURE CP32.81

82. The uniform 30 mT magnetic field in FIGURE CP32.82 points in the positive z -direction. An electron enters the region of magnetic field with a speed of 5.0×10^6 m/s and at an angle of 30° above the xy -plane. Find the radius r and the pitch p of the electron's spiral trajectory.

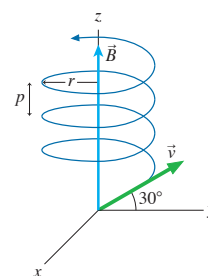


FIGURE CP32.82

STOP TO THINK ANSWERS

Stop to Think 32.1: Not at all. The charge exerts weak, attractive polarization forces on both ends of the compass needle, but in this configuration the forces will balance and have no net effect.

Stop to Think 32.2: d. Point your right thumb in the direction of the current and curl your fingers around the wire.

Stop to Think 32.3: b. Point your right thumb out of the page, in the direction of \vec{v} . Your fingers are pointing down as they curl around the left side.

Stop to Think 32.4: b. The right-hand rule gives a downward \vec{B} for a clockwise current. The north pole is on the side from which the field emerges.

Stop to Think 32.5: c. For a field pointing into the page, $\vec{v} \times \vec{B}$ is to the right. But the electron is negative, so the force is in the direction of $-(\vec{v} \times \vec{B})$.

Stop to Think 32.6: b. Repulsion indicates that the south pole of the loop is on the right, facing the bar magnet; the north pole is on the left. Then the right-hand rule gives the current direction.

Stop to Think 32.7: a or c. Any magnetic field to the right, whether leaving a north pole or entering a south pole, will align the magnetic domains as shown.

SUMMARY

The goal of Chapter 33 has been to understand and apply electromagnetic induction.

General Principles

Faraday's Law

MODEL Make simplifying assumptions.

VISUALIZE Use Lenz's law to determine the direction of the **induced current**.

SOLVE The **induced emf** is

$$\mathcal{E} = \left| \frac{d\Phi_m}{dt} \right|$$

Multiply by N for an N -turn coil.

The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Is the result reasonable?

Lenz's Law

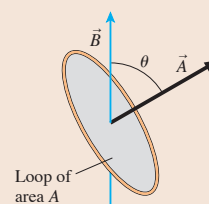
There is an induced current in a closed conducting loop if and only if the magnetic flux through the loop is changing.

The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

Magnetic flux

Magnetic flux measures the amount of magnetic field passing through a surface.

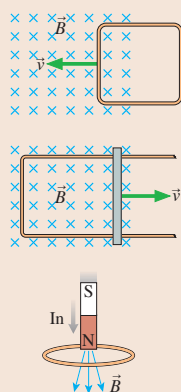
$$\Phi_m = \vec{A} \cdot \vec{B} = AB \cos \theta$$



Important Concepts

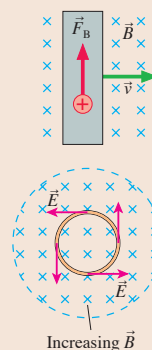
Three ways to change the flux

1. A loop moves into or out of a magnetic field.
2. The loop changes area or rotates.
3. The magnetic field through the loop increases or decreases.



Two ways to create an induced current

1. A **motional emf** is due to magnetic forces on moving charge carriers.
2. An induced electric field is due to a changing magnetic field.



Applications

Inductors



$$\text{Solenoid inductance } L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l}$$

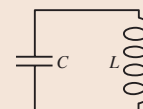
$$\text{Potential difference } \Delta V_L = -L \frac{dI}{dt}$$

$$\text{Energy stored } U_L = \frac{1}{2} LI^2$$

$$\text{Magnetic energy density } u_B = \frac{1}{2\mu_0} B^2$$

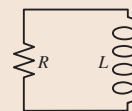
LC circuit

$$\text{Oscillates at } \omega = \sqrt{\frac{1}{LC}}$$



LR circuit

$$\text{Exponential change with } \tau = \frac{L}{R}$$



Terms and Notation

electromagnetic induction
induced current
motional emf
generator
eddy current
magnetic flux, Φ_m
weber, Wb

area vector, \vec{A}
Lenz's law
induced emf, \mathcal{E}
Faraday's law
induced electric field
Coulomb electric field
non-Coulomb electric field

induced magnetic field
electromagnetic wave
primary coil
secondary coil
transformer
inductance, L
henry, H

inductor
 LC circuit
 LR circuit
time constant, τ

CONCEPTUAL QUESTIONS

1. What is the direction of the induced current in **FIGURE Q33.1**?

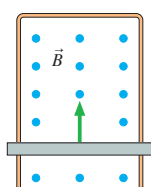


FIGURE Q33.1

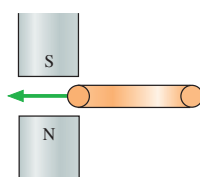


FIGURE Q33.2

2. You want to insert a loop of copper wire between the two permanent magnets in **FIGURE Q33.2**. Is there an attractive magnetic force that tends to *pull* the loop in, like a magnet pulls on a paper clip? Or do you need to *push* the loop in against a repulsive force? Explain.
3. A vertical, rectangular loop of copper wire is half in and half out of the horizontal magnetic field in **FIGURE Q33.3**. (The field is zero beneath the dashed line.) The loop is released and starts to fall. Is there a net magnetic force on the loop? If so, in which direction? Explain.

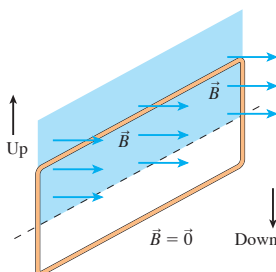


FIGURE Q33.3

4. Does the loop of wire in **FIGURE Q33.4** have a clockwise current, a counterclockwise current, or no current under the following circumstances? Explain.
- The magnetic field points out of the page and is increasing.
 - The magnetic field points out of the page and is constant.
 - The magnetic field points out of the page and is decreasing.

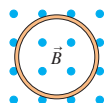


FIGURE Q33.4



FIGURE Q33.5

5. The two loops of wire in **FIGURE Q33.5** are stacked one above the other. Does the upper loop have a clockwise current, a counterclockwise current, or no current at the following times? Explain.
- Before the switch is closed.
 - Immediately after the switch is closed.
 - Long after the switch is closed.
 - Immediately after the switch is reopened.

6. **FIGURE Q33.6** shows a bar magnet being pushed toward a conducting loop from below, along the axis of the loop.
- What is the current direction in the loop? Explain.
 - Is there a magnetic force on the loop? If so, in which direction? Explain.
- Hint:** A current loop is a magnetic dipole.
- Is there a force on the magnet? If so, in which direction?

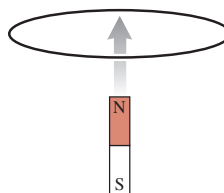


FIGURE Q33.6

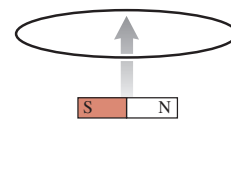


FIGURE Q33.7

7. A bar magnet is pushed toward a loop of wire as shown in **FIGURE Q33.7**. Is there a current in the loop? If so, in which direction? If not, why not?
8. **FIGURE Q33.8** shows a bar magnet, a coil of wire, and a current meter. Is the current through the meter right to left, left to right, or zero for the following circumstances? Explain.
- The magnet is inserted into the coil.
 - The magnet is held at rest inside the coil.
 - The magnet is withdrawn from the left side of the coil.

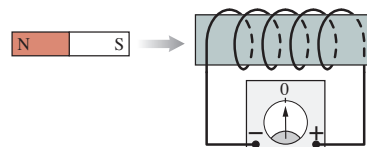


FIGURE Q33.8

9. Is the magnetic field strength in **FIGURE Q33.9** increasing, decreasing, or steady? Explain.

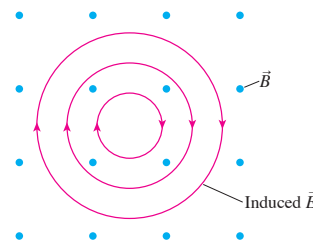


FIGURE Q33.9

10. An inductor with a 2.0 A current stores energy. At what current will the stored energy be twice as large?
11. a. Can you tell which of the inductors in **FIGURE Q33.11** has the larger current through it? If so, which one? Explain.
 b. Can you tell through which inductor the current is changing more rapidly? If so, which one? Explain.
 c. If the current enters the inductor from the bottom, can you tell if the current is increasing, decreasing, or staying the same? If so, which? Explain.

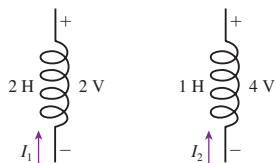


FIGURE Q33.11

12. An LC circuit oscillates at a frequency of 2000 Hz. What will the frequency be if the inductance is quadrupled?

13. Rank in order, from largest to smallest, the three time constants τ_a to τ_c for the three circuits in **FIGURE Q33.13**. Explain.

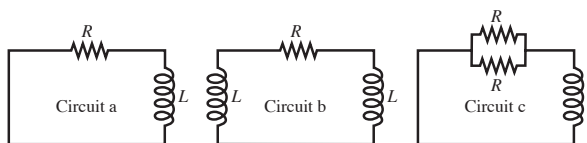


FIGURE Q33.13

14. For the circuit of **FIGURE Q33.14**:
- What is the battery current immediately after the switch closes? Explain.
 - What is the battery current after the switch has been closed a long time? Explain.

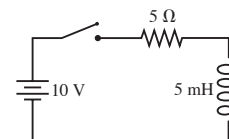


FIGURE Q33.14

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 33.2 Motional emf

- The earth's magnetic field strength is 5.0×10^{-5} T. How fast would you have to drive your car to create a 1.0 V motional emf along your 1.0-m-long radio antenna? Assume that the motion of the antenna is perpendicular to \vec{B} .
- A potential difference of 0.050 V is developed across the 10-cm-long wire of **FIGURE EX33.2** as it moves through a magnetic field perpendicular to the page. What are the strength and direction (in or out) of the magnetic field?
- A 10-cm-long wire is pulled along a U-shaped conducting rail in a perpendicular magnetic field. The total resistance of the wire and rail is 0.20Ω . Pulling the wire at a steady speed of 4.0 m/s causes 4.0 W of power to be dissipated in the circuit.
 - How big is the pulling force?
 - What is the strength of the magnetic field?

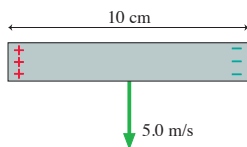


FIGURE EX33.2

Section 33.3 Magnetic Flux

- What is the magnetic flux through the loop shown in **FIGURE EX33.4**?
- FIGURE EX33.5** shows a 2.0-cm-diameter solenoid passing through the center of a 6.0-cm-diameter loop. The magnetic field inside the solenoid is 0.20 T. What is the magnetic flux through the loop when it is perpendicular to the solenoid and when it is tilted at a 60° angle?

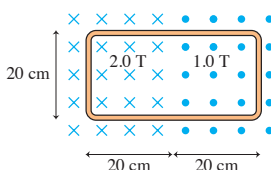


FIGURE EX33.4



FIGURE EX33.5

- What is the magnetic flux through the loop shown in **FIGURE EX33.6**?

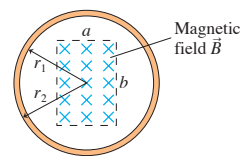


FIGURE EX33.6

Section 33.4 Lenz's Law

- There is a cw induced current in the conducting loop shown in **FIGURE EX33.7**. Is the magnetic field inside the loop increasing in strength, decreasing in strength, or steady?

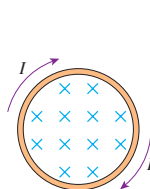


FIGURE EX33.7

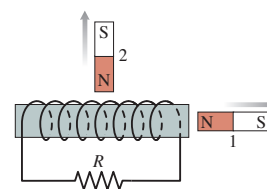


FIGURE EX33.8

- A solenoid is wound as shown in **FIGURE EX33.8**.
 - Is there an induced current as magnet 1 is moved away from the solenoid? If so, what is the current direction through resistor R ?
 - Is there an induced current as magnet 2 is moved away from the solenoid? If so, what is the current direction through resistor R ?

9. || The current in the solenoid of **FIGURE EX33.9** is increasing. The solenoid is surrounded by a conducting loop. Is there a current in the loop? If so, is the loop current cw or ccw?

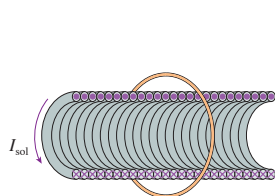


FIGURE EX33.9

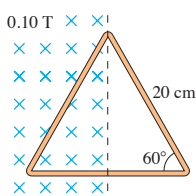


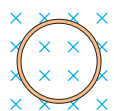
FIGURE EX33.10

10. | The metal equilateral triangle in **FIGURE EX33.10**, 20 cm on each side, is halfway into a 0.10 T magnetic field.
- What is the magnetic flux through the triangle?
 - If the magnetic field strength decreases, what is the direction of the induced current in the triangle?

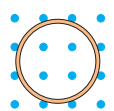
Section 33.5 Faraday's Law

11. | **FIGURE EX33.11** shows a 10-cm-diameter loop in three different magnetic fields. The loop's resistance is $0.20\ \Omega$. For each, what are the size and direction of the induced current?

(a) B increasing at $0.50\ \text{T/s}$



(b) B decreasing at $0.50\ \text{T/s}$



(c) B decreasing at $0.50\ \text{T/s}$

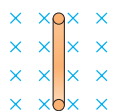


FIGURE EX33.11

12. | The loop in **FIGURE EX33.12** is being pushed into the $0.20\ \text{T}$ magnetic field at $50\ \text{m/s}$. The resistance of the loop is $0.10\ \Omega$. What are the direction and the magnitude of the current in the loop?

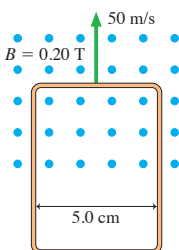


FIGURE EX33.12

13. || A 1000-turn coil of wire 1.0 cm in diameter is in a magnetic field that increases from $0.10\ \text{T}$ to $0.30\ \text{T}$ in $10\ \text{ms}$. The axis of the coil is parallel to the field. What is the emf of the coil?
14. | The resistance of the loop in **FIGURE EX33.14** is $0.20\ \Omega$. Is the magnetic field strength increasing or decreasing? At what rate (T/s)?

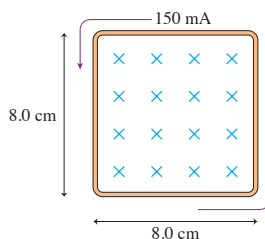


FIGURE EX33.14

Section 33.6 Induced Fields

15. || **FIGURE EX33.15** shows the current as a function of time through a 20-cm-long, 4.0-cm-diameter solenoid with 400 turns. Draw a graph of the induced electric field strength as a function of time at a point 1.0 cm from the axis of the solenoid.

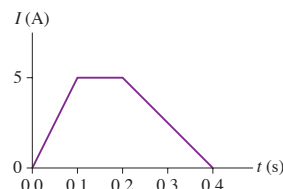


FIGURE EX33.15

16. || The magnetic field inside a 5.0-cm-diameter solenoid is $2.0\ \text{T}$ and decreasing at $4.0\ \text{T/s}$. What is the electric field strength inside the solenoid at a point (a) on the axis and (b) 2.0 cm from the axis?
17. || The magnetic field in **FIGURE EX33.17** is decreasing at the rate $0.10\ \text{T/s}$. What is the acceleration (magnitude and direction) of a proton initially at rest at points a to d?

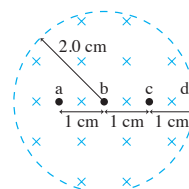


FIGURE EX33.17

Section 33.8 Inductors

18. | What is the potential difference across a $10\ \text{mH}$ inductor if the current through the inductor drops from $150\ \text{mA}$ to $50\ \text{mA}$ in $10\ \mu\text{s}$? What is the direction of this potential difference? That is, does the potential increase or decrease along the direction of the current?
19. | The maximum allowable potential difference across a $200\ \text{mH}$ inductor is $400\ \text{V}$. You need to raise the current through the inductor from $1.0\ \text{A}$ to $3.0\ \text{A}$. What is the minimum time you should allow for changing the current?
20. | A $100\ \text{mH}$ inductor whose windings have a resistance of $4.0\ \Omega$ is connected across a $12\ \text{V}$ battery having an internal resistance of $2.0\ \Omega$. How much energy is stored in the inductor?
21. || How much energy is stored in a 3.0-cm-diameter, 12-cm-long solenoid that has 200 turns of wire and carries a current of $0.80\ \text{A}$?

Section 33.9 LC Circuits

22. || An FM radio station broadcasts at a frequency of $100\ \text{MHz}$. What inductance should be paired with a $10\ \text{pF}$ capacitor to build a receiver circuit for this station?
23. | A $2.0\ \text{mH}$ inductor is connected in parallel with a variable capacitor. The capacitor can be varied from $100\ \text{pF}$ to $200\ \text{pF}$. What is the range of oscillation frequencies for this circuit?
24. | An MRI machine needs to detect signals that oscillate at very high frequencies. It does so with an LC circuit containing a $15\ \text{mH}$ coil. To what value should the capacitance be set to detect a $450\ \text{MHz}$ signal?

Section 33.10 LR Circuits

25. || What value of resistor R gives the circuit in FIGURE EX33.25 a time constant of $25\ \mu\text{s}$?

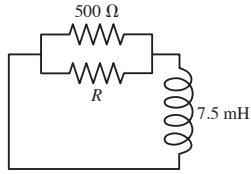


FIGURE EX33.25

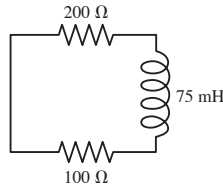


FIGURE EX33.26

26. || At $t = 0$ s, the current in the circuit in FIGURE EX33.26 is I_0 . At what time is the current $\frac{1}{2}I_0$?

Problems

27. || FIGURE P33.27 shows a $10\text{ cm} \times 10\text{ cm}$ square bent at a 90° angle. A uniform 0.050 T magnetic field points downward at a 45° angle. What is the magnetic flux through the loop?

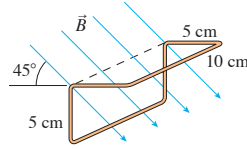


FIGURE P33.27

28. || A 5.0-cm -diameter coil has 20 turns and a resistance of $0.50\ \Omega$. A magnetic field perpendicular to the coil is $B = 0.020t + 0.010t^2$, where B is in tesla and t is in seconds.
- Find an expression for the induced current $I(t)$ as a function of time.
 - Evaluate I at $t = 5\text{ s}$ and $t = 10\text{ s}$.
29. || A $20\text{ cm} \times 20\text{ cm}$ square loop has a resistance of $0.10\ \Omega$. A magnetic field perpendicular to the loop is $B = 4t - 2t^2$, where B is in tesla and t is in seconds. What is the current in the loop at $t = 0.0\text{ s}$, $t = 1.0\text{ s}$, and $t = 2.0\text{ s}$?
30. || A 100-turn, 2.0-cm -diameter coil is at rest in a horizontal plane. A uniform magnetic field 60° away from vertical increases from 0.50 T to 1.50 T in 0.60 s . What is the induced emf in the coil?
31. || A 100-turn, 8.0-cm -diameter coil is made of 0.50-mm -diameter copper wire. A magnetic field is parallel to the axis of the coil. At what rate must B increase to induce a 2.0 A current in the coil?
32. || A circular loop made from a flexible, conducting wire is shrinking. Its radius as a function of time is $r = r_0 e^{-\beta t}$. The loop is perpendicular to a steady, uniform magnetic field B . Find an expression for the induced emf in the loop at time t .
33. || A $10\text{ cm} \times 10\text{ cm}$ square loop lies in the xy -plane. The magnetic field in this region of space is $B = (0.30t\hat{i} + 0.50t^2\hat{k})\text{ T}$, where t is in s. What is the emf induced in the loop at (a) $t = 0.5\text{ s}$ and (b) $t = 1.0\text{ s}$?
34. || A $20\text{ cm} \times 20\text{ cm}$ square loop of wire lies in the xy -plane with its bottom edge on the x -axis. The resistance of the loop is $0.50\ \Omega$. A magnetic field parallel to the z -axis is given by $B = 0.80y^2t$, where B is in tesla, y in meters, and t in seconds. What is the size of the induced current in the loop at $t = 0.50\text{ s}$?
35. || A $2.0\text{ cm} \times 2.0\text{ cm}$ square loop of wire with resistance $0.010\ \Omega$ has one edge parallel to a long straight wire. The near edge of the loop is 1.0 cm from the wire. The current in the wire is increasing at the rate of 100 A/s . What is the current in the loop?

36. || The rectangular loop in FIGURE P33.36 has $0.020\ \Omega$ resistance. What is the induced current in the loop at this instant?

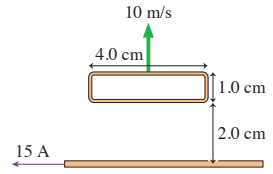


FIGURE P33.36

37. || FIGURE P33.37 shows a 4.0-cm -diameter loop with resistance $0.10\ \Omega$ around a 2.0-cm -diameter solenoid. The solenoid is 10 cm long, has 100 turns, and carries the current shown in the graph. A positive current is cw when seen from the left. Find the current in the loop at (a) $t = 0.5\text{ s}$, (b) $t = 1.5\text{ s}$, and (c) $t = 2.5\text{ s}$.

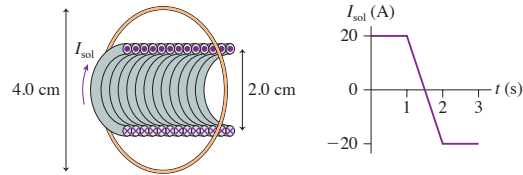


FIGURE P33.37

38. || FIGURE P33.38 shows a 1.0-cm -diameter loop with $R = 0.50\ \Omega$ inside a 2.0-cm -diameter solenoid. The solenoid is 8.0 cm long, has 120 turns, and carries the current shown in the graph. A positive current is cw when seen from the left. Determine the current in the loop at $t = 0.010\text{ s}$.

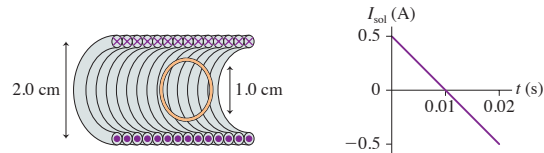


FIGURE P33.38

39. || FIGURE P33.39 shows two 20-turn coils tightly wrapped on the same 2.0-cm -diameter cylinder with 1.0-mm -diameter wire. The current through coil 1 is shown in the graph. Determine the current in coil 2 at (a) $t = 0.05\text{ s}$ and (b) $t = 0.25\text{ s}$. A positive current is into the page at the top of a loop. Assume that the magnetic field of coil 1 passes entirely through coil 2.

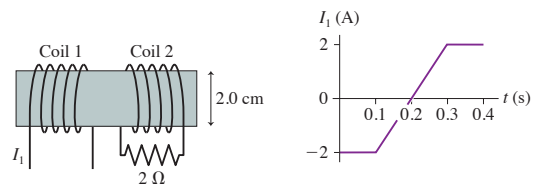
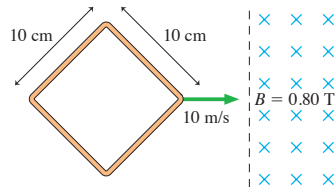


FIGURE P33.39

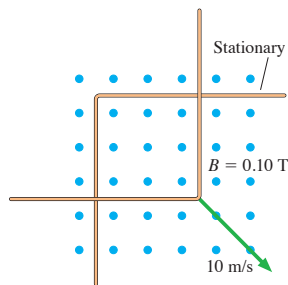
40. || A 50-turn, 4.0-cm -diameter coil with $R = 0.50\ \Omega$ surrounds a 2.0-cm -diameter solenoid. The solenoid is 20 cm long and has 200 turns. The 60 Hz current through the solenoid is $I_{\text{sol}} = (0.50\text{ A})\sin(2\pi ft)$. Find an expression for I_{coil} , the induced current in the coil as a function of time.
41. || A loop antenna, such as is used on older televisions to pick up UHF broadcasts, is 25 cm in diameter. The plane of the loop is perpendicular to the oscillating magnetic field of a 150 MHz electromagnetic wave. The magnetic field through the loop is $B = (20\text{ nT})\sin\omega t$.

- a. What is the maximum emf induced in the antenna?
 b. What is the maximum emf if the loop is turned 90° to be perpendicular to the oscillating electric field?
42. || A 40-turn, 4.0-cm-diameter coil with $R = 0.40\ \Omega$ surrounds a 3.0-cm-diameter solenoid. The solenoid is 20 cm long and has 200 turns. The 60 Hz current through the solenoid is $I = I_0 \sin(2\pi ft)$. What is I_0 if the maximum induced current in the coil is 0.20 A?
43. | Electricity is distributed from electrical substations to neighborhoods at 15,000 V. This is a 60 Hz oscillating (AC) voltage. Neighborhood transformers, seen on utility poles, step this voltage down to the 120 V that is delivered to your house.
- a. How many turns does the primary coil on the transformer have if the secondary coil has 100 turns?
 b. No energy is lost in an ideal transformer, so the output power P_{out} from the secondary coil equals the input power P_{in} to the primary coil. Suppose a neighborhood transformer delivers 250 A at 120 V. What is the current in the 15,000 V line from the substation?
44. || A small, 2.0-mm-diameter circular loop with $R = 0.020\ \Omega$ is at the center of a large 100-mm-diameter circular loop. Both loops lie in the same plane. The current in the outer loop changes from +1.0 A to -1.0 A in 0.10 s. What is the induced current in the inner loop?
45. || The square loop shown in **FIGURE P33.45** moves into a 0.80 T magnetic field at a constant speed of 10 m/s. The loop has a resistance of $0.10\ \Omega$, and it enters the field at $t = 0$ s.
- a. Find the induced current in the loop as a function of time. Give your answer as a graph of I versus t from $t = 0$ s to $t = 0.020$ s.
 b. What is the maximum current? What is the position of the loop when the current is maximum?

FIGURE P33.45



46. || The L-shaped conductor in **FIGURE P33.46** moves at 10 m/s across a stationary L-shaped conductor in a 0.10 T magnetic field. The two vertices overlap, so that the enclosed area is zero, at $t = 0$ s. The conductor has a resistance of 0.010 ohms *per meter*.
- a. What is the direction of the induced current?
 b. Find expressions for the induced emf and the induced current as functions of time.
 c. Evaluate \mathcal{E} and I at $t = 0.10$ s.
47. || A 4.0-cm-long slide wire moves outward with a speed of 100 m/s in a 1.0 T magnetic field. (See Figure 33.26.) At the instant the circuit forms a 4.0 cm \times 4.0 cm square, with $R = 0.010\ \Omega$ on each side, what are
- a. The induced emf?
 b. The induced current?
 c. The potential difference between the two ends of the moving wire?



48. || A 20-cm-long, zero-resistance slide wire moves outward, on zero-resistance rails, at a steady speed of 10 m/s in a 0.10 T magnetic field. (See Figure 33.26.) On the opposite side, a $1.0\ \Omega$ carbon resistor completes the circuit by connecting the two rails. The mass of the resistor is 50 mg.
- a. What is the induced current in the circuit?
 b. How much force is needed to pull the wire at this speed?
 c. If the wire is pulled for 10 s, what is the temperature increase of the carbon? The specific heat of carbon is 710 J/kg K.
49. || Your camping buddy has an idea for a light to go inside your tent. He happens to have a powerful (and heavy!) horseshoe magnet that he bought at a surplus store. This magnet creates a 0.20 T field between two pole tips 10 cm apart. His idea is to build the hand-cranked generator shown in **FIGURE P33.49**. He thinks you can make enough current to fully light a $1.0\ \Omega$ lightbulb rated at 4.0 W. That's not super bright, but it should be plenty of light for routine activities in the tent.
- a. Find an expression for the induced current as a function of time if you turn the crank at frequency f . Assume that the semicircle is at its highest point at $t = 0$ s.
 b. With what frequency will you have to turn the crank for the maximum current to fully light the bulb? Is this feasible?

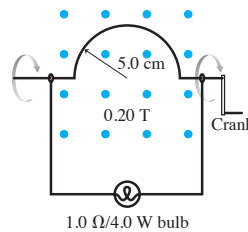


FIGURE P33.49

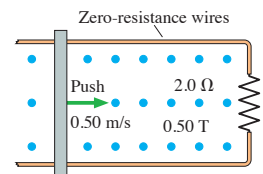


FIGURE P33.50

50. | The 10-cm-wide, zero-resistance slide wire shown in **FIGURE P33.50** is pushed toward the $2.0\ \Omega$ resistor at a steady speed of 0.50 m/s. The magnetic field strength is 0.50 T.
- a. How big is the pushing force?
 b. How much power does the pushing force supply to the wire?
 c. What are the direction and magnitude of the induced current?
 d. How much power is dissipated in the resistor?
51. || One way to determine a magnetic field strength is to measure the emf induced in a rotating coil. To calibrate a large magnet in your laboratory, you attach a 2.0-cm-diameter, 100-turn coil to the end of a motor-driven shaft, place the coil between the pole tips of the magnet, and rotate it at different frequencies. The emf oscillates, so you use a voltmeter that measures its amplitude. The table shows your data:

Frequency (Hz)	Voltage (mV)
10	380
15	610
20	780
25	1020
30	1160

Use an appropriate graph of the data to determine the magnetic field strength.

52. || You've decided to make the magnetic projectile launcher shown in **FIGURE P33.52** for your science project. An aluminum

bar of length l slides along metal rails through a magnetic field B . The switch closes at $t = 0$ s, while the bar is at rest, and a battery of emf \mathcal{E}_{bat} starts a current flowing around the loop. The battery has internal resistance r . The resistance of the rails and the bar are effectively zero.

- Show that the bar reaches a terminal speed v_{term} , and find an expression for v_{term} .
- Evaluate v_{term} for $\mathcal{E}_{\text{bat}} = 1.0$ V, $r = 0.10$ Ω , $l = 6.0$ cm, and $B = 0.50$ T.

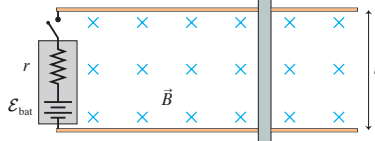


FIGURE P33.52

- A slide wire of length l , mass m , and resistance R slides down a U-shaped metal track that is tilted upward at angle θ . The track has zero resistance and no friction. A vertical magnetic field B fills the loop formed by the track and the slide wire.

 - Find an expression for the induced current I when the slide wire moves at speed v .
 - Show that the slide wire reaches a terminal speed v_{term} , and find an expression for v_{term} .
- FIGURE P33.54 shows a U-shaped conducting rail that is oriented vertically in a horizontal magnetic field. The rail has no electric resistance and does not move. A slide wire with mass m and resistance R can slide up and down without friction while maintaining electrical contact with the rail. The slide wire is released from rest.

 - Show that the slide wire reaches a terminal speed v_{term} , and find an expression for v_{term} .
 - Determine the value of v_{term} if $l = 20$ cm, $m = 10$ g, $R = 0.10$ Ω , and $B = 0.50$ T.

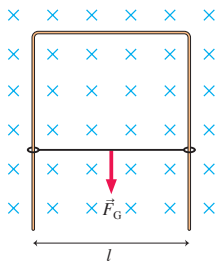


FIGURE P33.54

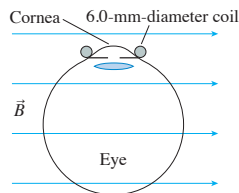


FIGURE P33.55

- Experiments to study vision often need to track the movements of a subject's eye. One way of doing so is to have the subject sit in a magnetic field while wearing special contact lenses with a coil of very fine wire circling the edge. A current is induced in the coil each time the subject rotates his eye. Consider the experiment of FIGURE P33.55 in which a 20-turn, 6.0-mm-diameter coil of wire circles the subject's cornea while a 1.0 T magnetic field is directed as shown. The subject begins by looking straight ahead. What emf is induced in the coil if the subject shifts his gaze by 5° in 0.20 s?
- A 10-turn coil of wire having a diameter of 1.0 cm and a resistance of 0.20 Ω is in a 1.0 mT magnetic field, with the coil oriented for maximum flux. The coil is connected to an uncharged 1.0 μF capacitor rather than to a current meter. The coil is quickly pulled out of the magnetic field. Afterward, what is the voltage across the capacitor?

Hint: Use $I = dq/dt$ to relate the net change of flux to the amount of charge that flows to the capacitor.

- The magnetic field at one place on the earth's surface is 55 μT in strength and tilted 60° down from horizontal. A 200-turn coil having a diameter of 4.0 cm and a resistance of 2.0 Ω is connected to a 1.0 μF capacitor rather than to a current meter. The coil is held in a horizontal plane and the capacitor is discharged. Then the coil is quickly rotated 180° so that the side that had been facing up is now facing down. Afterward, what is the voltage across the capacitor? See the Hint in Problem 56.
- The magnetic field inside a 4.0-cm-diameter superconducting solenoid varies sinusoidally between 8.0 T and 12.0 T at a frequency of 10 Hz.

 - What is the maximum electric field strength at a point 1.5 cm from the solenoid axis?
 - What is the value of B at the instant E reaches its maximum value?
- Equation 33.26 is an expression for the induced electric field inside a solenoid ($r < R$). Find an expression for the induced electric field outside a solenoid ($r > R$) in which the magnetic field is changing at the rate dB/dt .
- A solenoid inductor has an emf of 0.20 V when the current through it changes at the rate 10.0 A/s. A steady current of 0.10 A produces a flux of 5.0 μWb per turn. How many turns does the inductor have?
- What is the magnetic energy density at the center of a 4.0-cm-diameter loop carrying a current of 1.0 A?
 - What current in a straight wire gives the magnetic energy density you found in part a at a point 2.0 cm from the wire?
- MRI (magnetic resonance imaging) is a medical technique that produces detailed "pictures" of the interior of the body. The patient is placed into a solenoid that is 40 cm in diameter and 1.0 m long. A 100 A current creates a 5.0 T magnetic field inside the solenoid. To carry such a large current, the solenoid wires are cooled with liquid helium until they become superconducting (no electric resistance).

 - How much magnetic energy is stored in the solenoid? Assume that the magnetic field is uniform within the solenoid and quickly drops to zero outside the solenoid.
 - How many turns of wire does the solenoid have?
- One possible concern with MRI (see Problem 62) is turning the magnetic field on or off too quickly. Bodily fluids are conductors, and a changing magnetic field could cause electric currents to flow through the patient. Suppose a typical patient has a maximum cross-section area of 0.060 m^2 . What is the smallest time interval in which a 5.0 T magnetic field can be turned on or off if the induced emf around the patient's body must be kept to less than 0.10 V?
- FIGURE P33.64 shows the current through a 10 mH inductor. Draw a graph showing the potential difference ΔV_L across the inductor for these 6 ms.

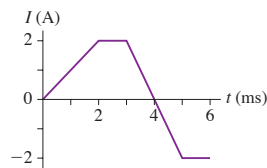


FIGURE P33.64

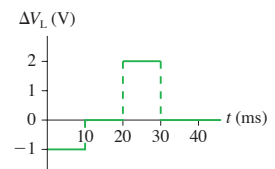


FIGURE P33.65

- FIGURE P33.65 shows the potential difference across a 50 mH inductor. The current through the inductor at $t = 0$ s is 0.20 A.

Draw a graph showing the current through the inductor from $t = 0$ s to $t = 40$ ms.

66. || The current through inductance L is given by $I = I_0 \sin \omega t$.
- Find an expression for the potential difference ΔV_L across the inductor.
 - The maximum voltage across the inductor is 0.20 V when $L = 50 \mu\text{H}$ and $f = 500$ kHz. What is I_0 ?
67. || The current through inductance L is given by $I = I_0 e^{-t/\tau}$.
- Find an expression for the potential difference ΔV_L across the inductor.
 - Evaluate ΔV_L at $t = 0, 1, 2$, and 3 ms if $L = 20$ mH, $I_0 = 50$ mA, and $\tau = 1.0$ ms.
68. || An LC circuit is built with a 20 mH inductor and an 8.0 pF capacitor. The capacitor voltage has its maximum value of 25 V at $t = 0$ s.
- How long is it until the capacitor is first fully discharged?
 - What is the inductor current at that time?
69. || An LC circuit has a 10 mH inductor. The current has its maximum value of 0.60 A at $t = 0$ s. A short time later the capacitor reaches its maximum potential difference of 60 V. What is the value of the capacitance?
70. || An electric oscillator is made with a 0.10 μF capacitor and a 1.0 mH inductor. The capacitor is initially charged to 5.0 V. What is the maximum current through the inductor as the circuit oscillates?
71. || In recent years it has been possible to buy a 1.0 F capacitor. This is an enormously large amount of capacitance. Suppose you want to build a 1.0 Hz oscillator with a 1.0 F capacitor. You have a spool of 0.25-mm-diameter wire and a 4.0-cm-diameter plastic cylinder. How long must your inductor be if you wrap it with 2 layers of closely spaced turns?
72. || For your final exam in electronics, you're asked to build an LC circuit that oscillates at 10 kHz. In addition, the maximum current must be 0.10 A and the maximum energy stored in the capacitor must be 1.0×10^{-5} J. What values of inductance and capacitance must you use?
73. || The switch in **FIGURE P33.73** has been in position 1 for a long time. It is changed to position 2 at $t = 0$ s.
- What is the maximum current through the inductor?
 - What is the first time at which the current is maximum?

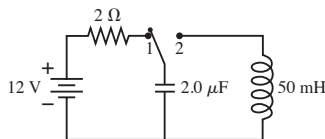


FIGURE P33.73

74. || The 300 μF capacitor in **FIGURE P33.74** is initially charged to 100 V, the 1200 μF capacitor is uncharged, and the switches are both open.
- What is the maximum voltage to which you can charge the 1200 μF capacitor by the proper closing and opening of the two switches?
 - How would you do it? Describe the sequence in which you would close and open switches and the times at which you would do so. The first switch is closed at $t = 0$ s.

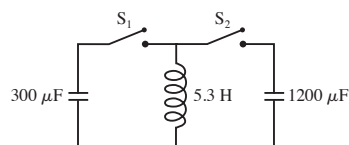


FIGURE P33.74

75. | The switch in **FIGURE P33.75** has been open for a long time. It is closed at $t = 0$ s.
- What is the current through the battery immediately after the switch is closed?
 - What is the current through the battery after the switch has been closed a long time?

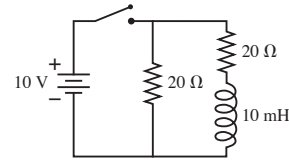


FIGURE P33.75

76. || The switch in **FIGURE P33.76** has been open for a long time. It is closed at $t = 0$ s. What is the current through the 20 Ω resistor
- immediately after the switch is closed?
 - after the switch has been closed a long time?
 - immediately after the switch is reopened?

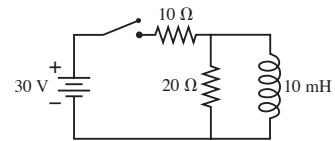


FIGURE P33.76

77. || The switch in **FIGURE P33.77** has been open for a long time. It is closed at $t = 0$ s.
- After the switch has been closed for a long time, what is the current in the circuit? Call this current I_0 .
 - Find an expression for the current I as a function of time. Write your expression in terms of I_0 , R , and L .
 - Sketch a current-versus-time graph from $t = 0$ s until the current is no longer changing.

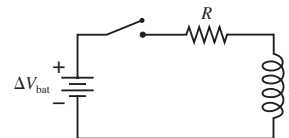


FIGURE P33.77

78. || To determine the inductance of an unmarked inductor, you set up the circuit shown in **FIGURE P33.78**. After moving the switch from a to b at $t = 0$ s, you monitor the resistor voltage with an oscilloscope. Your data are as follows:

Time (μs)	Voltage (V)
0	9.0
10	6.7
20	4.6
30	3.2
40	2.5

Use an appropriate graph of the data to determine the inductance.

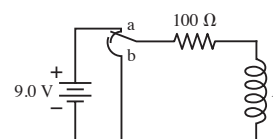


FIGURE P33.78

Challenge Problems

79. The metal wire in **FIGURE CP33.79** moves with speed v parallel to a straight wire that is carrying current I . The distance between the two wires is d . Find an expression for the potential difference between the two ends of the moving wire.

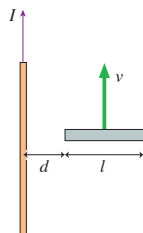


FIGURE CP33.79

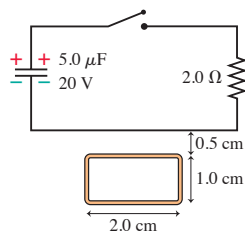


FIGURE CP33.80

80. A rectangular metal loop with $0.050\ \Omega$ resistance is placed next to one wire of the RC circuit shown in **FIGURE CP33.80**. The capacitor is charged to 20 V with the polarity shown, then the switch is closed at $t = 0$ s.

- What is the direction of current in the loop for $t > 0$ s?
- What is the current in the loop at $t = 5.0\ \mu\text{s}$? Assume that only the circuit wire next to the loop is close enough to produce a significant magnetic field.

81. A closed, square loop is formed with 40 cm of wire having $R = 0.10\ \Omega$, as shown in **FIGURE CP33.81**. A 0.50 T magnetic field is perpendicular to the loop. At $t = 0$ s, two diagonally opposite corners of the loop begin to move apart at 0.293 m/s.

- How long does it take the loop to collapse to a straight line?
- Find an expression for the induced current I as a function of time while the loop is collapsing. Assume that the sides remain straight lines during the collapse.
- Evaluate I at four or five times during the collapse, then draw a graph of I versus t .

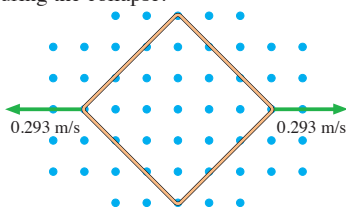


FIGURE CP33.81

82. Let's look at the details of eddy-current braking. A square loop, length l on each side, is shot with velocity v_0 into a uniform magnetic field B . The field is perpendicular to the plane of the loop. The loop has mass m and resistance R , and it enters the field at $t = 0$ s. Assume that the loop is moving to the right along the x -axis and that the field begins at $x = 0$ m.

- Find an expression for the loop's velocity as a function of time as it enters the magnetic field. You can ignore gravity, and you can assume that the back edge of the loop has not entered the field.
- Calculate and draw a graph of v over the interval $0 \leq t \leq 0.04$ s for the case that $v_0 = 10$ m/s, $l = 10$ cm, $m = 1.0$ g, $R = 0.0010\ \Omega$, and $B = 0.10$ T. The back edge of the loop does not reach the field during this time interval.

83. An $8.0\text{ cm} \times 8.0\text{ cm}$ square loop is halfway into a magnetic field perpendicular to the plane of the loop. The loop's mass is 10 g and its resistance is $0.010\ \Omega$. A switch is closed at $t = 0$ s, causing the magnetic field to increase from 0 to 1.0 T in 0.010 s.

- What is the induced current in the square loop?
- With what speed is the loop "kicked" away from the magnetic field?

Hint: What is the impulse on the loop?

84. A 2.0-cm-diameter solenoid is wrapped with 1000 turns per meter. 0.50 cm from the axis, the strength of an induced electric field is 5.0×10^{-4} V/m. What is the rate dI/dt with which the current through the solenoid is changing?

85. High-frequency signals are often transmitted along a *coaxial cable*, such as the one shown in **FIGURE CP33.85**. For example, the cable TV hookup coming into your home is a coaxial cable. The signal is carried on a wire of radius r_1 while the outer conductor of radius r_2 is grounded. A soft, flexible insulating material fills the space between them, and an insulating plastic coating goes around the outside.

- Find an expression for the inductance per meter of a coaxial cable. To do so, consider the flux through a rectangle of length l that spans the gap between the inner and outer conductors.
- Evaluate the inductance per meter of a cable having $r_1 = 0.50$ mm and $r_2 = 3.0$ mm.

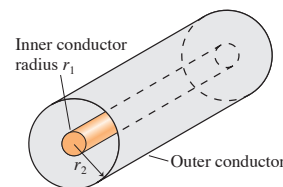


FIGURE CP33.85

Stop to Think 33.1: d. According to the right-hand rule, the magnetic force on a positive charge carrier is to the right.

Stop to Think 33.2: No. The charge carriers in the wire move parallel to \vec{B} . There's no magnetic force on a charge moving parallel to a magnetic field.

Stop to Think 33.3: $F_b = F_d > F_a = F_c$. \vec{F}_a is zero because there's no field. \vec{F}_c is also zero because there's no current around the loop. The charge carriers in both the right and left edges are pushed to the bottom of the loop, creating a motional emf but no current. The currents at b and d are in opposite directions, but the forces on the segments in the field are both to the left and of equal magnitude.

Stop to Think 33.4: Clockwise. The wire's magnetic field as it passes through the loop is into the page. The flux through the loop decreases

into the page as the wire moves away. To oppose this decrease, the induced magnetic field needs to point into the page.

Stop to Think 33.5: d. The flux is increasing into the loop. To oppose this increase, the induced magnetic field needs to point out of the page. This requires a ccw induced current. Using the right-hand rule, the magnetic force on the current in the left edge of the loop is to the right, away from the field. The magnetic forces on the top and bottom segments of the loop are in opposite directions and cancel each other.

Stop to Think 33.6: b or f. The potential decreases in the direction of increasing current and increases in the direction of decreasing current.

Stop to Think 33.7: $\tau_c > \tau_a > \tau_b$. $\tau = L/R$, so smaller total resistance gives a larger time constant. The parallel resistors have total resistance $R/2$. The series resistors have total resistance $2R$.

STOP TO THINK ANSWERS

SUMMARY

The goal of Chapter 34 has been to study the properties of electromagnetic fields and waves.

General Principles

Maxwell's Equations

These equations govern electromagnetic fields:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \quad \text{Gauss's law}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's law for magnetism}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \quad \text{Faraday's law}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad \text{Ampère-Maxwell law}$$

Maxwell's equations tell us that:

An electric field can be created by

- Charged particles
- A changing magnetic field

A magnetic field can be created by

- A current
- A changing electric field

Lorentz Force

This force law governs the interaction of charged particles with electromagnetic fields:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

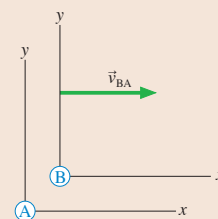
- An electric field exerts a force on any charged particle.
- A magnetic field exerts a force on a moving charged particle.

Field Transformations

Fields measured in reference frame A to be \vec{E}_A and \vec{B}_A are found in frame B to be

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A$$

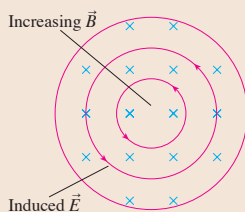
$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A$$



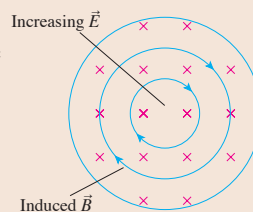
Important Concepts

Induced fields

An induced electric field is created by a changing magnetic field.



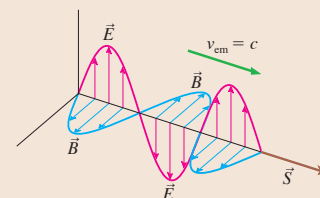
An induced magnetic field is created by a changing electric field.



These fields can exist independently of charges and currents.

An **electromagnetic wave** is a self-sustaining electromagnetic field.

- An em wave is a transverse wave with \vec{E} , \vec{B} , and \vec{v}_{em} mutually perpendicular.
- An em wave propagates with speed $v_{\text{em}} = c = 1/\sqrt{\epsilon_0 \mu_0}$.
- The electric and magnetic field strengths are related by $E = cB$.
- The **Poynting vector** $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$ is the energy transfer in the direction of travel.
- The wave **intensity** is $I = P/A = (1/2c\mu_0)E_0^2 = (c\epsilon_0/2)E_0^2$.



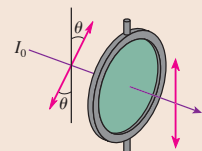
Applications

Polarization

The electric field and the Poynting vector define the **plane of polarization**. The intensity of polarized light transmitted through a polarizing filter is given by Malus's law:

$$I = I_0 \cos^2 \theta$$

where θ is the angle between the electric field and the polarizer axis.



Terms and Notation

Galilean field transformation equations
displacement current
Maxwell's equations

Poynting vector, \vec{S}
intensity, I
radiation pressure, p_{rad}

antenna
plane of polarization
Malus's law

CONCEPTUAL QUESTIONS

- Andre is flying his spaceship to the left through the laboratory magnetic field of **FIGURE Q34.1**.
 - Does Andre see a magnetic field? If so, in which direction does it point?
 - Does Andre see an electric field? If so, in which direction does it point?

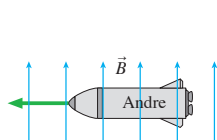


FIGURE Q34.1

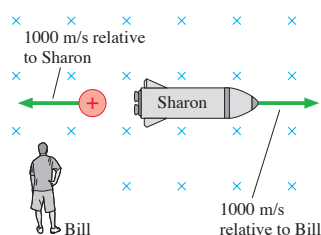


FIGURE Q34.2

- Sharon drives her rocket through the magnetic field of **FIGURE Q34.2** traveling to the right at a speed of 1000 m/s as measured by Bill. As she passes Bill, she shoots a positive charge backward at a speed of 1000 m/s relative to her.
 - According to Sharon, what kind of force or forces act on the charge? In which directions? Explain.
 - According to Bill, what kind of force or forces act on the charge? In which directions? Explain.
- If you curl the fingers of your right hand as shown, are the electric fluxes in **FIGURE Q34.3** positive or negative?

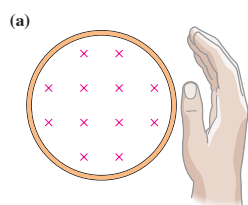
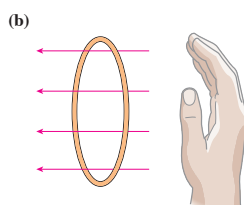


FIGURE Q34.3



- What is the current through surface S in **FIGURE Q34.4** if you curl your right fingers in the direction of the arrow?

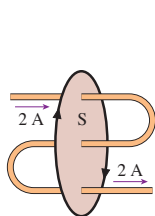


FIGURE Q34.4

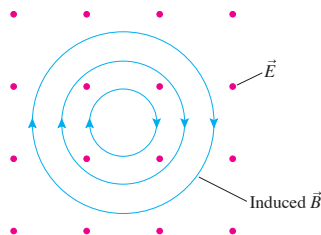


FIGURE Q34.5

- Is the electric field strength in **FIGURE Q34.5** increasing, decreasing, or not changing? Explain.
- Do the situations in **FIGURE Q34.6** represent possible electromagnetic waves? If not, why not?

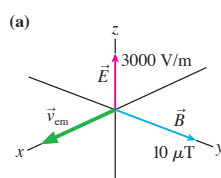
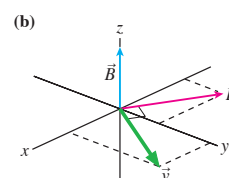


FIGURE Q34.6



- In what directions are the electromagnetic waves traveling in **FIGURE Q34.7**?

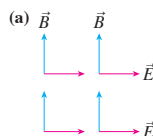
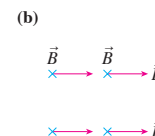


FIGURE Q34.7



- The intensity of an electromagnetic wave is 10 W/m^2 . What will the intensity be if:
 - The amplitude of the electric field is doubled?
 - The amplitude of the magnetic field is doubled?
 - The amplitudes of both the electric and the magnetic fields are doubled?
 - The frequency is doubled?
- Older televisions used a *loop antenna* like the one in **FIGURE Q34.9**. How does this antenna work?

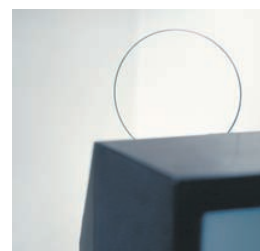


FIGURE Q34.9

- A vertically polarized electromagnetic wave passes through the five polarizers in **FIGURE Q34.10**. Rank in order, from largest to smallest, the transmitted intensities I_a to I_e .

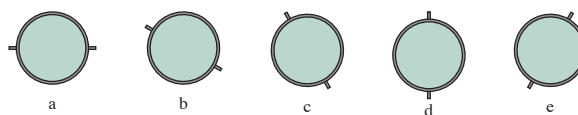







FIGURE Q34.10

EXERCISES AND PROBLEMS

Problems labeled  integrate material from earlier chapters.

Exercises

Section 34.1 *E* or *B*? It Depends on Your Perspective

1.  A rocket cruises past a laboratory at 1.00×10^6 m/s in the positive x -direction just as a proton is launched with velocity (in the laboratory frame) $\vec{v} = (1.41 \times 10^6 \hat{i} + 1.41 \times 10^6 \hat{j})$ m/s. What are the proton's speed and its angle from the y -axis in (a) the laboratory frame and (b) the rocket frame?
2.  **FIGURE EX34.2** shows the electric and magnetic field in frame A. A rocket in frame B travels parallel to one of the axes of the A coordinate system. Along which axis must the rocket travel, and in which direction, in order for the rocket scientists to measure (a) $B_B > B_A$, (b) $B_B = B_A$, and (c) $B_B < B_A$?
3.  Scientists in the laboratory create a uniform electric field $\vec{E} = 1.0 \times 10^6 \hat{k}$ V/m in a region of space where $\vec{B} = \vec{0}$. What are the fields in the reference frame of a rocket traveling in the positive x -direction at 1.0×10^6 m/s?
4.  Laboratory scientists have created the electric and magnetic fields shown in **FIGURE EX34.4**. These fields are also seen by scientists that zoom past in a rocket traveling in the x -direction at 1.0×10^6 m/s. According to the rocket scientists, what angle does the electric field make with the axis of the rocket?

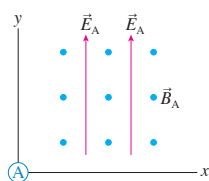


FIGURE EX34.2

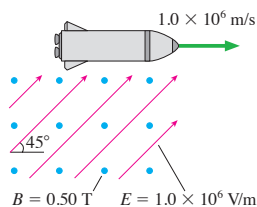


FIGURE EX34.4

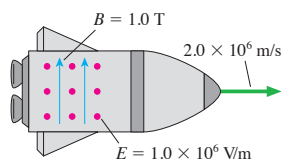




FIGURE EX34.5

5.  A rocket zooms past the earth at $v = 2.0 \times 10^6$ m/s. Scientists on the rocket have created the electric and magnetic fields shown in **FIGURE EX34.5**. What are the fields measured by an earthbound scientist?

Section 34.2 The Field Laws Thus Far

Section 34.3 The Displacement Current

6.  The magnetic field is uniform over each face of the box shown in **FIGURE EX34.6**. What are the magnetic field strength and direction on the front surface?

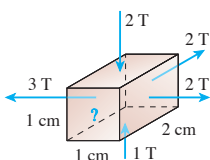


















FIGURE EX34.6

7.  Show that the quantity $\epsilon_0(d\Phi_e/dt)$ has units of current.
8.  Show that the displacement current inside a parallel-plate capacitor can be written $C(dV_C/dt)$.
9.  What capacitance, in μF , has its potential difference increasing at 1.0×10^6 V/s when the displacement current in the capacitor is 1.0 A?
10.  A 10-cm-diameter parallel-plate capacitor has a 1.0 mm spacing. The electric field between the plates is increasing at the rate 1.0×10^6 V/m.s. What is the magnetic field strength (a) on the axis, (b) 3.0 cm from the axis, and (c) 7.0 cm from the axis?
11.  A 5.0-cm-diameter parallel-plate capacitor has a 0.50 mm gap. What is the displacement current in the capacitor if the potential difference across the capacitor is increasing at 500,000 V/s?

Section 34.5 Electromagnetic Waves

12.  What is the electric field amplitude of an electromagnetic wave whose magnetic field amplitude is 2.0 mT?
13.  What is the magnetic field amplitude of an electromagnetic wave whose electric field amplitude is 10 V/m?
14.  The magnetic field of an electromagnetic wave in a vacuum is $B_z = (3.00 \mu\text{T}) \sin((1.00 \times 10^7)x - \omega t)$, where x is in m and t is in s. What are the wave's (a) wavelength, (b) frequency, and (c) electric field amplitude?
15.  The electric field of an electromagnetic wave in a vacuum is $E_y = (20.0 \text{ V/m}) \cos((6.28 \times 10^8)x - \omega t)$, where x is in m and t is in s. What are the wave's (a) wavelength, (b) frequency, and (c) magnetic field amplitude?

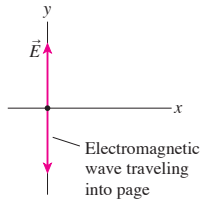
Section 34.6 Properties of Electromagnetic Waves

16.  A radio wave is traveling in the negative y -direction. What is the direction of \vec{E} at a point where \vec{B} is in the positive x -direction?
17.  a. What is the magnetic field amplitude of an electromagnetic wave whose electric field amplitude is 100 V/m?
b. What is the intensity of the wave?
18.  A radio receiver can detect signals with electric field amplitudes as small as $300 \mu\text{V/m}$. What is the intensity of the smallest detectable signal?
19.  A helium-neon laser emits a 1.0-mm-diameter laser beam with a power of 1.0 mW. What are the amplitudes of the electric and magnetic fields of the light wave?
20.  A 200 MW laser pulse is focused with a lens to a diameter of $2.0 \mu\text{m}$.
a. What is the laser beam's electric field amplitude at the focal point?
b. What is the ratio of the laser beam's electric field to the electric field that keeps the electron bound to the proton of a hydrogen atom? The radius of the electron orbit is 0.053 nm.
21.  A radio antenna broadcasts a 1.0 MHz radio wave with 25 kW of power. Assume that the radiation is emitted uniformly in all directions.
a. What is the wave's intensity 30 km from the antenna?
b. What is the electric field amplitude at this distance?
22.  At what distance from a 10 W point source of electromagnetic waves is the magnetic field amplitude $1.0 \mu\text{T}$?

23. I A 1000 W carbon-dioxide laser emits light with a wavelength of $10\text{ }\mu\text{m}$ into a 3.0-mm-diameter laser beam. What force does the laser beam exert on a completely absorbing target?

Section 34.7 Polarization

24. I **FIGURE EX34.24** shows a vertically polarized radio wave of frequency $1.0 \times 10^6\text{ Hz}$ traveling into the page. The maximum electric field strength is 1000 V/m. What are
- The maximum magnetic field strength?
 - The magnetic field strength and direction at a point where $\vec{E} = 500\text{ V/m, down}$?
25. II Only 25% of the intensity of a polarized light wave passes through a polarizing filter. What is the angle between the electric field and the axis of the filter?
26. II A 200 mW vertically polarized laser beam passes through a polarizing filter whose axis is 35° from horizontal. What is the power of the laser beam as it emerges from the filter?
27. II Unpolarized light with intensity 350 W/m^2 passes first through a polarizing filter with its axis vertical, then through a second polarizing filter. It emerges from the second filter with intensity 131 W/m^2 . What is the angle from vertical of the axis of the second polarizing filter?



Problems

28. II What is the force (magnitude and direction) on the proton in **FIGURE P34.28**? Give the direction as an angle cw or ccw from vertical.

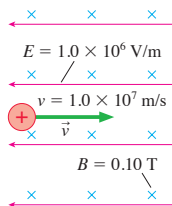


FIGURE P34.28

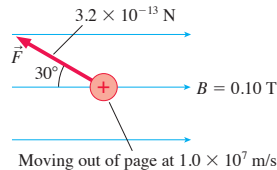


FIGURE P34.29

29. II What are the electric field strength and direction at the position of the proton in **FIGURE P34.29**?
30. I What electric field strength and direction will allow the electron in **FIGURE P34.30** to pass through this region of space without being deflected?

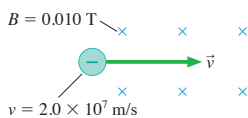


FIGURE P34.30

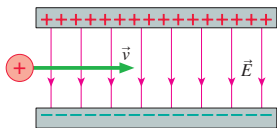


FIGURE P34.31

31. I A proton is fired with a speed of $1.0 \times 10^6\text{ m/s}$ through the parallel-plate capacitor shown in **FIGURE P34.31**. The capacitor's electric field is $\vec{E} = (1.0 \times 10^5\text{ V/m, down})$.
- What magnetic field \vec{B} , both strength and direction, must be applied to allow the proton to pass through the capacitor with no change in speed or direction?

- Find the electric and magnetic fields in the proton's reference frame.
- How does an experimenter in the proton's frame explain that the proton experiences no force as the charged plates fly by?

32. III An electron travels with $\vec{v} = 5.0 \times 10^6\text{ m/s}$ through a point in space where $\vec{E} = (2.0 \times 10^5\text{ i} - 2.0 \times 10^5\text{ j})\text{ V/m}$ and $\vec{B} = -0.10\text{ k T}$. What is the force on the electron?
33. II A very long, 1.0-mm-diameter wire carries a 2.5 A current from left to right. Thin plastic insulation on the wire is positively charged with linear charge density 2.5 nC/cm . A mosquito 1.0 cm from the center of the wire would like to move in such a way as to experience an electric field but no magnetic field. How fast and which direction should she fly?
34. II In **FIGURE P34.34**, a circular loop of radius r travels with speed v along a charged wire having linear charge density λ . The wire is at rest in the laboratory frame, and it passes through the center of the loop.
- What are \vec{E} and \vec{B} at a point on the loop as measured by a scientist in the laboratory? Include both strength and direction.
 - What are the fields \vec{E} and \vec{B} at a point on the loop as measured by a scientist in the frame of the loop?
 - Show that an experimenter in the loop's frame sees a current $I = \lambda v$ passing through the center of the loop.
 - What electric and magnetic fields would an experimenter in the loop's frame calculate at distance r from the current of part c?
 - Show that your fields of parts b and d are the same.

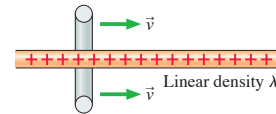


FIGURE P34.34

35. II The magnetic field inside a 4.0-cm-diameter superconducting solenoid varies sinusoidally between 8.0 T and 12.0 T at a frequency of 10 Hz.
- What is the maximum electric field strength at a point 1.5 cm from the solenoid axis?
 - What is the value of B at the instant E reaches its maximum value?
36. II A simple series circuit consists of a $150\text{ }\Omega$ resistor, a 25 V battery, a switch, and a 2.5 pF parallel-plate capacitor (initially uncharged) with plates 5.0 mm apart. The switch is closed at $t = 0\text{ s}$.
- After the switch is closed, find the maximum electric flux and the maximum displacement current through the capacitor.
 - Find the electric flux and the displacement current at $t = 0.50\text{ ns}$.
37. II A wire with conductivity σ carries current I . The current is increasing at the rate dI/dt .
- Show that there is a displacement current in the wire equal to $(\epsilon_0/\sigma)(dI/dt)$.
 - Evaluate the displacement current for a copper wire in which the current is increasing at $1.0 \times 10^6\text{ A/s}$.
38. II A 10 A current is charging a 1.0-cm-diameter parallel-plate capacitor.
- What is the magnetic field strength at a point 2.0 mm radially from the center of the wire leading to the capacitor?
 - What is the magnetic field strength at a point 2.0 mm radially from the center of the capacitor?

39. || FIGURE P34.39 shows the voltage across a $0.10\text{ }\mu\text{F}$ capacitor. Draw a graph showing the displacement current through the capacitor as a function of time.

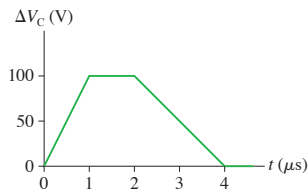


FIGURE P34.39

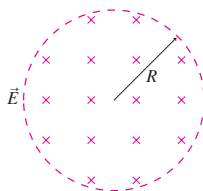


FIGURE P34.40

40. || FIGURE P34.40 shows the electric field inside a cylinder of radius $R = 3.0\text{ mm}$. The field strength is increasing with time as $E = 1.0 \times 10^8 t^2\text{ V/m}$, where t is in s. The electric field outside the cylinder is always zero, and the field inside the cylinder was zero for $t < 0$.
- Find an expression for the electric flux Φ_E through the entire cylinder as a function of time.
 - Draw a picture showing the magnetic field lines inside and outside the cylinder. Be sure to include arrowheads showing the field's direction.
 - Find an expression for the magnetic field strength as a function of time at a distance $r < R$ from the center. Evaluate the magnetic field strength at $r = 2.0\text{ mm}$, $t = 2.0\text{ s}$.
 - Find an expression for the magnetic field strength as a function of time at a distance $r > R$ from the center. Evaluate the magnetic field strength at $r = 4.0\text{ mm}$, $t = 2.0\text{ s}$.
41. || A $1.0\text{ }\mu\text{F}$ capacitor is discharged, starting at $t = 0\text{ s}$. The displacement current through the plates is $I_{\text{disp}} = (10\text{ A})\exp(-t/2.0\text{ }\mu\text{s})$. What was the capacitor's initial voltage ($\Delta V_C)_0$?
42. || At one instant, the electric and magnetic fields at one point of an electromagnetic wave are $\vec{E} = (200\hat{i} + 300\hat{j} - 50\hat{k})\text{ V/m}$ and $\vec{B} = B_0(7.3\hat{i} - 7.3\hat{j} + a\hat{k})\text{ }\mu\text{T}$.
- What are the values of a and B_0 ?
 - What is the Poynting vector at this time and position?
43. || a. Show that u_E and u_B , the energy densities of the electric and magnetic fields, are equal to each other in an electromagnetic wave. In other words, show that the wave's energy is divided equally between the electric field and the magnetic field.
- b. What is the total energy density in an electromagnetic wave of intensity 1000 W/m^2 ?
44. || Assume that a 7.0-cm-diameter , 100 W lightbulb radiates all its energy as a single wavelength of visible light. Estimate the electric and magnetic field strengths at the surface of the bulb.
45. | The intensity of sunlight reaching the earth is 1360 W/m^2 .
- What is the power output of the sun?
 - What is the intensity of sunlight on Mars?
46. || A cube of water 10 cm on a side is placed in a microwave beam having $E_0 = 11\text{ kV/m}$. The microwaves illuminate one face of the cube, and the water absorbs 80% of the incident energy. How long will it take to raise the water temperature by 50°C ? Assume that the water has no heat loss during this time.
47. || A laser beam passes through a converging lens with a focal length f . At what distance past the lens has the laser beam's (a) intensity and (b) electric field strength increased by a factor of 4?
48. | When the Voyager 2 spacecraft passed Neptune in 1989, it was $4.5 \times 10^9\text{ km}$ from the earth. Its radio transmitter, with which it sent back data and images, broadcast with a mere 21 W of power. Assuming that the transmitter broadcast equally in all directions,
- What signal intensity was received on the earth?
 - What electric field amplitude was detected?
- The received signal was somewhat stronger than your result because the spacecraft used a directional antenna, but not by much.
49. || In reading the instruction manual that came with your garage-door opener, you see that the transmitter unit in your car produces a 250 mW signal and that the receiver unit is supposed to respond to a radio wave of the correct frequency if the electric field amplitude exceeds 0.10 V/m . You wonder if this is really true. To find out, you put fresh batteries in the transmitter and start walking away from your garage while opening and closing the door. Your garage door finally fails to respond when you're 42 m away. Are the manufacturer's claims true?
50. || The maximum electric field strength in air is 3.0 MV/m . Stronger electric fields ionize the air and create a spark. What is the maximum power that can be delivered by a 1.0-cm-diameter laser beam propagating through air?
51. || A LASIK vision-correction system uses a laser that emits 10-ns-long pulses of light, each with 2.5 mJ of energy. The laser beam is focused to a 0.85-mm-diameter circle on the cornea. What is the electric field amplitude of the light wave at the cornea?
52. || The intensity of sunlight reaching the earth is 1360 W/m^2 . Assuming all the sunlight is absorbed, what is the radiation-pressure force on the earth? Give your answer (a) in newtons and (b) as a fraction of the sun's gravitational force on the earth.
53. || For radio and microwaves, the depth of penetration into the human body is proportional to $\lambda^{1/2}$. If 27 MHz radio waves penetrate to a depth of 14 cm , how far do 2.4 GHz microwaves penetrate?
54. || A laser beam shines straight up onto a flat, black foil of mass m .
- Find an expression for the laser power P needed to levitate the foil.
 - Evaluate P for a foil with a mass of $25\text{ }\mu\text{g}$.
55. | For a science project, you would like to horizontally suspend an $8.5\text{ by }11\text{ inch}$ sheet of black paper in a vertical beam of light whose dimensions exactly match the paper. If the mass of the sheet is 1.0 g , what light intensity will you need?
56. || You've recently read about a chemical laser that generates a 20-cm-diameter , 25 MW laser beam. One day, after physics class, you start to wonder if you could use the radiation pressure from this laser beam to launch small payloads into orbit. To see if this might be feasible, you do a quick calculation of the acceleration of a 20-cm-diameter , 100 kg , perfectly absorbing block. What speed would such a block have if pushed horizontally 100 m along a frictionless track by such a laser?
57. || An 80 kg astronaut has gone outside his space capsule to do some repair work. Unfortunately, he forgot to lock his safety tether in place, and he has drifted 5.0 m away from the capsule. Fortunately, he has a 1000 W portable laser with fresh batteries that will operate it for 1.0 h . His only chance is to accelerate himself toward the space capsule by firing the laser in the opposite direction. He has a 10-h supply of oxygen. How long will it take him to reach safety?

58. || Unpolarized light of intensity I_0 is incident on three polarizing filters. The axis of the first is vertical, that of the second is 45° from vertical, and that of the third is horizontal. What light intensity emerges from the third filter?

Challenge Problems

59. An electron travels with $\vec{v} = 5.0 \times 10^6 \hat{i}$ m/s through a point in space where $\vec{B} = 0.10 \hat{j}$ T. The force on the electron at this point is $\vec{F} = (9.6 \times 10^{-14} \hat{i} - 9.6 \times 10^{-14} \hat{k})$ N. What is the electric field?
60. A 4.0-cm-diameter parallel-plate capacitor with a 1.0 mm spacing is charged to 1000 V. A switch closes at $t = 0$ s, and the capacitor is discharged through a wire with 0.20Ω resistance.
- Find an expression for the magnetic field strength inside the capacitor at $r = 1.0$ cm as a function of time.
 - Draw a graph of B versus t .
61. The radar system at an airport broadcasts 11 GHz microwaves with 150 kW of power. An approaching airplane with a 31 m^2 cross section is 30 km away. Assume that the radar broadcasts uniformly in all directions and that the airplane scatters microwaves uniformly in all directions. What is the electric field strength of the microwave signal received back at the airport 200 μs later?
62. Large quantities of dust should have been left behind after the creation of the solar system. Larger dust particles, comparable in size to soot and sand grains, are common. They create shooting stars when they collide with the earth's atmosphere. But very small dust particles are conspicuously absent. Astronomers believe that the very small dust particles have been blown out of the solar system by the sun. By comparing the forces on dust particles, determine the diameter of the smallest dust particles that can remain in the solar system over long periods of time. Assume that the dust particles are spherical, black, and have a density of 2000 kg/m^3 . The sun emits electromagnetic radiation with power $3.9 \times 10^{26} \text{ W}$.
63. Consider current I passing through a resistor of radius r , length L , and resistance R .
- Determine the electric and magnetic fields at the surface of the resistor. Assume that the electric field is uniform throughout, including at the surface.
 - Determine the strength and direction of the Poynting vector at the surface of the resistor.
 - Show that the flux of the Poynting vector (i.e., the integral of $\vec{S} \cdot d\vec{A}$) over the surface of the resistor is $I^2 R$. Then give an interpretation of this result.
64. Unpolarized light of intensity I_0 is incident on a stack of 7 polarizing filters, each with its axis rotated 15° cw with respect to the previous filter. What light intensity emerges from the last filter?

STOP TO THINK ANSWERS

Stop to Think 34.1: b. \vec{v}_{AB} is parallel to \vec{B}_A hence $\vec{v}_{AB} \times \vec{B}_A$ is zero. Thus $\vec{E}_B = \vec{E}_A$ and points in the positive z -direction. $\vec{v}_{AB} \times \vec{E}_A$ points down, in the negative y -direction, so $-\vec{v}_{AB} \times \vec{E}_A/c^2$ points in the positive y -direction and causes \vec{B}_B to be angled upward.

Stop to Think 34.2: $B_c > B_a > B_d > B_b$. The induced magnetic field strength depends on the rate dE/dt at which the electric field is changing. Steeper slopes on the graph correspond to larger magnetic fields.

Stop to Think 34.3: e. \vec{E} is perpendicular to \vec{B} and to \vec{v} , so it can only be along the z -axis. According to the Ampère-Maxwell law, $d\Phi_e/dt$ has the same sign as the line integral of $\vec{B} \cdot d\vec{s}$ around the closed curve. The integral is positive for a cw integration. Thus, from the right-hand rule, \vec{E} is either into the page (negative z -direction) and increasing, or out of the page (positive z -direction) and decreasing. We can see from the figure that B is decreasing in strength as the wave moves from left

to right, so E must also be decreasing. Thus \vec{E} points along the positive z -axis.

Stop to Think 34.4: a. The Poynting vector $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$ points in the direction of travel, which is the positive y -direction. \vec{B} must point in the positive x -direction in order for $\vec{E} \times \vec{B}$ to point upward.

Stop to Think 34.5: b. The intensity along a line from the antenna decreases inversely with the square of the distance, so the intensity at 20 km is $\frac{1}{4}$ that at 10 km. But the intensity depends on the square of the electric field amplitude, or, conversely, E_0 is proportional to $I^{1/2}$. Thus E_0 at 20 km is $\frac{1}{2}$ that at 10 km.

Stop to Think 34.6: $I_d > I_a > I_b = I_c$. The intensity depends on $\cos^2 \theta$, where θ is the angle between the axes of the two filters. The filters in d have $\theta = 0^\circ$. The two filters in both b and c are crossed ($\theta = 90^\circ$) and transmit no light at all.

