SUMMARY

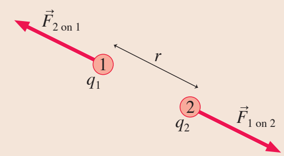
The goal of Chapter 25 has been to describe electric phenomena in terms of charges, forces, and fields.

General Principles

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}$$



These forces are an action/reaction pair directed along the line joining the particles.

- The forces are repulsive for two like charges, attractive for two opposite charges.
- The net force on a charge is the sum of the forces from all other charges.
- The unit of charge is the coulomb (C).
- The electrostatic constant is $K = 9.0 \times 10^9\ \text{N m}^2/\text{C}^2$.

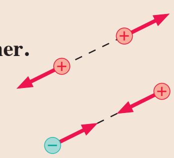
Important Concepts

The Charge Model

There are two kinds of charge, positive and negative.

- Fundamental charges are protons and electrons, with charge $\pm e$ where $e = 1.60 \times 10^{-19}$ C.
- Objects are charged by adding or removing electrons.
- The amount of charge is $q = (N_p - N_e)e$.
- An object with an equal number of protons and electrons is neutral, meaning no *net* charge.

Charged objects exert electric forces on each other.



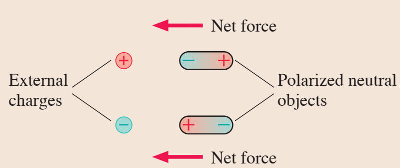
- Like charges repel, opposite charges attract.
- The force increases as the charge increases.
- The force decreases as the distance increases.

There are two types of material, insulators and conductors.

- Charge remains fixed in or on an insulator.
- Charge moves easily through or along conductors.
- Charge is transferred by contact between objects.

Charged objects attract neutral objects.

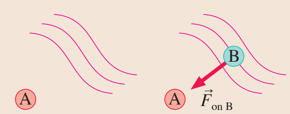
- Charge polarizes metal by shifting the electron sea.
- Charge polarizes atoms, creating electric dipoles.
- The polarization force is always an attractive force.



The Field Model

Charges interact with each other via the electric field $\vec{E}$.

- Charge A alters the space around it by creating an electric field.

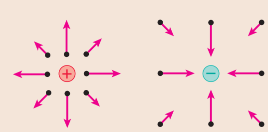


- The field is the agent that exerts a force. The force on charge q_B is $\vec{F}_{\text{on B}} = q_B\vec{E}$.

An electric field is identified and measured in terms of the force on a **probe charge** q:

$$\vec{E} = \vec{F}_{\text{on }q}/q$$

- The electric field exists at all points in space.
- An electric field vector shows the field only at one point, the point at the tail of the vector.



The electric field of a **point charge** is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Terms and Notation

neutral	electron cloud	electrostatic equilibrium	coulomb, C
charging	fundamental unit of charge, e	grounded	permittivity constant, ϵ_0
charge model	charge quantization	charge polarization	field
charge, q or Q	ionization	polarization force	electric field, $\vec{E}$
like charges	law of conservation of charge	electric dipole	field model
opposite charges	sea of electrons	charging by induction	source charge
discharging	ion core	Coulomb's law	electric field strength, E
conductor	current	electrostatic constant, K	field diagram
insulator	charge carriers	point charge	

CONCEPTUAL QUESTIONS

1. Can an insulator be charged? If so, how would you charge an insulator? If not, why not?
2. Can a conductor be charged? If so, how would you charge a conductor? If not, why not?
3. Four lightweight balls A, B, C, and D are suspended by threads. Ball A has been touched by a plastic rod that was rubbed with wool. When the balls are brought close together, without touching, the following observations are made:
   - Balls B, C, and D are attracted to ball A.
   - Balls B and D have no effect on each other.
   - Ball B is attracted to ball C.

   What are the charge states (glass, plastic, or neutral) of balls A, B, C, and D? Explain.
4. Charged plastic and glass rods hang by threads.
   a. An object repels the plastic rod. Can you predict what it will do to the glass rod? If so, what? If not, why not?
   b. A different object attracts the plastic rod. Can you predict what it will do to the glass rod? If so, what? If not, why not?
5. A lightweight metal ball hangs by a thread. When a charged rod is held near, the ball moves toward the rod, touches the rod, then quickly "flies away" from the rod. Explain this behavior.
6. Suppose there exists a third type of charge in addition to the two types we've called glass and plastic. Call this third type X charge. What experiment or series of experiments would you use to test whether an object has X charge? State clearly how each possible outcome of the experiments is to be interpreted.
7. A negatively charged electroscope has separated leaves.
   a. Suppose you bring a negatively charged rod close to the top of the electroscope, but not touching. How will the leaves respond? Use both charge diagrams and words to explain.
   b. How will the leaves respond if you bring a positively charged rod close to the top of the electroscope, but not touching? Use both charge diagrams and words to explain.
8. The two oppositely charged metal spheres in FIGURE Q25.8 have equal quantities of charge. They are brought into contact with a neutral metal rod. What is the final charge state of each sphere and of the rod? Use both charge diagrams and words to explain.

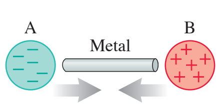


FIGURE Q25.8

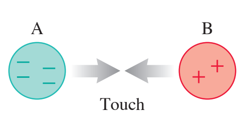


FIGURE Q25.9

9. Metal sphere A in FIGURE Q25.9 has 4 units of negative charge and metal sphere B has 2 units of positive charge. The two spheres are brought into contact. What is the final charge state of each sphere? Explain.
10. Metal spheres A and B in FIGURE Q25.10 are initially neutral and are touching. A positively charged rod is brought near A, but not touching. Is A now positive, negative, or neutral? Use both charge diagrams and words to explain.

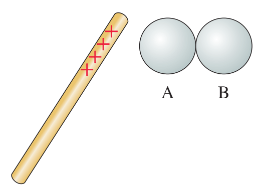


FIGURE Q25.10

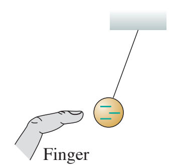


FIGURE Q25.11

11. If you bring your finger near a lightweight, negatively charged hanging ball, the ball swings over toward your finger as shown in FIGURE Q25.11. Use charge diagrams and words to explain this observation.
12. Reproduce FIGURE Q25.12 on your paper. Then draw a dot (or dots) on the figure to show the position (or positions) where an electron would experience no net force.

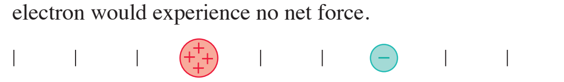


FIGURE Q25.12

13. Charges A and B in FIGURE Q25.13 are equal. Each charge exerts a force on the other of magnitude F. Suppose the charge of B is increased by a factor of 4, but everything else is unchanged. In terms of F, (a) what is the magnitude of the force on A, and (b) what is the magnitude of the force on B?

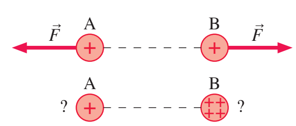


FIGURE Q25.13

14. The electric field strength at one point near a point charge is 1000 N/C. What is the field strength if (a) the distance from the point charge is doubled, and (b) the distance from the point charge is halved?
15. The electric force on a charged particle in an electric field is F. What will be the force if the particle's charge is tripled and the electric field strength is halved?

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 25.1 Developing a Charge Model

Section 25.2 Charge

1. | A plastic rod is charged to −12 nC by rubbing.
   a. Have electrons been added to the rod or protons removed? Explain.
   b. How many electrons have been added or protons removed?
2. | A glass rod is charged to +8.0 nC by rubbing.
   a. Have electrons been removed from the rod or protons added? Explain.
   b. How many electrons have been removed or protons added?
3. | A glass rod that has been charged to +12 nC touches a metal sphere. Afterward, the rod's charge is +8.0 nC.
   a. What kind of charged particle was transferred between the rod and the sphere, and in which direction? That is, did it move from the rod to the sphere or from the sphere to the rod?
   b. How many charged particles were transferred?
4. | A plastic rod that has been charged to −15 nC touches a metal sphere. Afterward, the rod's charge is −10 nC.
   a. What kind of charged particle was transferred between the rod and the sphere, and in which direction? That is, did it move from the rod to the sphere or from the sphere to the rod?
   b. How many charged particles were transferred?
5. || What is the total charge of all the protons in 1.0 mol of He gas?
6. ||| What is the total charge of all the electrons in 1.0 L of liquid water?

Section 25.3 Insulators and Conductors

7. | Figure 25.8 showed how an electroscope becomes negatively charged. The leaves will also repel each other if you touch the electroscope with a positively charged glass rod. Use a series of charge diagrams to explain what happens and why the leaves repel each other.
8. | A plastic balloon that has been rubbed with wool will stick to a wall.
   a. Can you conclude that the wall is charged? If not, why not? If so, where does the charge come from?
   b. Draw a series of charge diagrams showing how the balloon is held to the wall.
9. | Two neutral metal spheres on wood stands are touching. A negatively charged rod is held directly above the top of the left sphere, not quite touching it. While the rod is there, the right sphere is moved so that the spheres no longer touch. Then the rod is withdrawn. Afterward, what is the charge state of each sphere? Use charge diagrams to explain your answer.
10. || You have two neutral metal spheres on wood stands. Devise a procedure for charging the spheres so that they will have like charges of *exactly* equal magnitude. Use charge diagrams to explain your procedure.
11. || You have two neutral metal spheres on wood stands. Devise a procedure for charging the spheres so that they will have opposite charges of *exactly* equal magnitude. Use charge diagrams to explain your procedure.

Section 25.4 Coulomb's Law

12. || Two 1.0 kg masses are 1.0 m apart (center to center) on a frictionless table. Each has +10 μC of charge.
    a. What is the magnitude of the electric force on one of the masses?
    b. What is the initial acceleration of this mass if it is released and allowed to move?
13. || Two small plastic spheres each have a mass of 2.0 g and a charge of −50.0 nC. They are placed 2.0 cm apart (center to center).
    a. What is the magnitude of the electric force on each sphere?
    b. By what factor is the electric force on a sphere larger than its weight?
14. || A small glass bead has been charged to +20 nC. A metal ball bearing 1.0 cm above the bead feels a 0.018 N downward electric force. What is the charge on the ball bearing?
15. | Two protons are 2.0 fm apart.
    a. What is the magnitude of the electric force on one proton due to the other proton?
    b. What is the magnitude of the gravitational force on one proton due to the other proton?
    c. What is the ratio of the electric force to the gravitational force?
16. | What is the net electric force on charge A in FIGURE EX25.16?

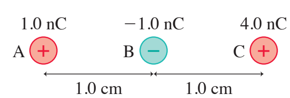


FIGURE EX25.16

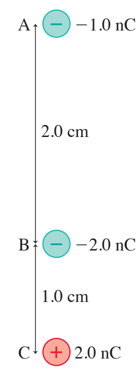


FIGURE EX25.17

17. | What is the net electric force on charge B in FIGURE EX25.17?
18. | Object A, which has been charged to +4.0 nC, is at the origin. Object B, which has been charged to −8.0 nC, is at $(x, y) = (0.0 \text{ cm}, 2.0 \text{ cm})$. Determine the electric force on each object. Write each force vector in component form.
19. | A small plastic bead has been charged to −15 nC. What are the magnitude and direction of the acceleration of (a) a proton and (b) an electron that is 1.0 cm from the center of the bead?

Section 25.5 The Field Model

20. | What are the strength and direction of the electric field 1.0 mm from (a) a proton and (b) an electron?
21. | The electric field at a point in space is $\vec{E} = (400\hat{\imath} + 100\hat{\jmath})$ N/C.
    a. What is the electric force on a proton at this point? Give your answer in component form.
    b. What is the electric force on an electron at this point? Give your answer in component form.
    c. What is the magnitude of the proton's acceleration?
    d. What is the magnitude of the electron's acceleration?

22. || What magnitude charge creates a 1.0 N/C electric field at a point 1.0 m away?
23. | What are the strength and direction of the electric field 4.0 cm from a small plastic bead that has been charged to −8.0 nC?
24. || The electric field 2.0 cm from a small object points away from the object with a strength of 270,000 N/C. What is the object's charge?
25. || What are the strength and direction of an electric field that will balance the weight of a 1.0 g plastic sphere that has been charged to −3.0 nC?
26. || A +12 nC charge is located at the origin.
   a. What are the electric fields at the positions $(x, y) = (5.0\text{ cm}, 0\text{ cm}), (-5.0\text{ cm}, 5.0\text{ cm})$, and $(-5.0\text{ cm}, -5.0\text{ cm})$? Write each electric field vector in component form.
   b. Draw a field diagram showing the electric field vectors at these points.
27. || A –12 nC charge is located at $(x, y) = (1.0\text{ cm}, 0\text{ cm})$. What are the electric fields at the positions $(x, y) = (5.0\text{ cm}, 0\text{ cm})$, $(-5.0\text{ cm}, 0\text{ cm})$, and $(0\text{ cm}, 5.0\text{ cm})$? Write each electric field vector in component form.

Problems

28. ||| Pennies today are copper-covered zinc, but older pennies are 3.1 g of solid copper. What are the total positive charge and total negative charge in a solid copper penny that is electrically neutral?
29. | A 2.0 g plastic bead charged to −4.0 nC and a 4.0 g glass bead charged to +8.0 nC are 2.0 cm apart (center to center). What are the accelerations of (a) the plastic bead and (b) the glass bead?
30. || The nucleus of a ^{125}Xe atom (an isotope of the element xenon with mass 125 u) is 6.0 fm in diameter. It has 54 protons and charge $q = +54e$.
   a. What is the electric force on a proton 2.0 fm from the surface of the nucleus?
   b. What is the proton's acceleration?
   Hint: Treat the spherical nucleus as a point charge.
31. || Two 1.0 g spheres are charged equally and placed 2.0 cm apart. When released, they begin to accelerate at 150 m/s^2. What is the magnitude of the charge on each sphere?
32. || Objects A and B are both positively charged. Both have a mass of 100 g, but A has twice the charge of B. When A and B are placed 10 cm apart, B experiences an electric force of 0.45 N.
   a. What is the charge on A?
   b. If the objects are released, what is the initial acceleration of A?
33. || What is the force $\vec{F}$ on the 1.0 nC charge in FIGURE P25.33? Give your answer as a magnitude and a direction.

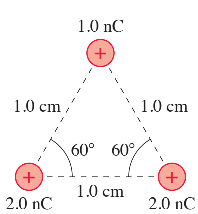


FIGURE P25.33

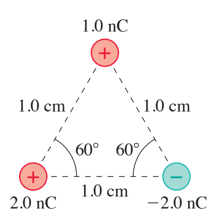


FIGURE P25.34

34. || What is the force $\vec{F}$ on the 1.0 nC charge in FIGURE P25.34? Give your answer as a magnitude and a direction.
35. || What is the force $\vec{F}$ on the −10 nC charge in FIGURE P25.35? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the $+x$-axis.

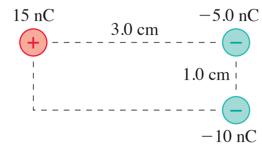


FIGURE P25.35

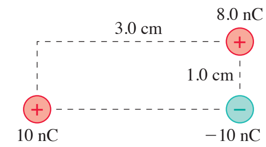


FIGURE P25.36

36. || What is the force $\vec{F}$ on the −10 nC charge in FIGURE P25.36? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the $+x$-axis.
37. || What is the force $\vec{F}$ on the 5.0 nC charge in FIGURE P25.37? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the $+x$-axis.

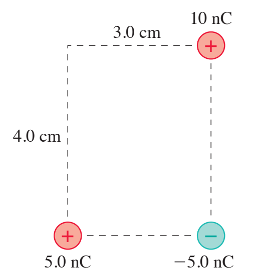


FIGURE P25.37

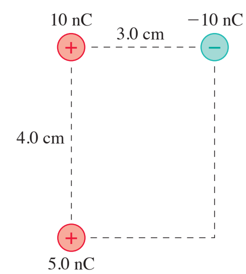


FIGURE P25.38

38. || What is the force $\vec{F}$ on the 5.0 nC charge in FIGURE P25.38? Give your answer as a magnitude and an angle measured cw or ccw (specify which) from the $+x$-axis.
39. ||| What is the force $\vec{F}$ on the 1.0 nC charge in the middle of FIGURE P25.39 due to the four other charges? Give your answer in component form.

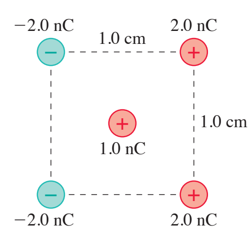


FIGURE P25.39

40. || What is the force $\vec{F}$ on the 1.0 nC charge at the bottom in FIGURE P25.40? Give your answer in component form.

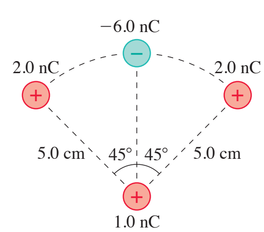


FIGURE P25.40

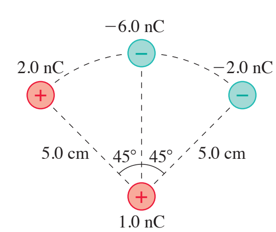


FIGURE P25.41

41. || What is the force $\vec{F}$ on the 1.0 nC charge at the bottom in FIGURE P25.41? Give your answer in component form.

42. ||| A +2.0 nC charge is at the origin and a −4.0 nC charge is at $x = 1.0$ cm.
   a. At what x-coordinate could you place a proton so that it would experience no net force?
   b. Would the net force be zero for an electron placed at the same position? Explain.

43. || The net force on the 1.0 nC charge in **FIGURE P25.43** is zero. What is q?

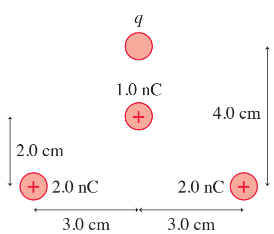


FIGURE P25.43

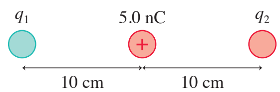


FIGURE P25.44

44. || Charge q_2 in **FIGURE P25.44** is in static equilibrium. What is q_1?

45. || A positive point charge Q is located at $x = a$ and a negative point charge $-Q$ is at $x = -a$. A positive charge q can be placed anywhere on the y-axis. Find an expression for $(F_{\text{net}})_x$, the x-component of the net force on q.

46. || A positive point charge Q is located at $x = a$ and a negative point charge $-Q$ is at $x = -a$. A positive charge q can be placed anywhere on the x-axis. Find an expression for $(F_{\text{net}})_x$, the x-component of the net force on q, when (a) $|x| < a$ and (b) $|x| > a$.

47. ||| **FIGURE P25.47** shows four charges at the corners of a square of side L. What is the magnitude of the net force on q?

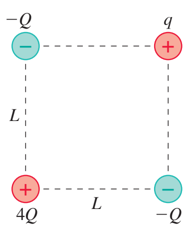


FIGURE P25.47

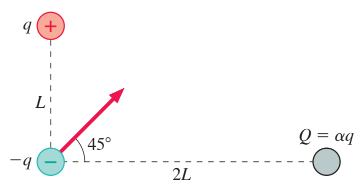


FIGURE P25.48

48. || **FIGURE P25.48** shows three charges and the net force on charge $-q$. Charge Q is some multiple α of q. What is α?

49. || Two positive point charges q and $4q$ are at $x = 0$ and $x = L$, respectively, and free to move. A third charge is placed so that the entire three-charge system is in static equilibrium. What are the magnitude, sign, and x-coordinate of the third charge?

50. ||| Suppose the magnitude of the proton charge differs from the magnitude of the electron charge by a mere 1 part in 10^9.
   a. What would be the force between two 2.0-mm-diameter copper spheres 1.0 cm apart? Assume that each copper atom has an equal number of electrons and protons.
   b. Would this amount of force be detectable? What can you conclude from the fact that no such forces are observed?

51. || In a simple model of the hydrogen atom, the electron moves in a circular orbit of radius 0.053 nm around a stationary proton. How many revolutions per second does the electron make?

52. || You have two small, 2.0 g balls that have been given equal but opposite charges, but you don't know the magnitude of the charge. To find out, you place the balls distance d apart on a slippery horizontal surface, release them, and use a motion detector to measure the initial acceleration of one of the balls toward the other. After repeating this for several different separation distances, your data are as follows:

Distance (cm)	Acceleration (m/s²)
2.0	0.74
3.0	0.30
4.0	0.19
5.0	0.10

Use an appropriate graph of the data to determine the magnitude of the charge.

53. || A 0.10 g honeybee acquires a charge of +23 pC while flying.
BIO a. The earth's electric field near the surface is typically (100 N/C, downward). What is the ratio of the electric force on the bee to the bee's weight?
   b. What electric field (strength and direction) would allow the bee to hang suspended in the air?

54. || As a science project, you've invented an "electron pump" that moves electrons from one object to another. To demonstrate your invention, you bolt a small metal plate to the ceiling, connect the pump between the metal plate and yourself, and start pumping electrons from the metal plate to you. How many electrons must be moved from the metal plate to you in order for you to hang suspended in the air 2.0 m below the ceiling? Your mass is 60 kg. **Hint:** Assume that both you and the plate can be modeled as point charges.

55. || You have a lightweight spring whose unstretched length is 4.0 cm. First, you attach one end of the spring to the ceiling and hang a 1.0 g mass from it. This stretches the spring to a length of 5.0 cm. You then attach two small plastic beads to the opposite ends of the spring, lay the spring on a frictionless table, and give each plastic bead the same charge. This stretches the spring to a length of 4.5 cm. What is the magnitude of the charge (in nC) on each bead?

56. || An electric dipole consists of two opposite charges $\pm q$ separated by a small distance s. The product $p = qs$ is called the *dipole moment*. **FIGURE P25.56** shows an electric dipole perpendicular to an electric field $\vec{E}$. Find an expression in terms of p and E for the magnitude of the torque that the electric field exerts on the dipole.

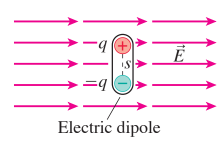


FIGURE P25.56

57. || You sometimes create a spark when you touch a doorknob after shuffling your feet on a carpet. Why? The air always has a few free electrons that have been kicked out of atoms by cosmic rays. If an electric field is present, a free electron is accelerated until it collides with an air molecule. It will transfer its kinetic energy to the molecule, then accelerate, then collide, then accelerate, collide, and so on. If the electron's kinetic energy just before a collision is 2.0×10^{-18} J or more, it has sufficient energy to kick an electron out of the molecule it hits. Where there was one free electron, now there are two! Each of these can then

accelerate, hit a molecule, and kick out another electron. Then there will be four free electrons. In other words, as FIGURE P25.57 shows, a sufficiently strong electric field causes a "chain reaction" of electron production. This is called a *breakdown* of the air. The current of moving electrons is what gives you the shock, and a spark is generated when the electrons recombine with the positive ions and give off excess energy as a burst of light.

a. The average distance an electron travels between collisions is 2.0 μm. What acceleration must an electron have to gain 2.0×10^{-18} J of kinetic energy in this distance?
b. What force must act on an electron to give it the acceleration found in part a?
c. What strength electric field will exert this much force on an electron? This is the *breakdown field strength*. **Note:** The measured breakdown field strength is a little less than your calculated value because our model of the process is a bit too simple. Even so, your calculated value is close.
d. Suppose a free electron in air is 1.0 cm away from a point charge. What minimum charge must this point charge have to cause a breakdown of the air and create a spark?

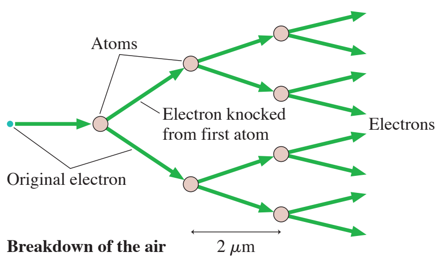


FIGURE P25.57

58. ‖ Two 5.0 g point charges on 1.0-m-long threads repel each other after being charged to +100 nC, as shown in FIGURE P25.58. What is the angle θ? You can assume that θ is a small angle.

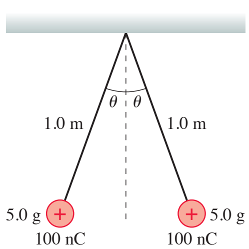


FIGURE P25.58

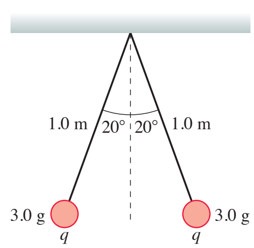


FIGURE P25.59

59. ‖ Two 3.0 g point charges on 1.0-m-long threads repel each other after being equally charged, as shown in FIGURE P25.59. What is the charge q?

60. ‖ What are the electric fields at points 1, 2, and 3 in FIGURE P25.60? Give your answer in component form.

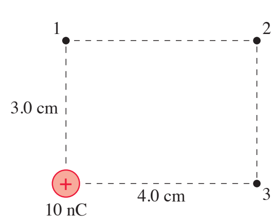


FIGURE P25.60

61. ‖ What are the electric fields at points 1 and 2 in FIGURE P25.61? Give your answer as a magnitude and direction.

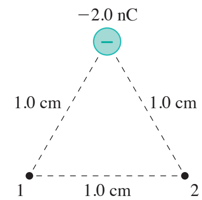


FIGURE P25.61

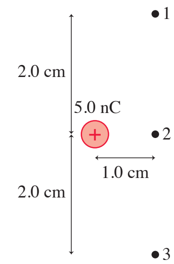


FIGURE P25.62

62. ‖ What are the electric fields at points 1, 2, and 3 in FIGURE P25.62? Give your answer in component form.

63. ‖ A −10.0 nC charge is located at position $(x, y) = (2.0 \text{ cm}, 1.0 \text{ cm})$. At what (x, y) position(s) is the electric field
a. $-225{,}000\,\hat{\imath}$ N/C?
b. $(161{,}000\,\hat{\imath} - 80{,}500\,\hat{\jmath})$ N/C?
c. $(28{,}800\,\hat{\imath} + 21{,}600\,\hat{\jmath})$ N/C?

64. ‖ A 10.0 nC charge is located at position $(x, y) = (1.0 \text{ cm}, 2.0 \text{ cm})$. At what (x, y) position(s) is the electric field
a. $-225{,}000\,\hat{\imath}$ N/C?
b. $(161{,}000\,\hat{\imath} + 80{,}500\,\hat{\jmath})$ N/C?
c. $(21{,}600\,\hat{\imath} - 28{,}800\,\hat{\jmath})$ N/C?

65. ‖ Three 1.0 nC charges are placed as shown in FIGURE P25.65. Each of these charges creates an electric field $\vec{E}$ at a point 3.0 cm in front of the middle charge.

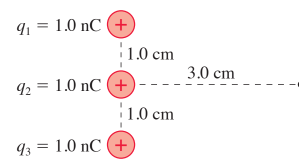


FIGURE P25.65

a. What are the three fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ created by the three charges? Write your answer for each as a vector in component form.
b. Do you think that electric fields obey a principle of superposition? That is, is there a "net field" at this point given by $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$? Use what you learned in this chapter and previously in our study of forces to argue why this is or is not true.
c. If it is true, what is $\vec{E}_{net}$?

66. ‖ An electric field $\vec{E} = 100{,}000\,\hat{\imath}$ N/C causes the 5.0 g point charge in FIGURE P25.66 to hang at a 20° angle. What is the charge on the ball?

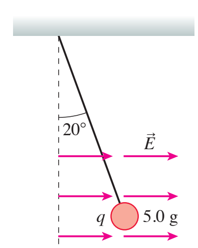


FIGURE P25.66

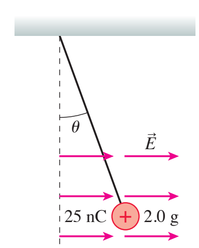


FIGURE P25.67

67. ‖ An electric field $\vec{E} = 200{,}000\,\hat{\imath}$ N/C causes the point charge in FIGURE P25.67 to hang at an angle. What is θ?

In Problems 68 through 71 you are given the equation(s) used to solve a problem. For each of these,

a. Write a realistic problem for which this is the correct equation(s).
b. Finish the solution of the problem.

68. $$\frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2}) \times N \times (1.60 \times 10^{-19}\ \mathrm{C})}{(1.0 \times 10^{-6}\ \mathrm{m})^2} = 1.5 \times 10^6\ \mathrm{N/C}$$

69. $$\frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})q^2}{(0.0150\ \mathrm{m})^2} = 0.020\ \mathrm{N}$$

70. $$\frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})(15 \times 10^{-9}\ \mathrm{C})}{r^2} = 54{,}000\ \mathrm{N/C}$$

71. $$\sum F_x = 2 \times \frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})(1.0 \times 10^{-9}\ \mathrm{C})q}{\left((0.020\ \mathrm{m})/\sin 30°\right)^2} \times \cos 30° = 5.0 \times 10^{-5}\ \mathrm{N}$$
$$\sum F_y = 0\ \mathrm{N}$$

Challenge Problems

72. A 2.0-mm-diameter copper ball is charged to +50 nC. What fraction of its electrons have been removed?
73. Three 3.0 g balls are tied to 80-cm-long threads and hung from a *single* fixed point. Each of the balls is given the same charge q. At equilibrium, the three balls form an equilateral triangle in a horizontal plane with 20 cm sides. What is q?
74. The identical small spheres shown in FIGURE CP25.74 are charged to +100 nC and −100 nC. They hang as shown in a 100,000 N/C electric field. What is the mass of each sphere?

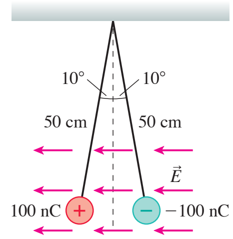


FIGURE CP25.74

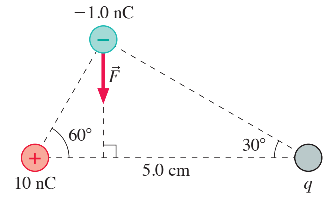


FIGURE CP25.75

75. The force on the −1.0 nC charge is as shown in FIGURE CP25.75. What is the magnitude of this force?
76. In Section 25.3 we claimed that a charged object exerts a net attractive force on an electric dipole. Let's investigate this. FIGURE CP25.76 shows a *permanent* electric dipole consisting of charges $+q$ and $-q$ separated by the fixed distance s. Charge $+Q$ is distance r from the center of the dipole. We'll assume, as is usually the case in practice, that $s \ll r$.
    a. Write an expression for the net force exerted on the dipole by charge $+Q$.
    b. Is this force toward $+Q$ or away from $+Q$? Explain.
    c. Use the *binomial approximation* $(1 + x)^{-n} \approx 1 - nx$ if $x \ll 1$ to show that your expression from part a can be written $F_{net} = 2KqQs/r^3$.
    d. How can an electric force have an inverse-cube dependence? Doesn't Coulomb's law say that the electric force depends on the inverse square of the distance? Explain.

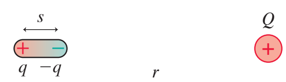


FIGURE CP25.76

STOP TO THINK ANSWERS

Stop to Think 25.1: b. Charged objects are attracted to neutral objects, so an attractive force is inconclusive. Repulsion is the only sure test.

Stop to Think 25.2: $q_e(+3e) > q_a(+1e) > q_d(0) > q_b(-1e) > q_c(-2e)$.

Stop to Think 25.3: a. The negative plastic rod will polarize the electroscope by pushing electrons down toward the leaves. This will partially neutralize the positive charge the leaves had acquired from the glass rod.

Stop to Think 25.4: b. The two forces are an action/reaction pair, opposite in direction but *equal* in magnitude.

Stop to Think 25.5: c. There's an electric field at *all* points, whether an $\vec{E}$ vector is shown or not. The electric field at the dot is to the right. But an electron is a negative charge, so the force of the electric field on the electron is to the left.

Stop to Think 25.6: $E_b > E_a > E_d > E_c$.

SUMMARY

The goal of Chapter 26 has been to learn how to calculate and use the electric field.

General Principles

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$

Continuous distribution of charge

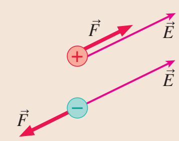
- Divide the charge into segments ΔQ for which you already know the field.
- Find the field of each ΔQ.
- Find $\vec{E}$ by summing the fields of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

Consequences of $\vec{E}$

The electric field exerts a force on a charged particle:

$$\vec{F} = q\vec{E}$$



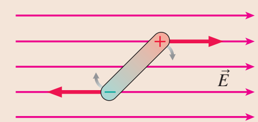
The force causes acceleration:

$$\vec{a} = (q/m)\vec{E}$$

Trajectories of charged particles are calculated with kinematics.

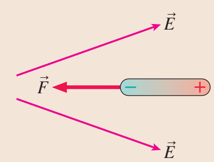
The electric field exerts a torque on a dipole:

$$\tau = pE\sin\theta$$



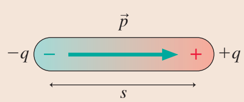
The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.



Applications

Electric dipole



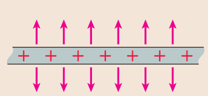
The electric dipole moment is

$\vec{p} = (qs$, from negative to positive)

Field on axis: $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{2\vec{p}}{r^3}$

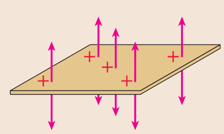
Field in bisecting plane: $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{p}}{r^3}$

Infinite line of charge with linear charge density λ



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}, \text{ perpendicular to line}\right)$$

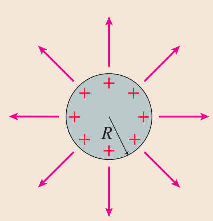
Infinite plane of charge with surface charge density η



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{ perpendicular to plane}\right)$$

Sphere of charge

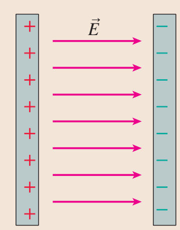
Same as a point charge Q for $r > R$



Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field:**

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$



A real capacitor has a weak **fringe field** around it.

Terms and Notation

dipole moment, $\vec{p}$	uniformly charged	plane of charge	fringe field
electric field line	line of charge	sphere of charge	uniform electric field
linear charge density, λ	electrode	parallel-plate capacitor	charge-to-mass ratio, q/m
surface charge density, η			

CONCEPTUAL QUESTIONS

1. You've been assigned the task of determining the magnitude and direction of the electric field at a point in space. Give a step-by-step procedure of how you will do so. List any objects you will use, any measurements you will make, and any calculations you will need to perform. Make sure that your measurements do not disturb the charges that are creating the field.
2. Reproduce FIGURE Q26.2 on your paper. For each part, draw a dot or dots on the figure to show any position or positions (other than infinity) where $\vec{E} = \vec{0}$.

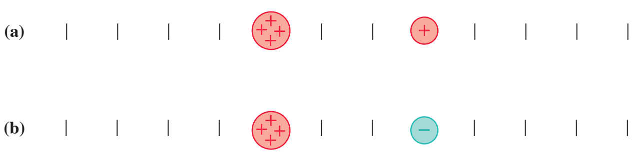


FIGURE Q26.2

3. Rank in order, from largest to smallest, the electric field strengths E_1 to E_4 at points 1 to 4 in FIGURE Q26.3. Explain.

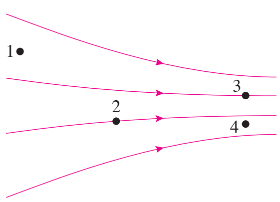


FIGURE Q26.3

FIGURE Q26.4

4. A small segment of wire in FIGURE Q26.4 contains 10 nC of charge.
   a. The segment is shrunk to one-third of its original length. What is the ratio λ_f/λ_i, where λ_i and λ_f are the initial and final linear charge densities?
   b. A proton is very far from the wire. What is the ratio F_f/F_i of the electric force on the proton after the segment is shrunk to the force before the segment was shrunk?
   c. Suppose the original segment of wire is stretched to 10 times its original length. How much charge must be *added* to the wire to keep the linear charge density unchanged?
5. An electron experiences a force of magnitude F when it is 1 cm from a very long, charged wire with linear charge density λ. If the charge density is doubled, at what distance from the wire will a proton experience a force of the same magnitude F?
6. FIGURE Q26.6 shows a hollow soda straw that has been uniformly charged with positive charge. What is the electric field at the center (inside) of the straw? Explain.

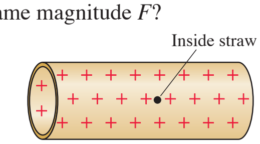


FIGURE Q26.6

7. The irregularly shaped area of charge in FIGURE Q26.7 has surface charge density η_i. Each dimension (x and y) of the area is reduced by a factor of 3.163.
   a. What is the ratio η_f/η_i, where η_f is the final surface charge density?
   b. An electron is very far from the area. What is the ratio F_f/F_i of the electric force on the electron after the area is reduced to the force before the area was reduced?

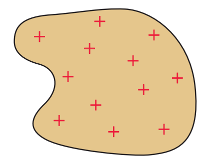


FIGURE Q26.7

8. A circular disk has surface charge density 8 nC/cm^2. What will the surface charge density be if the radius of the disk is doubled?
9. A sphere of radius R has charge Q. The electric field strength at distance $r > R$ is E_i. What is the ratio E_f/E_i of the final to initial electric field strengths if (a) Q is halved, (b) R is halved, and (c) r is halved (but is still $> R$)? Each part changes only one quantity; the other quantities have their initial values.
10. The ball in FIGURE Q26.10 is suspended from a large, uniformly charged positive plate. It swings with period T. If the ball is discharged, will the period increase, decrease, or stay the same? Explain.

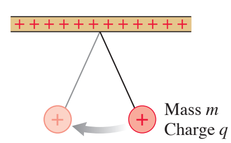


FIGURE Q26.10

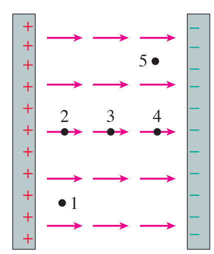


FIGURE Q26.11

11. Rank in order, from largest to smallest, the electric field strengths E_1 to E_5 at the five points in FIGURE Q26.11. Explain.
12. A parallel-plate capacitor consists of two square plates, size $L \times L$, separated by distance d. The plates are given charge $\pm Q$. What is the ratio E_f/E_i of the final to initial electric field strengths if (a) Q is doubled, (b) L is doubled, and (c) d is doubled? Each part changes only one quantity; the other quantities have their initial values.
13. A small object is released at point 3 in the center of the capacitor in FIGURE Q26.11. For each situation, does the object move to the right, to the left, or remain in place? If it moves, does it accelerate or move at constant speed?
    a. A positive object is released from rest.
    b. A neutral but polarizable object is released from rest.
    c. A negative object is released from rest.

14. A proton and an electron are released from rest in the center of a capacitor.
    a. Is the force ratio F_p/F_e greater than 1, less than 1, or equal to 1? Explain.
    b. Is the acceleration ratio a_p/a_e greater than 1, less than 1, or equal to 1? Explain.

15. Three charges are placed at the corners of the triangle in FIGURE Q26.15. The ++ charge has twice the quantity of charge of the two – charges; the net charge is zero. Is the triangle in equilibrium? If so, explain why. If not, draw the equilibrium orientation.

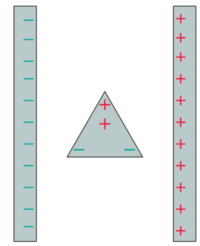


FIGURE Q26.15

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 26.2 The Electric Field of Multiple Point Charges

1. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.1? Specify the direction as an angle above or below horizontal.

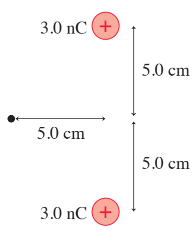


FIGURE EX26.1

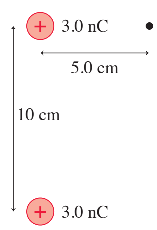


FIGURE EX26.2

2. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.2? Specify the direction as an angle above or below horizontal.
3. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.3? Specify the direction as an angle above or below horizontal.

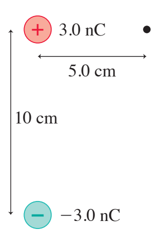


FIGURE EX26.3

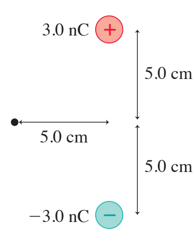


FIGURE EX26.4

4. ‖ What are the strength and direction of the electric field at the position indicated by the dot in FIGURE EX26.4? Specify the direction as an angle above or below horizontal.
5. ‖ An electric dipole is formed from ± 1.0 nC charges spaced 2.0 mm apart. The dipole is at the origin, oriented along the x-axis. What is the electric field strength at the points (a) $(x, y) = (10 \text{ cm}, 0 \text{ cm})$ and (b) $(x, y) = (0 \text{ cm}, 10 \text{ cm})$?
6. ‖ An electric dipole is formed from two charges, $\pm q$, spaced 1.0 cm apart. The dipole is at the origin, oriented along the y-axis. The electric field strength at the point $(x, y) = (0 \text{ cm}, 10 \text{ cm})$ is 360 N/C.
    a. What is the charge q? Give your answer in nC.
    b. What is the electric field strength at the point $(x, y) = (10 \text{ cm}, 0 \text{ cm})$?

Section 26.3 The Electric Field of a Continuous Charge Distribution

7. | The electric field strength 10.0 cm from a very long charged wire is 2000 N/C. What is the electric field strength 5.0 cm from the wire?
8. ‖ A 10-cm-long thin glass rod uniformly charged to +10 nC and a 10-cm-long thin plastic rod uniformly charged to −10 nC are placed side by side, 4.0 cm apart. What are the electric field strengths E_1 to E_3 at distances 1.0 cm, 2.0 cm, and 3.0 cm from the glass rod along the line connecting the midpoints of the two rods?
9. ‖ Two 10-cm-long thin glass rods uniformly charged to +10 nC are placed side by side, 4.0 cm apart. What are the electric field strengths E_1 to E_3 at distances 1.0 cm, 2.0 cm, and 3.0 cm to the right of the rod on the left along the line connecting the midpoints of the two rods?
10. ‖ A small glass bead charged to +6.0 nC is 4.0 cm from a thin, uniformly charged, 10-cm-long glass rod. The bead is repelled from the rod with a force of 840 μN. What is the total charge on the rod?

Section 26.4 The Electric Fields of Rings, Disks, Planes, and Spheres

11. | Two 10-cm-diameter charged rings face each other, 20 cm apart. The left ring is charged to −20 nC and the right ring is charged to +20 nC.
    a. What is the electric field $\vec{E}$, both magnitude and direction, at the midpoint between the two rings?
    b. What is the force $\vec{F}$ on a −1.0 nC charge placed at the midpoint?
12. ‖ Two 10-cm-diameter charged rings face each other, 20 cm apart. Both rings are charged to +20 nC. What is the electric field strength at (a) the midpoint between the two rings and (b) the center of the left ring?
13. ‖ Two 10-cm-diameter charged disks face each other, 20 cm apart. The left disk is charged to −50 nC and the right disk is charged to +50 nC.
    a. What is the electric field $\vec{E}$, both magnitude and direction, at the midpoint between the two disks?
    b. What is the force $\vec{F}$ on a −1.0 nC charge placed at the midpoint?

14. || Two 10-cm-diameter charged disks face each other, 20 cm apart. Both disks are charged to $+50$ nC. What is the electric field strength at (a) the midpoint between the two disks and (b) a point on the axis 5.0 cm from one disk?
15. || The electric field strength 2.0 cm from a 10-cm-diameter metal ball is 50,000 N/C. What is the charge (in nC) on the ball?
16. || A $20\text{ cm} \times 20\text{ cm}$ horizontal metal electrode is uniformly charged to $+80$ nC. What is the electric field strength 2.0 mm above the center of the electrode?

Section 26.5 The Parallel-Plate Capacitor

17. || Two circular disks spaced 0.50 mm apart form a parallel-plate capacitor. Transferring 3.0×10^9 electrons from one disk to the other causes the electric field strength to be 2.0×10^5 N/C. What are the diameters of the disks?
18. || A parallel-plate capacitor is formed from two 6.0-cm-diameter electrodes spaced 2.0 mm apart. The electric field strength inside the capacitor is 1.0×10^6 N/C. What is the charge (in nC) on each electrode?
19. || Air "breaks down" when the electric field strength reaches 3.0×10^6 N/C, causing a spark. A parallel-plate capacitor is made from two $4.0\text{ cm} \times 4.0\text{ cm}$ disks. How many electrons must be transferred from one disk to the other to create a spark between the disks?

Section 26.6 Motion of a Charged Particle in an Electric Field

20. || A 0.10 g glass bead is charged by the removal of 1.0×10^{10} electrons. What electric field $\vec{E}$ (strength and direction) will cause the bead to hang suspended in the air?
21. | Two 2.0-cm-diameter disks face each other, 1.0 mm apart. They are charged to ± 10 nC.
   a. What is the electric field strength between the disks?
   b. A proton is shot from the negative disk toward the positive disk. What launch speed must the proton have to just barely reach the positive disk?
22. || An electron in a uniform electric field increases its speed from 2.0×10^7 m/s to 4.0×10^7 m/s over a distance of 1.2 cm. What is the electric field strength?
23. || The surface charge density on an infinite charged plane is $-2.0 \times 10^{-6}\text{ C/m}^2$. A proton is shot straight away from the plane at 2.0×10^6 m/s. How far does the proton travel before reaching its turning point?
24. || A 1.0-μm-diameter oil droplet (density 900 kg/m^3) is negatively charged with the addition of 25 extra electrons. It is released from rest 2.0 mm from a very wide plane of positive charge, after which it accelerates toward the plane and collides with a speed of 3.5 m/s. What is the surface charge density of the plane?

Section 26.7 Motion of a Dipole in an Electric Field

25. | The permanent electric dipole moment of the water molecule (H_2O) is 6.2×10^{-30} C m. What is the maximum possible torque on a water molecule in a 5.0×10^8 N/C electric field?
26. ||| A point charge Q is distance r from the center of a dipole consisting of charges $\pm q$ separated by distance s. The charge is located in the plane that bisects the dipole. At this instant, what are (a) the force (magnitude and direction) and (b) the magnitude of the torque on the dipole? You can assume $r \gg s$.
27. || An ammonia molecule (NH_3) has a permanent electric dipole moment 5.0×10^{-30} C m. A proton is 2.0 nm from the molecule in the plane that bisects the dipole. What is the electric force of the molecule on the proton?

Problems

28. || What are the strength and direction of the electric field at the position indicated by the dot in FIGURE P26.28? Give your answer (a) in component form and (b) as a magnitude and angle measured cw or ccw (specify which) from the positive x-axis.

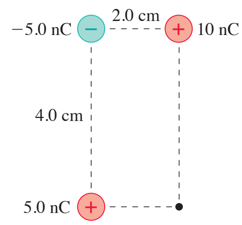


FIGURE P26.28

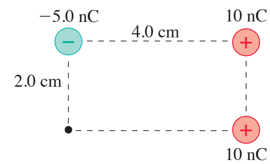


FIGURE P26.29

29. || What are the strength and direction of the electric field at the position indicated by the dot in FIGURE P26.29? Give your answer (a) in component form and (b) as a magnitude and angle measured cw or ccw (specify which) from the positive x-axis.
30. || What are the strength and direction of the electric field at the position indicated by the dot in FIGURE P26.30? Give your answer (a) in component form and (b) as a magnitude and angle measured cw or ccw (specify which) from the positive x-axis.

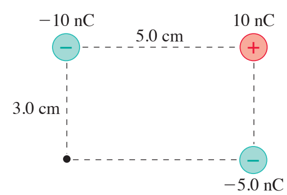


FIGURE P26.30

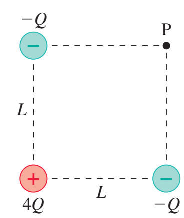


FIGURE P26.31

31. || FIGURE P26.31 shows three charges at the corners of a square. Write the electric field at point P in component form.
32. || Charges $-q$ and $+2q$ in FIGURE P26.32 are located at $x = \pm a$. Determine the electric field at points 1 to 4. Write each field in component form.

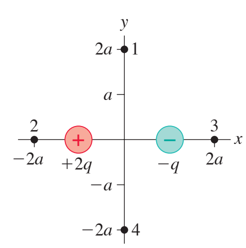


FIGURE P26.32

33. || Two positive charges q are on the y-axis at $y = \pm\frac{1}{2}s$.
   a. Find an expression for the electric field strength at distance x on the axis that bisects the two charges.
   b. For $q = 1.0$ nC and $s = 6.0$ mm, evaluate E at $x = 0, 2, 4, 6,$ and 10 mm.

34. ‖ Derive Equation 26.12 for the field $\vec{E}_{\text{dipole}}$ in the plane that bisects an electric dipole.

35. ‖ Three charges are on the y-axis. Charges $-q$ are at $y = \pm d$ and charge $+2q$ is at $y = 0$.
   a. Find an expression for the electric field $\vec{E}$ at a point on the x-axis.
   b. Verify that your answer to part a has the expected behavior as x becomes very small and very large.

36. ‖ FIGURE P26.36 is a cross section of two infinite lines of charge that extend out of the page. Both have linear charge density λ. Find an expression for the electric field strength E at height y above the midpoint between the lines.

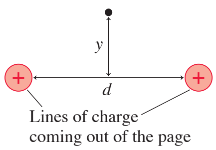


FIGURE P26.36

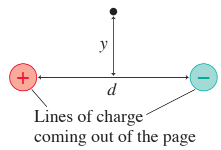


FIGURE P26.37

37. ||| FIGURE P26.37 is a cross section of two infinite lines of charge that extend out of the page. The linear charge densities are $\pm\lambda$. Find an expression for the electric field strength E at height y above the midpoint between the lines.

38. ‖ Two infinite lines of charge, each with linear charge density λ, lie along the x- and y-axes, crossing at the origin. What is the electric field strength at position (x, y)?

39. ‖ The electric field 5.0 cm from a very long charged wire is (2000 N/C, toward the wire). What is the charge (in nC) on a 1.0-cm-long segment of the wire?

40. ‖ FIGURE P26.40 shows a thin rod of length L with total charge Q.
   a. Find an expression for the electric field strength at point P on the axis of the rod at distance r from the center.
   b. Verify that your expression has the expected behavior if $r \gg L$.
   c. Evaluate E at $r = 3.0$ cm if $L = 5.0$ cm and $Q = 3.0$ nC.

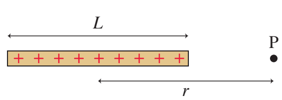


FIGURE P26.40

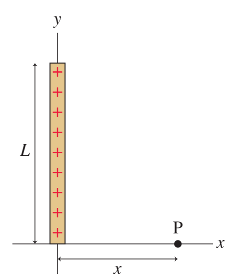


FIGURE P26.41

41. ||| FIGURE P26.41 shows a thin rod of length L with total charge Q. Find an expression for the electric field $\vec{E}$ at point P. Give your answer in component form.

42. ‖ Show that the on-axis electric field of a ring of charge has the expected behavior when $z \ll R$ and when $z \gg R$.

43. ‖ A ring of radius R has total charge Q.
   a. At what distance along the z-axis is the electric field strength a maximum?
   b. What is the electric field strength at this point?

44. ‖ Charge Q is uniformly distributed along a thin, flexible rod of length L. The rod is then bent into the semicircle shown in FIGURE P26.44.
   a. Find an expression for the electric field $\vec{E}$ at the center of the semicircle.
   Hint: A small piece of arc length Δs spans a small angle $\Delta\theta = \Delta s/R$, where R is the radius.
   b. Evaluate the field strength if $L = 10$ cm and $Q = 30$ nC.

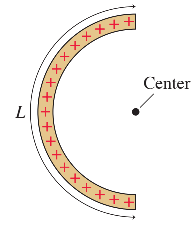


FIGURE P26.44

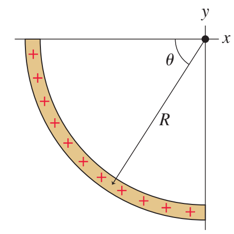


FIGURE P26.45

45. ‖ A plastic rod with linear charge density λ is bent into the quarter circle shown in FIGURE P26.45. We want to find the electric field at the origin.
   a. Write expressions for the x- and y-components of the electric field at the origin due to a small piece of charge at angle θ.
   b. Write, but do not evaluate, definite integrals for the x- and y-components of the net electric field at the origin.
   c. Evaluate the integrals and write $\vec{E}_{\text{net}}$ in component form.

46. ‖ You've hung two very large sheets of plastic facing each other with distance d between them, as shown in FIGURE P26.46. By rubbing them with wool and silk, you've managed to give one sheet a uniform surface charge density $\eta_1 = -\eta_0$ and the other a uniform surface charge density $\eta_2 = +3\eta_0$. What are the electric field vectors at points 1, 2, and 3?

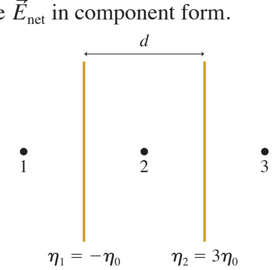


FIGURE P26.46

47. ‖ Two 2.0-cm-diameter insulating spheres have a 6.0 cm space between them. One sphere is charged to +10 nC, the other to −15 nC. What is the electric field strength at the midpoint between the two spheres?

48. ‖ Two parallel plates 1.0 cm apart are equally and oppositely charged. An electron is released from rest at the surface of the negative plate and simultaneously a proton is released from rest at the surface of the positive plate. How far from the negative plate is the point at which the electron and proton pass each other?

49. ‖ A parallel-plate capacitor has 2.0 cm × 2.0 cm electrodes with surface charge densities $\pm 1.0 \times 10^{-6}$ C/m^2. A proton traveling parallel to the electrodes at 1.0×10^6 m/s enters the center of the gap between them. By what distance has the proton been deflected sideways when it reaches the far edge of the capacitor? Assume the field is uniform inside the capacitor and zero outside the capacitor.

50. ‖ An electron is launched at a 45° angle and a speed of 5.0×10^6 m/s from the positive plate of the parallel-plate capacitor shown in FIGURE P26.50. The electron lands 4.0 cm away.
   a. What is the electric field strength inside the capacitor?
   b. What is the smallest possible spacing between the plates?

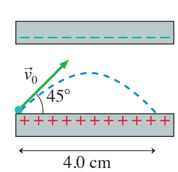


FIGURE P26.50

51. ||| The two parallel plates in FIGURE P26.51 are 2.0 cm apart and the electric field strength between them is 1.0×10^4 N/C. An electron is launched at a 45° angle from the positive plate. What is the maximum initial speed v_0 the electron can have without hitting the negative plate?

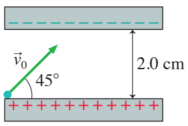


FIGURE P26.51

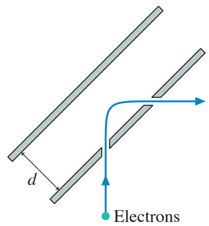


FIGURE P26.52

52. || A problem of practical interest is to make a beam of electrons turn a 90° corner. This can be done with the parallel-plate capacitor shown in FIGURE P26.52. An electron with kinetic energy 3.0×10^{-17} J enters through a small hole in the bottom plate of the capacitor.

a. Should the bottom plate be charged positive or negative relative to the top plate if you want the electron to turn to the right? Explain.

b. What strength electric field is needed if the electron is to emerge from an exit hole 1.0 cm away from the entrance hole, traveling at right angles to its original direction?

Hint: The difficulty of this problem depends on how you choose your coordinate system.

c. What minimum separation d_{min} must the capacitor plates have?

53. || The combustion of fossil fuels produces micron-sized particles of soot, one of the major components of air pollution. The terminal speeds of these particles are extremely small, so they remain suspended in air for very long periods of time. Furthermore, very small particles almost always acquire small amounts of charge from cosmic rays and various atmospheric effects, so their motion is influenced not only by gravity but also by the earth's weak electric field. Consider a small spherical particle of radius r, density ρ, and charge q. A small sphere moving with speed v experiences a drag force $F_{drag} = 6\pi\eta r v$, where η is the viscosity of the air. (This differs from the drag force you learned in Chapter 6 because there we considered macroscopic rather than microscopic objects.)

a. A particle falling at its terminal speed v_{term} is in dynamic equilibrium with no net force. Write Newton's first law for this particle falling in the presence of a *downward* electric field of strength E, then solve to find an expression for v_{term}.

b. Soot is primarily carbon, and carbon in the form of graphite has a density of 2200 kg/m^3. In the absence of an electric field, what is the terminal speed in mm/s of a 1.0-μm-diameter graphite particle? The viscosity of air at 20°C is 1.8×10^{-5} kg/m s.

c. The earth's electric field is typically (150 N/C, downward). In this field, what is the terminal speed in mm/s of a 1.0-μm-diameter graphite particle that has acquired 250 extra electrons?

54. || A 2.0-mm-diameter glass sphere has a charge of +1.0 nC. What speed does an electron need to orbit the sphere 1.0 mm above the surface?

55. || In a classical model of the hydrogen atom, the electron orbits the proton in a circular orbit of radius 0.053 nm. What is the orbital frequency? The proton is so much more massive than the electron that you can assume the proton is at rest.

56. ||| In a classical model of the hydrogen atom, the electron orbits a stationary proton in a circular orbit. What is the radius of the orbit for which the orbital frequency is 1.0×10^{12} s^{-1}?

57. || An electric field can *induce* an electric dipole in a neutral atom or molecule by pushing the positive and negative charges in opposite directions. The dipole moment of an induced dipole is directly proportional to the electric field. That is, $\vec{p} = \alpha\vec{E}$, where α is called the *polarizability* of the molecule. A bigger field stretches the molecule farther and causes a larger dipole moment.

a. What are the units of α?

b. An ion with charge q is distance r from a molecule with polarizability α. Find an expression for the force $\vec{F}_{\text{ion on dipole}}$.

58. || Show that an infinite line of charge with linear charge density λ exerts an attractive force on an electric dipole with magnitude $F = 2\lambda p/4\pi\epsilon_0 r^2$. Assume that r is much larger than the charge separation in the dipole.

In Problems 59 through 62 you are given the equation(s) used to solve a problem. For each of these

a. Write a realistic problem for which this is the correct equation(s).

b. Finish the solution of the problem.

59. $$(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\frac{(2.0 \times 10^{-9}\text{ C})s}{(0.025\text{ m})^3} = 1150 \text{ N/C}$$

60. $$(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)\frac{2(2.0 \times 10^{-7}\text{ C/m})}{r} = 25{,}000 \text{ N/C}$$

61. $$\frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right] = \frac{1}{2}\frac{\eta}{2\epsilon_0}$$

62. $$2.0 \times 10^{12}\text{ m/s}^2 = \frac{(1.60 \times 10^{-19}\text{ C})E}{(1.67 \times 10^{-27}\text{ kg})}$$

$$E = \frac{Q}{(8.85 \times 10^{-12}\text{ C}^2/\text{N m}^2)(0.020\text{ m})^2}$$

Challenge Problems

63. Your physics assignment is to figure out a way to use electricity to launch a small 6.0-cm-long plastic drink stirrer. You decide that you'll charge the little plastic rod by rubbing it with fur, then hold it near a long, charged wire, as shown in FIGURE CP26.63. When you let go, the electric force of the wire on the plastic rod will shoot it away. Suppose you can uniformly charge the plastic stirrer to 10 nC and that the linear charge density of the long wire is 1.0×10^{-7} C/m. What is the net electric force on the plastic stirrer if the end closest to the wire is 2.0 cm away?

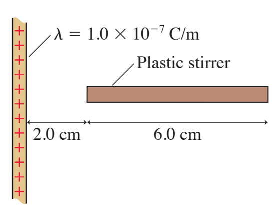


FIGURE CP26.63

64. Three 10-cm-long rods form an equilateral triangle in a plane. Two of the rods are charged to $+10$ nC, the third to -10 nC. What is the electric field strength at the center of the triangle?
65. A rod of length L lies along the y-axis with its center at the origin. The rod has a nonuniform linear charge density $\lambda = a|y|$, where a is a constant with the units C/m^2.
    a. Draw a graph of λ versus y over the length of the rod.
    b. Determine the constant a in terms of L and the rod's total charge Q.

    Hint: This requires an integration. Think about how to handle the absolute value sign.

    c. Find the electric field strength of the rod at distance x on the x-axis.
66. a. An infinitely long *sheet* of charge of width L lies in the xy-plane between $x = -L/2$ and $x = L/2$. The surface charge density is η. Derive an expression for the electric field $\vec{E}$ at height z above the centerline of the sheet.
    b. Verify that your expression has the expected behavior if $z \ll L$ and if $z \gg L$.
    c. Draw a graph of field strength E versus z.
67. a. An infinitely long *sheet* of charge of width L lies in the xy-plane between $x = -L/2$ and $x = L/2$. The surface charge density is η. Derive an expression for the electric field $\vec{E}$ along the x-axis for points outside the sheet ($x > L/2$).
    b. Verify that your expression has the expected behavior if $x \gg L$.

    Hint: $\ln(1 + u) \approx u$ if $u \ll 1$.

    c. Draw a graph of field strength E versus x for $x > L/2$.
68. One type of ink-jet printer, called an electrostatic ink-jet printer, forms the letters by using deflecting electrodes to steer charged ink drops up and down vertically as the ink jet sweeps horizontally across the page. The ink jet forms 30-μm-diameter drops of ink, charges them by spraying 800,000 electrons on the surface, and shoots them toward the page at a speed of 20 m/s. Along the way, the drops pass through two horizontal, parallel electrodes that are 6.0 mm long, 4.0 mm wide, and spaced 1.0 mm apart. The distance from the center of the electrodes to the paper is 2.0 cm. To form the tallest letters, which have a height of 6.0 mm, the drops need to be deflected upward (or downward) by 3.0 mm. What electric field strength is needed between the electrodes to achieve this deflection? Ink, which consists of dye particles suspended in alcohol, has a density of $800\ \text{kg/m}^3$.
69. A proton orbits a long charged wire, making 1.0×10^6 revolutions per second. The radius of the orbit is 1.0 cm. What is the wire's linear charge density?
70. A *positron* is an elementary particle identical to an electron except that its charge is $+e$. An electron and a positron can rotate about their center of mass as if they were a dumbbell connected by a massless rod. What is the orbital frequency for an electron and a positron 1.0 nm apart?
71. You have a summer intern position with a company that designs and builds nanomachines. An engineer with the company is designing a microscopic oscillator to help keep time, and you've been assigned to help him analyze the design. He wants to place a negative charge at the center of a very small, positively charged metal ring. His claim is that the negative charge will undergo simple harmonic motion at a frequency determined by the amount of charge on the ring.
    a. Consider a negative charge near the center of a positively charged ring centered on the z-axis. Show that there is a restoring force on the charge if it moves along the z-axis but stays close to the center of the ring. That is, show there's a force that tries to keep the charge at $z = 0$.
    b. Show that for *small* oscillations, with amplitude $\ll R$, a particle of mass m with charge $-q$ undergoes simple harmonic motion with frequency

    $$f = \frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\epsilon_0 m R^3}}$$

    R and Q are the radius and charge of the ring.

    c. Evaluate the oscillation frequency for an electron at the center of a 2.0-μm-diameter ring charged to 1.0×10^{-13} C.

STOP TO THINK ANSWERS

Stop to Think 26.1: c. From symmetry, the fields of the positive charges cancel. The net field is that of the negative charge, which is toward the charge.

Stop to Think 26.2: $\eta_c = \eta_b = \eta_a$. All pieces of a uniformly charged surface have the same surface charge density.

Stop to Think 26.3: b, e, and h. b and e both increase the linear charge density λ.

Stop to Think 26.4: $E_a = E_b = E_c = E_d = E_e$. The field strength of a charged plane is the same at all distances from the plane. An electric field diagram shows the electric field vectors at only a few points; the field exists at all points.

Stop to Think 26.5: $F_a = F_b = F_c = F_d = F_e$. The field strength inside a capacitor is the same at all points, hence the force on a charge is the same at all points. The electric field exists at all points whether or not a vector is shown at that point.

Stop to Think 26.6: c. Parabolic trajectories require *constant* acceleration and thus a *uniform* electric field. The proton has an initial velocity component to the left, but it's being pushed back to the right.

SUMMARY

The goal of Chapter 27 has been to understand and apply Gauss's law.

General Principles

Gauss's Law

For any *closed* surface enclosing net charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

The electric flux Φ_e is the same for *any* closed surface enclosing charge Q_{in}.

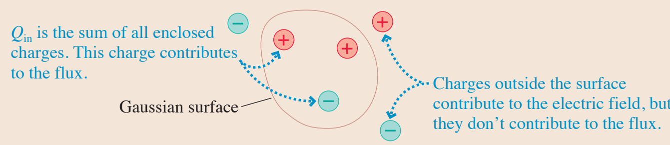
Symmetry

The symmetry of the electric field must match the symmetry of the charge distribution.

In practice, Φ_e is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

Important Concepts

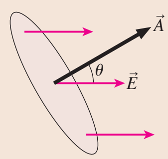
Charge creates the electric field that is responsible for the electric flux.



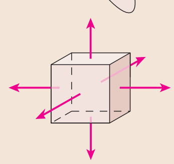
Flux is the amount of electric field passing through a surface of area A:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

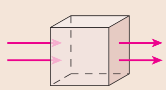
where $\vec{A}$ is the **area vector.**



For closed surfaces:
A net flux in or out indicates that the surface encloses a net charge.



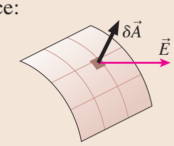
Field lines through but with no *net* flux mean that the surface encloses no *net* charge.



Surface integrals calculate the flux by summing the fluxes through many small pieces of the surface:

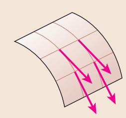
$$\Phi_e = \sum \vec{E} \cdot \delta\vec{A}$$

$$\rightarrow \int \vec{E} \cdot d\vec{A}$$



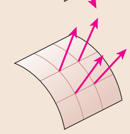
Two important situations:
If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$



If the electric field is everywhere perpendicular to the surface *and* has the same strength E at all points, then

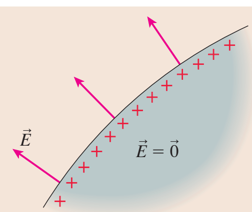
$$\Phi_e = EA$$



Applications

Conductors in electrostatic equilibrium

- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.



Terms and Notation

symmetric
Gaussian surface
electric flux, Φ_e
area vector, $\vec{A}$
surface integral
Gauss's law
screening

CONCEPTUAL QUESTIONS

1. Suppose you have the uniformly charged cube in FIGURE Q27.1. Can you use symmetry alone to deduce the *shape* of the cube's electric field? If so, sketch and describe the field shape. If not, why not?

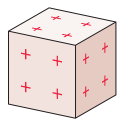


FIGURE Q27.1

2. FIGURE Q27.2 shows cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page. However, the electric field is everywhere parallel to the page, so there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. For each, does the surface enclose a net positive charge, a net negative charge, or no net charge? Explain.

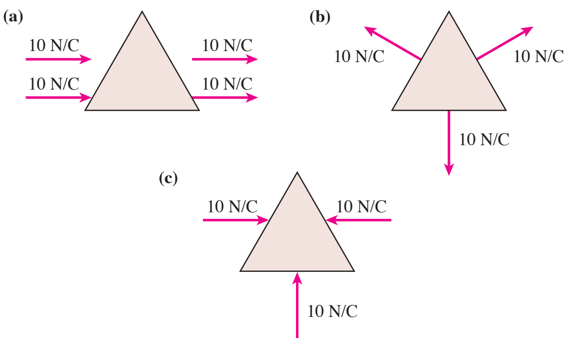


FIGURE Q27.2

3. The square and circle in FIGURE Q27.3 are in the same uniform field. The diameter of the circle equals the edge length of the square. Is Φ_{square} larger than, smaller than, or equal to Φ_{circle}? Explain.

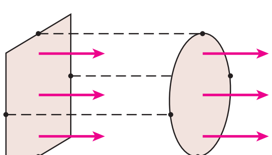


FIGURE Q27.3

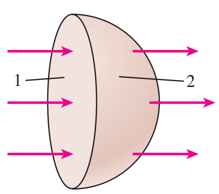


FIGURE Q27.4

4. In FIGURE Q27.4, where the field is uniform, is Φ_1 larger than, smaller than, or equal to Φ_2? Explain.
5. What is the electric flux through each of the surfaces in FIGURE Q27.5? Give each answer as a multiple of q/ϵ_0.

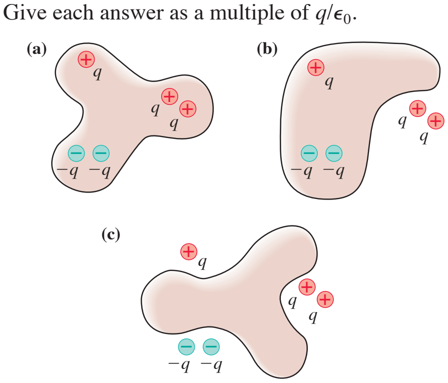


FIGURE Q27.5

6. What is the electric flux through each of the surfaces A to E in FIGURE Q27.6? Give each answer as a multiple of q/ϵ_0.

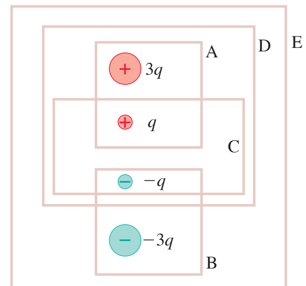


FIGURE Q27.6

7. The charged balloon in FIGURE Q27.7 expands as it is blown up, increasing in size from the initial to final diameters shown. Do the electric field strengths at points 1, 2, and 3 increase, decrease, or stay the same? Explain your reasoning for each.

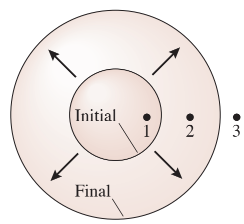


FIGURE Q27.7

8. The two spheres in FIGURE Q27.8 surround equal charges. Three students are discussing the situation.
Student 1: The fluxes through spheres A and B are equal because they enclose equal charges.
Student 2: But the electric field on sphere B is weaker than the electric field on sphere A. The flux depends on the electric field strength, so the flux through A is larger than the flux through B.
Student 3: I thought we learned that flux was about surface area. Sphere B is larger than sphere A, so I think the flux through B is larger than the flux through A.
Which of these students, if any, do you agree with? Explain.

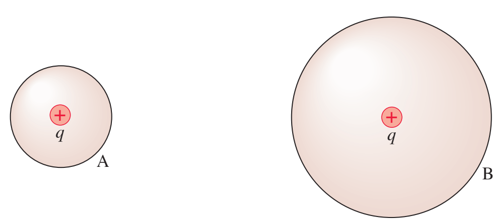


FIGURE Q27.8

9. The sphere and ellipsoid in FIGURE Q27.9 surround equal charges. Four students are discussing the situation.
   Student 1: The fluxes through A and B are equal because the average radius is the same.
   Student 2: I agree that the fluxes are equal, but that's because they enclose equal charges.
   Student 3: The electric field is not perpendicular to the surface for B, and that makes the flux through B less than the flux through A.
   Student 4: I don't think that Gauss's law even applies to a situation like B, so we can't compare the fluxes through A and B.
   Which of these students, if any, do you agree with? Explain.

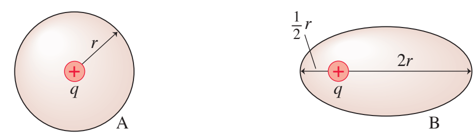


FIGURE Q27.9

10. A small, metal sphere hangs by an insulating thread within the larger, hollow conducting sphere of FIGURE Q27.10. A conducting wire extends from the small sphere through, but not touching, a small hole in the hollow sphere. A charged rod is used to transfer positive charge to the protruding wire. After the charged rod has touched the wire and been removed, are the following surfaces positive, negative, or not charged? Explain.
    a. The small sphere.
    b. The inner surface of the hollow sphere.
    c. The outer surface of the hollow sphere.

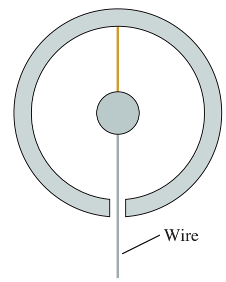


FIGURE Q27.10

EXERCISES AND PROBLEMS

Exercises

Section 27.1 Symmetry

1. | FIGURE EX27.1 shows two cross sections of two infinitely long coaxial cylinders. The inner cylinder has a positive charge, the outer cylinder has an equal negative charge. Draw this figure on your paper, then draw electric field vectors showing the shape of the electric field.

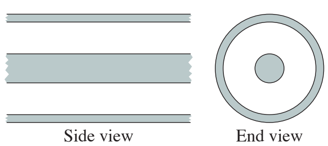


FIGURE EX27.1

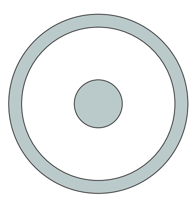


FIGURE EX27.2

2. | FIGURE EX27.2 shows a cross section of two concentric spheres. The inner sphere has a negative charge. The outer sphere has a positive charge larger in magnitude than the charge on the inner sphere. Draw this figure on your paper, then draw electric field vectors showing the shape of the electric field.
3. | FIGURE EX27.3 shows a cross section of two infinite parallel planes of charge. Draw this figure on your paper, then draw electric field vectors showing the shape of the electric field.

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FIGURE EX27.3

Section 27.2 The Concept of Flux

4. | The electric field is constant over each face of the cube shown in FIGURE EX27.4. Does the box contain positive charge, negative charge, or no charge? Explain.

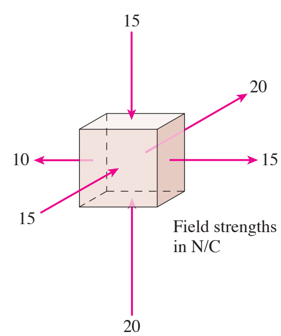


FIGURE EX27.4

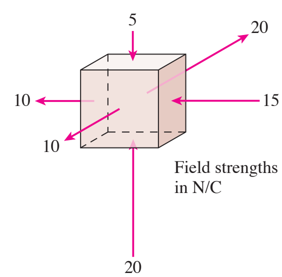


FIGURE EX27.5

5. | The electric field is constant over each face of the cube shown in FIGURE EX27.5. Does the box contain positive charge, negative charge, or no charge? Explain.
6. | The cube in FIGURE EX27.6 contains negative charge. The electric field is constant over each face of the cube. Does the missing electric field vector on the front face point in or out? What strength must this field exceed?

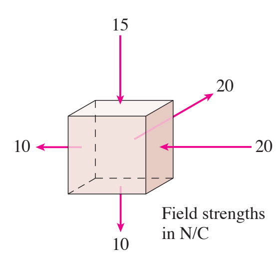


FIGURE EX27.6

7. | The cube in **FIGURE EX27.7** contains negative charge. The electric field is constant over each face of the cube. Does the missing electric field vector on the front face point in or out? What strength must this field exceed?

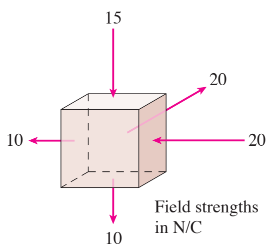


FIGURE EX27.7

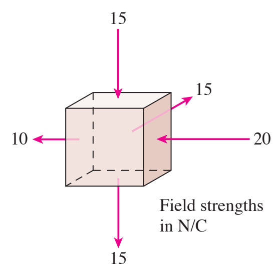


FIGURE EX27.8

8. | The cube in **FIGURE EX27.8** contains no net charge. The electric field is constant over each face of the cube. Does the missing electric field vector on the front face point in or out? What is the field strength?

Section 27.3 Calculating Electric Flux

9. || What is the electric flux through the surface shown in **FIGURE EX27.9**?

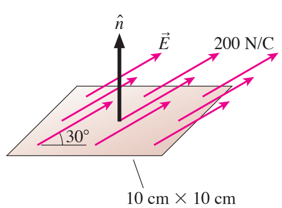


FIGURE EX27.9

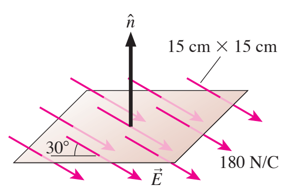


FIGURE EX27.10

10. || What is the electric flux through the surface shown in **FIGURE EX27.10**?

11. || The electric flux through the surface shown in **FIGURE EX27.11** is 25 N m^2/C. What is the electric field strength?

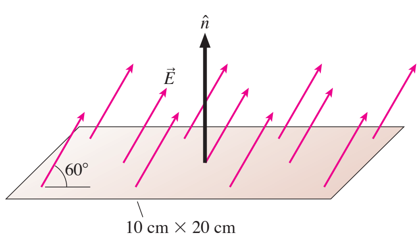


FIGURE EX27.11

12. || A 2.0 cm × 3.0 cm rectangle lies in the xy-plane. What is the electric flux through the rectangle if
    a. $\vec{E} = (100\hat{\imath} + 50\hat{k})$ N/C?
    b. $\vec{E} = (100\hat{\imath} + 50\hat{\jmath})$ N/C?

13. || A 2.0 cm × 3.0 cm rectangle lies in the xz-plane. What is the electric flux through the rectangle if
    a. $\vec{E} = (100\hat{\imath} + 50\hat{k})$ N/C?
    b. $\vec{E} = (100\hat{\imath} + 50\hat{\jmath})$ N/C?

14. || A 3.0-cm-diameter circle lies in the xz-plane in a region where the electric field is $\vec{E} = (1500\hat{\imath} + 1500\hat{\jmath} - 1500\hat{k})$ N/C. What is the electric flux through the circle?

15. || A 1.0 cm × 1.0 cm × 1.0 cm box with its edges aligned with the xyz-axes is in the electric field $\vec{E} = (350x + 150)\hat{\imath}$ N/C, where x is in meters. What is the net electric flux through the box?

16. | What is the net electric flux through the two cylinders shown in **FIGURE EX27.16**? Give your answer in terms of R and E.

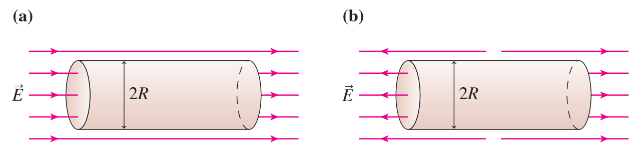
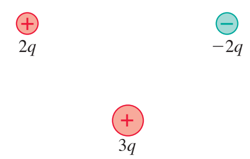


FIGURE EX27.16

Section 27.4 Gauss's Law

Section 27.5 Using Gauss's Law

17. | **FIGURE EX27.17** shows three charges. Draw these charges on your paper four times. Then draw two-dimensional cross sections of three-dimensional closed surfaces through which the electric flux is (a) $2q/\epsilon_0$, (b) q/ϵ_0, (c) 0, and (d) $5q/\epsilon_0$.



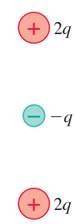
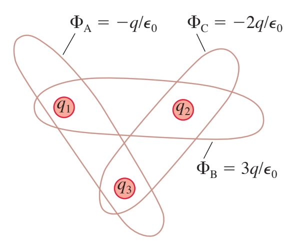


FIGURE EX27.17 FIGURE EX27.18

18. | **FIGURE EX27.18** shows three charges. Draw these charges on your paper four times. Then draw two-dimensional cross sections of three-dimensional closed surfaces through which the electric flux is (a) $-q/\epsilon_0$, (b) q/ϵ_0, (c) $3q/\epsilon_0$, and (d) $4q/\epsilon_0$.

19. | **FIGURE EX27.19** shows three Gaussian surfaces and the electric flux through each. What are the three charges q_1, q_2, and q_3?



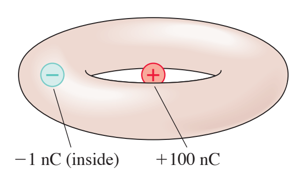


FIGURE EX27.19 FIGURE EX27.20

20. || What is the net electric flux through the torus (i.e., doughnut shape) of **FIGURE EX27.20**?

21. | What is the net electric flux through the cylinder of **FIGURE EX27.21**?

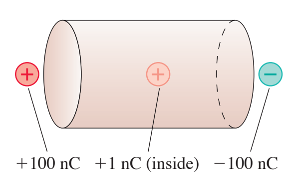


FIGURE EX27.21

22. || The net electric flux through an octahedron is −1000 N m^2/C. How much charge is enclosed within the octahedron?

23. || 55.3 million excess electrons are inside a closed surface. What is the net electric flux through the surface?

Section 27.6 Conductors in Electrostatic Equilibrium

24. | The electric field strength just above one face of a copper penny is 2000 N/C. What is the surface charge density on this face of the penny?

25. | A spark occurs at the tip of a metal needle if the electric field strength exceeds 3.0×10^6 N/C, the field strength at which air breaks down. What is the minimum surface charge density for producing a spark?

26. | The conducting box in FIGURE EX27.26 has been given an excess negative charge. The surface density of excess electrons at the center of the top surface is 5.0×10^{10} electrons/m^2. What are the electric field strengths E_1 to E_3 at points 1 to 3?

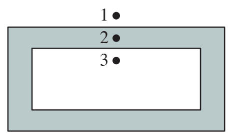


FIGURE EX27.26

27. | A thin, horizontal, 10-cm-diameter copper plate is charged to 3.5 nC. If the electrons are uniformly distributed on the surface, what are the strength and direction of the electric field
a. 0.1 mm above the center of the top surface of the plate?
b. at the plate's center of mass?
c. 0.1 mm below the center of the bottom surface of the plate?

28. || FIGURE EX27.28 shows a hollow cavity within a neutral conductor. A point charge Q is inside the cavity. What is the net electric flux through the closed surface that surrounds the conductor?

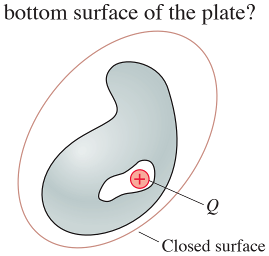


FIGURE EX27.28

Problems

29. | FIGURE P27.29 shows four sides of a 3.0 cm × 3.0 cm × 3.0 cm cube.
a. What are the electric fluxes Φ_1 to Φ_4 through sides 1 to 4?
b. What is the net flux through these four sides?

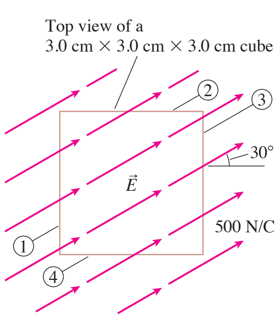


FIGURE P27.29

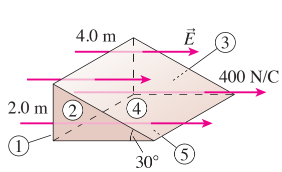


FIGURE P27.30

30. ||| Find the electric fluxes Φ_1 to Φ_5 through surfaces 1 to 5 in FIGURE P27.30.

31. || A tetrahedron has an equilateral triangle base with 20-cm-long edges and three equilateral triangle sides. The base is parallel to the ground, and a vertical uniform electric field of strength 200 N/C passes upward through the tetrahedron.
a. What is the electric flux through the base?
b. What is the electric flux through each of the three sides?

32. | Charges $q_1 = -4Q$ and $q_2 = +2Q$ are located at $x = -a$ and $x = +a$, respectively. What is the net electric flux through a sphere of radius $2a$ centered (a) at the origin and (b) at $x = 2a$?

33. || A 10 nC point charge is at the center of a 2.0 m × 2.0 m × 2.0 m cube. What is the electric flux through the top surface of the cube?

34. || The electric flux is 300 N m^2/C through two opposing faces of a 2.0 cm × 2.0 cm × 2.0 cm box. The flux through each of the other faces is 100 N m^2/C. How much charge is inside the box?

35. || A spherically symmetric charge distribution produces the electric field $\vec{E} = (200/r)\hat{r}$ N/C, where r is in m.
a. What is the electric field strength at $r = 10$ cm?
b. What is the electric flux through a 20-cm-diameter spherical surface that is concentric with the charge distribution?
c. How much charge is inside this 20-cm-diameter spherical surface?

36. || A spherically symmetric charge distribution produces the electric field $\vec{E} = (5000r^2)\hat{r}$ N/C, where r is in m.
a. What is the electric field strength at $r = 20$ cm?
b. What is the electric flux through a 40-cm-diameter spherical surface that is concentric with the charge distribution?
c. How much charge is inside this 40-cm-diameter spherical surface?

37. || A neutral conductor contains a hollow cavity in which there is a +100 nC point charge. A charged rod then transfers −50 nC to the conductor. Afterward, what is the charge (a) on the inner wall of the cavity wall, and (b) on the exterior surface of the conductor?

38. || A hollow metal sphere has inner radius a and outer radius b. The hollow sphere has charge $+2Q$. A point charge $+Q$ sits at the center of the hollow sphere.
a. Determine the electric fields in the three regions $r \le a$, $a < r < b$, and $r \ge b$.
b. How much charge is on the inside surface of the hollow sphere? On the exterior surface?

39. || A 20-cm-radius ball is uniformly charged to 80 nC.
a. What is the ball's volume charge density (C/m^3)?
b. How much charge is enclosed by spheres of radii 5, 10, and 20 cm?
c. What is the electric field strength at points 5, 10, and 20 cm from the center?

40. || FIGURE P27.40 shows a solid metal sphere at the center of a hollow metal sphere. What is the total charge on (a) the exterior of the inner sphere, (b) the inside surface of the hollow sphere, and (c) the exterior surface of the hollow sphere?

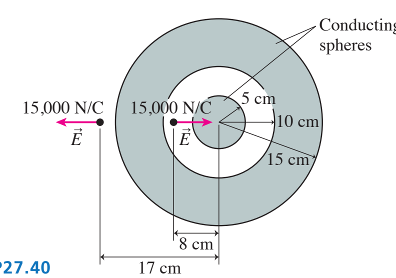


FIGURE P27.40

41. || The earth has a vertical electric field at the surface, pointing down, that averages 100 N/C. This field is maintained by various atmospheric processes, including lightning. What is the excess charge on the surface of the earth?

42. ‖ Figure 27.32b showed a conducting box inside a parallel-plate capacitor. The electric field inside the box is $\vec{E} = \vec{0}$. Suppose the surface charge on the exterior of the box could be frozen. Draw a picture of the electric field inside the box after the box, with its frozen charge, is removed from the capacitor.
Hint: Superposition.

43. ‖ A hollow metal sphere has 6 cm and 10 cm inner and outer radii, respectively. The surface charge density on the inside surface is $-100\ \text{nC/m}^2$. The surface charge density on the exterior surface is $+100\ \text{nC/m}^2$. What are the strength and direction of the electric field at points 4, 8, and 12 cm from the center?

44. ‖ A positive point charge q sits at the center of a hollow spherical shell. The shell, with radius R and negligible thickness, has net charge $-2q$. Find an expression for the electric field strength (a) inside the sphere, $r < R$, and (b) outside the sphere, $r > R$. In what direction does the electric field point in each case?

45. ‖ Find the electric field inside and outside a hollow plastic ball of radius R that has charge Q uniformly distributed on its outer surface.

46. ‖ A uniformly charged ball of radius a and charge $-Q$ is at the center of a hollow metal shell with inner radius b and outer radius c. The hollow sphere has net charge $+2Q$. Determine the electric field strength in the four regions $r \le a$, $a < r < b$, $b \le r \le c$, and $r > c$.

47. | The three parallel planes of charge shown in FIGURE P27.47 have surface charge densities $-\frac{1}{2}\eta$, η, and $-\frac{1}{2}\eta$. Find the electric fields $\vec{E}_1$ to $\vec{E}_4$ in regions 1 to 4.

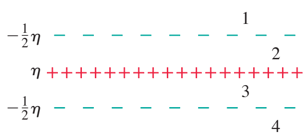


FIGURE P27.47

48. ‖ An infinite slab of charge of thickness $2z_0$ lies in the xy-plane between $z = -z_0$ and $z = +z_0$. The volume charge density ρ (C/m^3) is a constant.
   a. Use Gauss's law to find an expression for the electric field strength inside the slab ($-z_0 \le z \le z_0$).
   b. Find an expression for the electric field strength above the slab ($z \ge z_0$).
   c. Draw a graph of E from $z = 0$ to $z = 3z_0$.

49. ‖ FIGURE P27.49 shows an infinitely wide conductor parallel to and distance d from an infinitely wide plane of charge with surface charge density η. What are the electric fields $\vec{E}_1$ to $\vec{E}_4$ in regions 1 to 4?

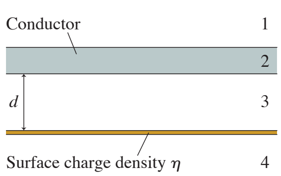


FIGURE P27.49

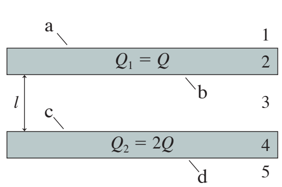


FIGURE P27.50

50. ‖ FIGURE P27.50 shows two very large slabs of metal that are parallel and distance l apart. Each slab has a total surface area (top + bottom) A. The thickness of each slab is so small in comparison to its lateral dimensions that the surface area around the sides is negligible. Metal 1 has total charge $Q_1 = Q$ and metal 2 has total charge $Q_2 = 2Q$. Assume Q is positive. In terms of Q and A, determine
   a. The electric field strengths E_1 to E_5 in regions 1 to 5.
   b. The surface charge densities η_a to η_d on the four surfaces a to d.

51. ‖ A long, thin straight wire with linear charge density λ runs down the center of a thin, hollow metal cylinder of radius R. The cylinder has a net linear charge density 2λ. Assume λ is positive. Find expressions for the electric field strength (a) inside the cylinder, $r < R$, and (b) outside the cylinder, $r > R$. In what direction does the electric field point in each of the cases?

52. ‖ A very long, uniformly charged cylinder has radius R and linear charge density λ. Find the cylinder's electric field (a) outside the cylinder, $r \ge R$, and (b) inside the cylinder, $r \le R$. (c) Show that your answers to parts a and b match at the boundary, $r = R$.

53. ‖ A spherical shell has inner radius R_{in} and outer radius R_{out}. The shell contains total charge Q, uniformly distributed. The interior of the shell is empty of charge and matter.
   a. Find the electric field outside the shell, $r \ge R_{out}$.
   b. Find the electric field in the interior of the shell, $r \le R_{in}$.
   c. Find the electric field within the shell, $R_{in} \le r \le R_{out}$.
   d. Show that your solutions match at both the inner and outer boundaries.

54. ‖ An early model of the atom, proposed by Rutherford after his discovery of the atomic nucleus, had a positive point charge $+Ze$ (the nucleus) at the center of a sphere of radius R with uniformly distributed negative charge $-Ze$. Z is the atomic number, the number of protons in the nucleus and the number of electrons in the negative sphere.
   a. Show that the electric field inside this atom is

$$E_{in} = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

   b. What is E at the surface of the atom? Is this the expected value? Explain.
   c. A uranium atom has $Z = 92$ and $R = 0.10$ nm. What is the electric field strength at $r = \frac{1}{2}R$?

Challenge Problems

55. All examples of Gauss's law have used highly symmetric surfaces where the flux integral is either zero or EA. Yet we've claimed that the net $\Phi_e = Q_{in}/\epsilon_0$ is independent of the surface. This is worth checking. FIGURE CP27.55 shows a cube of edge length L centered on a long thin wire with linear charge density λ. The flux through one face of the cube is *not* simply EA because, in this case, the electric field varies in both strength and direction. But you can calculate the flux by actually doing the flux integral.

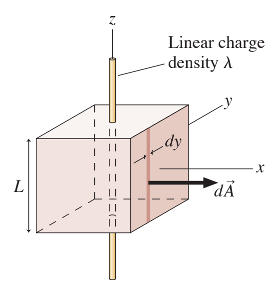


FIGURE CP27.55

a. Consider the face parallel to the yz-plane. Define area $d\vec{A}$ as a strip of width dy and height L with the vector pointing in the x-direction. One such strip is located at position y. Use the known electric field of a wire to calculate the electric flux $d\Phi$ through this little area. Your expression should be written in terms of y, which is a variable, and various constants. It should not explicitly contain any angles.
b. Now integrate $d\Phi$ to find the total flux through this face.
c. Finally, show that the net flux through the cube is $\Phi_e = Q_{in}/\epsilon_0$.

56. An infinite cylinder of radius R has a linear charge density λ. The volume charge density (C/m^3) within the cylinder ($r \le R$) is $\rho(r) = r\rho_0/R$, where ρ_0 is a constant to be determined.
a. Draw a graph of ρ versus x for an x-axis that crosses the cylinder perpendicular to the cylinder axis. Let x range from $-2R$ to $2R$.
b. The charge within a small volume dV is $dq = \rho\, dV$. The integral of $\rho\, dV$ over a cylinder of length L is the total charge $Q = \lambda L$ within the cylinder. Use this fact to show that $\rho_0 = 3\lambda/2\pi R^2$.
Hint: Let dV be a cylindrical shell of length L, radius r, and thickness dr. What is the volume of such a shell?
c. Use Gauss's law to find an expression for the electric field E inside the cylinder, $r \le R$.
d. Does your expression have the expected value at the surface, $r = R$? Explain.

57. A sphere of radius R has total charge Q. The volume charge density (C/m^3) within the sphere is $\rho(r) = C/r^2$, where C is a constant to be determined.
a. The charge within a small volume dV is $dq = \rho\, dV$. The integral of $\rho\, dV$ over the entire volume of the sphere is the total charge Q. Use this fact to determine the constant C in terms of Q and R.
Hint: Let dV be a spherical shell of radius r and thickness dr. What is the volume of such a shell?
b. Use Gauss's law to find an expression for the electric field E inside the sphere, $r \le R$.
c. Does your expression have the expected value at the surface, $r = R$? Explain.

58. A sphere of radius R has total charge Q. The volume charge density (C/m^3) within the sphere is

$$\rho = \rho_0\left(1 - \frac{r}{R}\right)$$

This charge density decreases linearly from ρ_0 at the center to zero at the edge of the sphere.
a. Show that $\rho_0 = 3Q/\pi R^3$.
b. Show that the electric field inside the sphere points radially outward with magnitude

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - 3\frac{r}{R}\right)$$

c. Show that your result of part b has the expected value at $r = R$.

59. A spherical ball of charge has radius R and total charge Q. The electric field strength inside the ball ($r \le R$) is $E(r) = E_{max}(r^4/R^4)$.
a. What is E_{max} in terms of Q and R?
b. Find an expression for the volume charge density $\rho(r)$ inside the ball as a function of r.
c. Verify that your charge density gives the total charge Q when integrated over the volume of the ball.

STOP TO THINK ANSWERS

Stop to Think 27.1: a and d. Symmetry requires the electric field to be unchanged if front and back are reversed, if left and right are reversed, or if the field is rotated about the wire's axis. Fields a and d both have the proper symmetry. Other factors would now need to be considered to determine the correct field.

Stop to Think 27.2: e. The net flux is into the box.

Stop to Think 27.3: c. There's no flux through the four sides. The flux is positive 1 N m^2/C through both the top and bottom because $\vec{E}$ and $\vec{A}$ both point outward.

Stop to Think 27.4: $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$. The flux through a closed surface depends only on the amount of enclosed charge, not the size or shape of the surface.

Stop to Think 27.5: d. A cube doesn't have enough symmetry to use Gauss's law. The electric field of a charged cube is *not* constant over the face of a cubic Gaussian surface, so we can't evaluate the surface integral for the flux.

SUMMARY

The goals of Chapter 28 have been to calculate and use the electric potential and electric potential energy.

General Principles

Sources of *V*

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

Consequences of *V*

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + qV_f = K_i + qV_i$$

The potential energy of **two point charges** separated by distance r is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

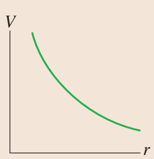
The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \rightarrow \infty$.

The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

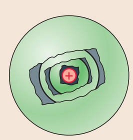
$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Applications

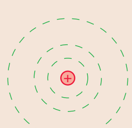
Graphical representations of the potential:



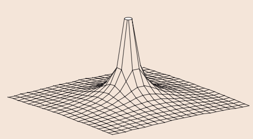
Potential graph



Equipotential surfaces



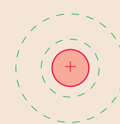
Contour map



Elevation graph

Sphere of charge *Q*

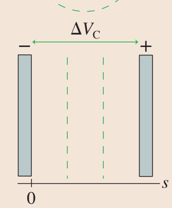
Same as a point charge if $r \geq R$



Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \frac{\Delta V_C}{d}$$



Units

Electric potential: 1 V = 1 J/C

Electric field: 1 V/m = 1 N/C

Terms and Notation

electric potential energy, U
electric potential, V
volt, V
potential difference, ΔV
voltage, ΔV
equipotential surface
contour map
elevation graph

CONCEPTUAL QUESTIONS

1. a. Charge q_1 is distance r from a positive point charge Q. Charge $q_2 = q_1/3$ is distance $2r$ from Q. What is the ratio U_1/U_2 of their potential energies due to their interactions with Q?
   b. Charge q_1 is distance s from the negative plate of a parallel-plate capacitor. Charge $q_2 = q_1/3$ is distance $2s$ from the negative plate. What is the ratio U_1/U_2 of their potential energies?
2. FIGURE Q28.2 shows the potential energy of a proton ($q = +e$) and a lead nucleus ($q = +82e$). The horizontal scale is in units of *femtometers*, where 1 fm $= 10^{-15}$ m.
   a. A proton is fired toward a lead nucleus from very far away. How much initial kinetic energy does the proton need to reach a turning point 10 fm from the nucleus? Explain.
   b. How much kinetic energy does the proton of part a have when it is 20 fm from the nucleus and moving toward it, before the collision?

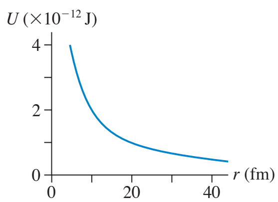


FIGURE Q28.2

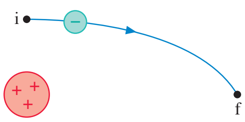


FIGURE Q28.3

3. An electron moves along the trajectory of FIGURE Q28.3 from i to f.
   a. Does the electric potential energy increase, decrease, or stay the same? Explain.
   b. Is the electron's speed at f greater than, less than, or equal to its speed at i? Explain.
4. Two protons are launched with the same speed from point 1 inside the parallel-plate capacitor of FIGURE Q28.4. Points 2 and 3 are the same distance from the negative plate.
   a. Is $\Delta U_{1\to 2}$, the change in potential energy along the path $1 \to 2$, larger than, smaller than, or equal to $\Delta U_{1\to 3}$?
   b. Is the proton's speed v_2 at point 2 larger than, smaller than, or equal to v_3? Explain.

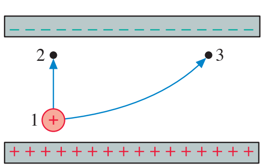


FIGURE Q28.4

5. Rank in order, from most positive to most negative, the potential energies U_a to U_f of the six electric dipoles in the uniform electric field of FIGURE Q28.5. Explain.

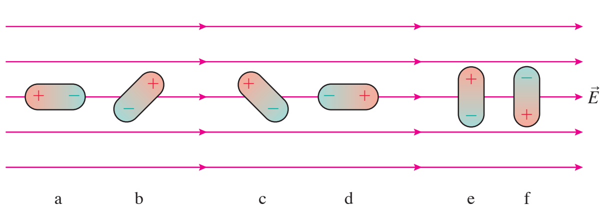


FIGURE Q28.5

6. FIGURE Q28.6 shows the electric potential along the x-axis.
   a. Draw a graph of the potential energy of a 0.1 C charged particle. Provide a numerical scale for both axes.
   b. If the charged particle is shot toward the right from $x = 1$ m with 1.0 J of kinetic energy, where is its turning point? Use your graph to explain.

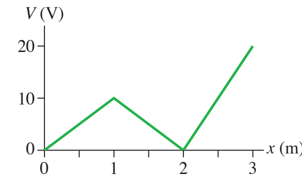


FIGURE Q28.6

7. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.
   a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?
   b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?
   c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?
8. Rank in order, from largest to smallest, the electric potentials V_a to V_e at points a to e in FIGURE Q28.8. Explain.

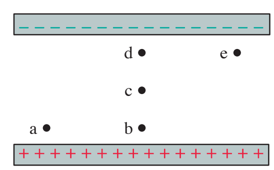


FIGURE Q28.8

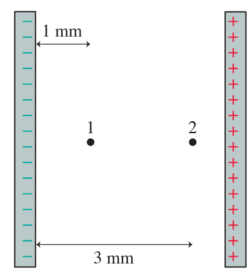


FIGURE Q28.9

9. FIGURE Q28.9 shows two points inside a capacitor. Let $V = 0$ V at the negative plate.
   a. What is the ratio V_2/V_1 of the electric potentials? Explain.
   b. What is the ratio E_2/E_1 of the electric field strengths?
10. FIGURE Q28.10 shows two points near a positive point charge.
   a. What is the ratio V_2/V_1 of the electric potentials? Explain.
   b. What is the ratio E_2/E_1 of the electric field strengths?

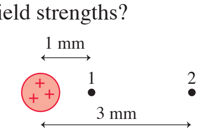


FIGURE Q28.10

11. FIGURE Q28.11 shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from most positive to most negative, the potentials V_a to V_c.

FIGURE Q28.11

12. Reproduce FIGURE Q28.12 on your paper. Then draw a dot (or dots) on the figure to show the position (or positions) at which the electric potential is zero.

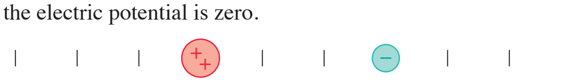


FIGURE Q28.12

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 28.1 Electric Potential Energy

1. The electric field strength is 50,000 N/C inside a parallel-plate capacitor with a 2.0 mm spacing. A proton is released from rest at the positive plate. What is the proton's speed when it reaches the negative plate?
2. The electric field strength is 20,000 N/C inside a parallel-plate capacitor with a 1.0 mm spacing. An electron is released from rest at the negative plate. What is the electron's speed when it reaches the positive plate?
3. A proton is released from rest at the positive plate of a parallel-plate capacitor. It crosses the capacitor and reaches the negative plate with a speed of 50,000 m/s. What will be the final speed of an electron released from rest at the negative plate?
4. A proton is released from rest at the positive plate of a parallel-plate capacitor. It crosses the capacitor and reaches the negative plate with a speed of 50,000 m/s. The experiment is repeated with a He^+ ion (charge e, mass 4 u). What is the ion's speed at the negative plate?

Section 28.2 The Potential Energy of Point Charges

5. What is the electric potential energy of the proton in FIGURE EX28.5? The electrons are fixed and cannot move.

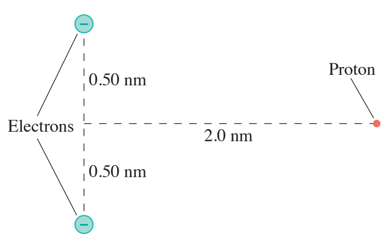


FIGURE EX28.5

6. What is the electric potential energy of the group of charges in FIGURE EX28.6?

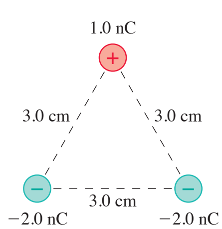


FIGURE EX28.6

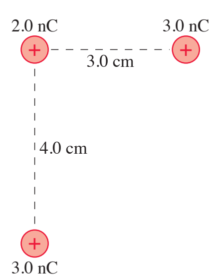


FIGURE EX28.7

7. What is the electric potential energy of the group of charges in FIGURE EX28.7?

Section 28.3 The Potential Energy of a Dipole

8. A water molecule perpendicular to an electric field has 1.0×10^{-21} J more potential energy than a water molecule aligned with the field. The dipole moment of a water molecule is 6.2×10^{-30} C m. What is the strength of the electric field?
9. FIGURE EX28.9 shows the potential energy of an electric dipole. Consider a dipole that oscillates between $\pm 60°$.
   a. What is the dipole's mechanical energy?
   b. What is the dipole's kinetic energy when it is aligned with the electric field?

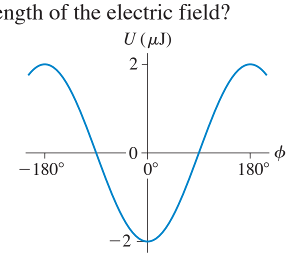


FIGURE EX28.9

Section 28.4 The Electric Potential

10. What is the speed of a proton that has been accelerated from rest through a potential difference of −1000 V?
11. What is the speed of an electron that has been accelerated from rest through a potential difference of 1000 V?
12. What potential difference is needed to accelerate an electron from rest to a speed of 2.0×10^6 m/s?
13. What potential difference is needed to accelerate a He^+ ion (charge $+e$, mass 4 u) from rest to a speed of 2.0×10^6 m/s?
14. A proton with an initial speed of 800,000 m/s is brought to rest by an electric field.
    a. Did the proton move into a region of higher potential or lower potential?
    b. What was the potential difference that stopped the proton?
15. An electron with an initial speed of 500,000 m/s is brought to rest by an electric field.
    a. Did the electron move into a region of higher potential or lower potential?
    b. What was the potential difference that stopped the electron?

Section 28.5 The Electric Potential Inside a Parallel-Plate Capacitor

16. Show that 1 V/m = 1 N/C.
17. a. What is the potential of an ordinary AA or AAA battery? (If you're not sure, find one and look at the label.)
    b. An AA battery is connected to a parallel-plate capacitor having 4.0 cm × 4.0 cm plates spaced 1.0 mm apart. How much charge does the battery supply to each plate?
18. Two 2.00 cm × 2.00 cm plates that form a parallel-plate capacitor are charged to ±0.708 nC. What are the electric field strength inside and the potential difference across the capacitor if the spacing between the plates is (a) 1.00 mm and (b) 2.00 mm?
19. A 3.0-cm-diameter parallel-plate capacitor has a 2.0 mm spacing. The electric field strength inside the capacitor is 1.0×10^5 V/m.
    a. What is the potential difference across the capacitor?
    b. How much charge is on each plate?

20. ‖ Two 2.0-cm-diameter disks spaced 2.0 mm apart form a parallel-plate capacitor. The electric field between the disks is 5.0×10^5 V/m.
   a. What is the voltage across the capacitor?
   b. An electron is launched from the negative plate. It strikes the positive plate at a speed of 2.0×10^7 m/s. What was the electron's speed as it left the negative plate?

Section 28.6 The Electric Potential of a Point Charge

21. | a. What is the electric potential at points A, B, and C in FIGURE EX28.21?
   b. What are the potential differences ΔV_{AB} and ΔV_{BC}?

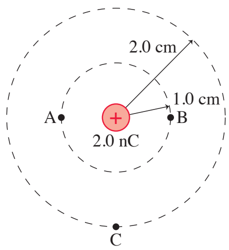


FIGURE EX28.21

22. ‖ A 1.0-mm-diameter ball bearing has 2.0×10^9 excess electrons. What is the ball bearing's potential?

23. | In a semiclassical model of the hydrogen atom, the electron orbits the proton at a distance of 0.053 nm.
   a. What is the electric potential of the proton at the position of the electron?
   b. What is the electron's potential energy?

Section 28.7 The Electric Potential of Many Charges

24. | What is the electric potential at the point indicated with the dot in FIGURE EX28.24?

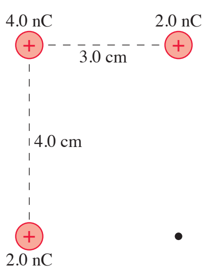


FIGURE EX28.24

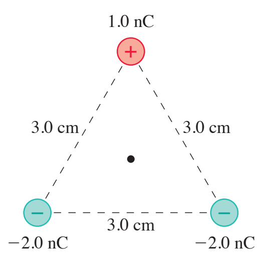


FIGURE EX28.25

25. | What is the electric potential at the point indicated with the dot in FIGURE EX28.25?

26. ‖ The electric potential at the dot in FIGURE EX28.26 is 3140 V. What is charge q?

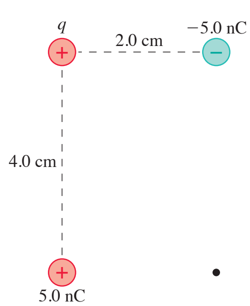


FIGURE EX28.26

27. ‖ A −2.0 nC charge and a +2.0 nC charge are located on the x-axis at $x = -1.0$ cm and $x = +1.0$ cm, respectively.
   a. Other than at infinity, is there a position or positions on the x-axis where the electric field is zero? If so, where?
   b. Other than at infinity, at what position or positions on the x-axis is the electric potential zero?
   c. Sketch graphs of the electric field strength and the electric potential along the x-axis.

28. ‖ Two point charges q_a and q_b are located on the x-axis at $x = a$ and $x = b$. FIGURE EX28.28 is a graph of E_x, the x-component of the electric field.
   a. What are the signs of q_a and q_b?
   b. What is the ratio $|q_a/q_b|$?
   c. Draw a graph of V, the electric potential, as a function of x.

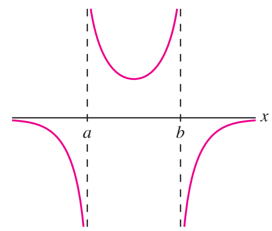


FIGURE EX28.28

29. ‖ Two point charges q_a and q_b are located on the x-axis at $x = a$ and $x = b$. FIGURE EX28.29 is a graph of V, the electric potential.
   a. What are the signs of q_a and q_b?
   b. What is the ratio $|q_a/q_b|$?
   c. Draw a graph of E_x, the x-component of the electric field, as a function of x.

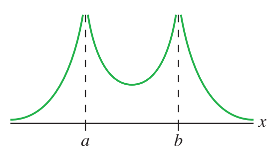


FIGURE EX28.29

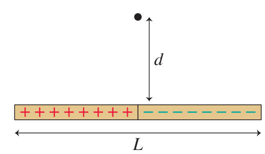


FIGURE EX28.30

30. | The two halves of the rod in FIGURE EX28.30 are uniformly charged to $\pm Q$. What is the electric potential at the point indicated by the dot?

Problems

31. | Two positive point charges are 5.0 cm apart. If the electric potential energy is 72 μJ, what is the magnitude of the force between the two charges?

32. ‖| Two point charges 2.0 cm apart have an electric potential energy −180 μJ. The total charge is 30 nC. What are the two charges?

33. ‖ A −10.0 nC point charge and a +20.0 nC point charge are 15.0 cm apart on the x-axis.
   a. What is the electric potential at the point on the x-axis where the electric field is zero?
   b. What is the magnitude of the electric field at the point on the x-axis, between the charges, where the electric potential is zero?

34. ‖| A +3.0 nC charge is at $x = 0$ cm and a −1.0 nC charge is at $x = 4$ cm. At what point or points on the x-axis is the electric potential zero?

35. ‖| A −3.0 nC charge is on the x-axis at $x = -9$ cm and a +4.0 nC charge is on the x-axis at $x = 16$ cm. At what point or points on the y-axis is the electric potential zero?

36. || Two small metal cubes with masses 2.0 g and 4.0 g are tied together by a 5.0-cm-long massless string and are at rest on a frictionless surface. Each is charged to $+2.0\ \mu C$.
   a. What is the energy of this system?
   b. What is the tension in the string?
   c. The string is cut. What is the speed of each cube when they are far apart?

   Hint: There are *two* conserved quantities. Make use of both.

37. || The four 1.0 g spheres shown in **FIGURE P28.37** are released simultaneously and allowed to move away from each other. What is the speed of each sphere when they are very far apart?

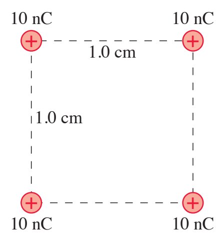


FIGURE P28.37

38. || A proton's speed as it passes point A is 50,000 m/s. It follows the trajectory shown in **FIGURE P28.38**. What is the proton's speed at point B?

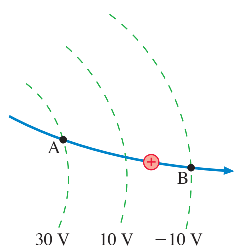


FIGURE P28.38

39. || Living cells "pump" singly ionized sodium ions, Na^+, from
BIO the inside of the cell to the outside to maintain a membrane potential $\Delta V_{membrane} = V_{in} - V_{out} = -70$ mV. It is called *pumping* because work must be done to move a positive ion from the negative inside of the cell to the positive outside, and it must go on continuously because sodium ions "leak" back through the cell wall by diffusion.
   a. How much work must be done to move one sodium ion from the inside of the cell to the outside?
   b. At rest, the human body uses energy at the rate of approximately 100 W to maintain basic metabolic functions. It has been estimated that 20% of this energy is used to operate the sodium pumps of the body. Estimate—to one significant figure—the number of sodium ions pumped per second.

40. || An arrangement of source charges produces the electric potential $V = 5000x^2$ along the x-axis, where V is in volts and x is in meters. What is the maximum speed of a 1.0 g, 10 nC charged particle that moves in this potential with turning points at ± 8.0 cm?

41. || A proton moves along the x-axis, where an arrangement of source charges has created the electric potential $V = 6000x^2$, where V is in volts and x is in meters. By exploiting the analogy with the potential energy of a mass on a spring, determine the proton's oscillation frequency.

42. || In **FIGURE P28.42**, a proton is fired with a speed of 200,000 m/s from the midpoint of the capacitor toward the positive plate.
   a. Show that this is insufficient speed to reach the positive plate.
   b. What is the proton's speed as it collides with the negative plate?

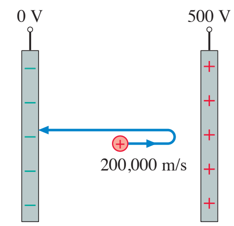


FIGURE P28.42

43. || The electron gun in an old TV picture tube accelerates electrons between two parallel plates 1.2 cm apart with a 25 kV potential difference between them. The electrons enter through a small hole in the negative plate, accelerate, then exit through a small hole in the positive plate. Assume that the holes are small enough not to affect the electric field or potential.
   a. What is the electric field strength between the plates?
   b. With what speed does an electron exit the electron gun if its entry speed is close to zero?

   NOTE ▶ The exit speed is so fast that we really need to use the theory of relativity to compute an accurate value. Your answer to part b is in the right range but a little too big. ◀

44. || An uncharged parallel-plate capacitor with spacing d is horizontal. A small bead with mass m and positive charge q is shot straight up from the bottom plate with speed v_0. It reaches maximum height y_{max} before falling back. Then the capacitor is charged with the bottom plate negative. Find an expression for the capacitor voltage ΔV_C for which the bead's maximum height is reduced to $\frac{1}{2}y_{max}$. Ignore air resistance.

45. || A room with 3.0-m-high ceilings has a metal plate on the floor with $V = 0$ V and a separate metal plate on the ceiling. A 1.0 g glass ball charged to +4.9 nC is shot straight up at 5.0 m/s. How high does the ball go if the ceiling voltage is (a) $+3.0 \times 10^6$ V and (b) -3.0×10^6 V?

46. || In *proton-beam therapy*, a high-energy beam of protons is
BIO fired at a tumor. As the protons stop in the tumor, their kinetic energy breaks apart the tumor's DNA, thus killing the tumor cells. For one patient, it is desired to deposit 0.10 J of proton energy in the tumor. To create the proton beam, protons are accelerated from rest through a 10,000 kV potential difference. What is the total charge of the protons that must be fired at the tumor?

47. || What is the escape speed of an electron launched from the surface of a 1.0-cm-diameter glass sphere that has been charged to 10 nC?

48. || An electric dipole consists of 1.0 g spheres charged to ± 2.0 nC at the ends of a 10-cm-long massless rod. The dipole rotates on a frictionless pivot at its center. The dipole is held perpendicular to a uniform electric field with field strength 1000 V/m, then released. What is the dipole's angular velocity at the instant it is aligned with the electric field?

49. ||| Three electrons form an equilateral triangle 1.0 nm on each side. A proton is at the center of the triangle. What is the potential energy of this group of charges?

50. ||| A 2.0-mm-diameter glass bead is positively charged. The potential difference between a point 2.0 mm from the bead and a point 4.0 mm from the bead is 500 V. What is the charge on the bead?

51. ||| Your lab assignment for the week is to measure the amount of charge on the 6.0-cm-diameter metal sphere of a Van de Graaff generator. To do so, you're going to use a spring with spring constant 0.65 N/m to launch a small, 1.5 g bead horizontally toward the sphere. You can reliably charge the bead to 2.5 nC, and your plan is to use a video camera to measure the bead's closest approach to the sphere as you change the compression of the spring. Your data are as follows:

Compression (cm)	Closest approach (cm)
1.6	5.5
1.9	2.6
2.2	1.6
2.5	0.4

Use an appropriate graph of the data to determine the sphere's charge in nC. You can assume that the bead's motion is entirely horizontal and that the spring is so far away that the bead has no interaction with the sphere as it's launched.

52. || A proton is fired from far away toward the nucleus of an iron atom. Iron is element number 26, and the diameter of the nucleus is 9.0 fm. What initial speed does the proton need to just reach the surface of the nucleus? Assume the nucleus remains at rest.

53. || A proton is fired from far away toward the nucleus of a mercury atom. Mercury is element number 80, and the diameter of the nucleus is 14.0 fm. If the proton is fired at a speed of 4.0×10^7 m/s, what is its closest approach to the surface of the nucleus? Assume the nucleus remains at rest.

54. || In the form of radioactive decay known as *alpha decay,* an unstable nucleus emits a helium-atom nucleus, which is called an *alpha particle*. An alpha particle contains two protons and two neutrons, thus having mass $m = 4$ u and charge $q = 2e$. Suppose a uranium nucleus with 92 protons decays into thorium, with 90 protons, and an alpha particle. The alpha particle is initially at rest at the surface of the thorium nucleus, which is 15 fm in diameter. What is the speed of the alpha particle when it is detected in the laboratory? Assume the thorium nucleus remains at rest.

55. || One form of nuclear radiation, *beta decay,* occurs when a neutron changes into a proton, an electron, and a neutral particle called a *neutrino:* $n \rightarrow p^+ + e^- + \nu$ where ν is the symbol for a neutrino. When this change happens to a neutron within the nucleus of an atom, the proton remains behind in the nucleus while the electron and neutrino are ejected from the nucleus. The ejected electron is called a *beta particle*. One nucleus that exhibits beta decay is the isotope of hydrogen ^{3}H, called *tritium,* whose nucleus consists of one proton (making it hydrogen) and two neutrons (giving tritium an atomic mass $m = 3$ u). Tritium is radioactive, and it decays to helium: $^3\text{H} \rightarrow {}^3\text{He} + e^- + \nu$.
   a. Is charge conserved in the beta decay process? Explain.
   b. Why is the final product a helium atom? Explain.
   c. The nuclei of both ^{3}H and ^{3}He have radii of 1.5×10^{-15} m. With what minimum speed must the electron be ejected if it is to escape from the nucleus and not fall back?

56. || The sun is powered by *fusion*, with four protons fusing together to form a helium nucleus (two of the protons turn into neutrons) and, in the process, releasing a large amount of thermal energy. The process happens in several steps, not all at once. In one step, two protons fuse together, with one proton then becoming a neutron, to form the "heavy hydrogen" isotope *deuterium* (^{2}H). A proton is essentially a 2.4-fm-diameter sphere of charge, and fusion occurs only if two protons come into contact with each other. This requires extraordinarily high temperatures due to the strong repulsion between the protons. Recall that the average kinetic energy of a gas particle is $\frac{3}{2}k_BT$.
   a. Suppose two protons, each with exactly the average kinetic energy, have a head-on collision. What is the minimum temperature for fusion to occur?
   b. Your answer to part a is much hotter than the 15 million K in the core of the sun. If the temperature were as high as you calculated, every proton in the sun would fuse almost instantly and the sun would explode. For the sun to last for billions of years, fusion can occur only in collisions between two protons with kinetic energies much higher than average. Only a very tiny fraction of the protons have enough kinetic energy to fuse when they collide, but that fraction is enough to keep the sun going. Suppose two protons with the same kinetic energy collide head-on and just barely manage to fuse. By what factor does each proton's energy exceed the average kinetic energy at 15 million K?

57. || Two 10-cm-diameter electrodes 0.50 cm apart form a parallel-plate capacitor. The electrodes are attached by metal wires to the terminals of a 15 V battery. After a long time, the capacitor is disconnected from the battery but is not discharged. What are the charge on each electrode, the electric field strength inside the capacitor, and the potential difference between the electrodes
   a. Right after the battery is disconnected?
   b. After insulating handles are used to pull the electrodes away from each other until they are 1.0 cm apart?
   c. After the original electrodes (not the modified electrodes of part b) are expanded until they are 20 cm in diameter?

58. || Two 10-cm-diameter electrodes 0.50 cm apart form a parallel-plate capacitor. The electrodes are attached by metal wires to the terminals of a 15 V battery. What are the charge on each electrode, the electric field strength inside the capacitor, and the potential difference between the electrodes
   a. While the capacitor is attached to the battery?
   b. After insulating handles are used to pull the electrodes away from each other until they are 1.0 cm apart? The electrodes remain connected to the battery during this process.
   c. After the original electrodes (not the modified electrodes of part b) are expanded until they are 20 cm in diameter while remaining connected to the battery?

59. || a. Find an algebraic expression for the electric field strength E_0 at the surface of a charged sphere in terms of the sphere's potential V_0 and radius R.
   b. What is the electric field strength at the surface of a 1.0-cm-diameter marble charged to 500 V?

60. || Two spherical drops of mercury each have a charge of 0.10 nC and a potential of 300 V at the surface. The two drops merge to form a single drop. What is the potential at the surface of the new drop?

61. || A Van de Graaff generator is a device for generating a large electric potential by building up charge on a hollow metal sphere. A typical classroom-demonstration model has a diameter of 30 cm.
   a. How much charge is needed on the sphere for its potential to be 500,000 V?
   b. What is the electric field strength just outside the surface of the sphere when it is charged to 500,000 V?

62. || A thin spherical shell of radius R has total charge Q. What is the electric potential at the center of the shell?

63. | FIGURE P28.63 shows two uniformly charged spheres. What is the potential difference between points a and b? Which point is at the higher potential?

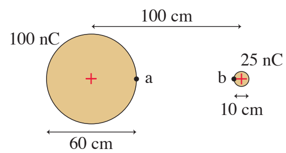


FIGURE P28.63

Hint: The potential at any point is the superposition of the potentials due to *all* charges.

64. || An electric dipole with dipole moment p is oriented along the y-axis.
   a. Find an expression for the electric potential on the y-axis at a point where y is much larger than the charge spacing s. Write your expression in terms of the dipole moment p.
   b. The dipole moment of a water molecule is 6.2×10^{-30} C m. What is the electric potential 1.0 nm from a water molecule along the axis of the dipole?

65. || Two positive point charges q are located on the y-axis at $y = \pm\frac{1}{2}s$.
   a. Find an expression for the potential along the x-axis.
   b. Draw a graph of V versus x for $-\infty < x < \infty$. For comparison, use a dotted line to show the potential of a point charge $2q$ located at the origin.

66. || The arrangement of charges shown in FIGURE P28.66 is called a *linear electric quadrupole*. The positive charges are located at $y = \pm s$. Notice that the net charge is zero. Find an expression for the electric potential on the y-axis at distances $y \gg s$.

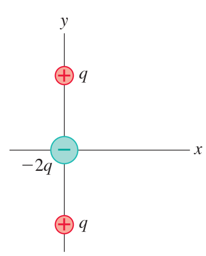


FIGURE P28.66

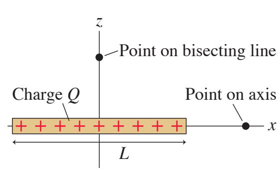


FIGURE P28.67

67. || FIGURE P28.67 shows a thin rod of length L and charge Q. Find an expression for the electric potential a distance x away from the center of the rod on the axis of the rod.

68. ||| FIGURE P28.67 showed a thin rod of length L and charge Q. Find an expression for the electric potential a distance z away from the center of rod on the line that bisects the rod.

69. | FIGURE P28.69 shows a thin rod with charge Q that has been bent into a semicircle of radius R. Find an expression for the electric potential at the center.

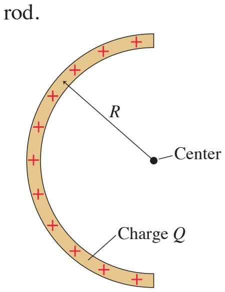


FIGURE P28.69

70. || A disk with a hole has inner radius R_{in} and outer radius R_{out}. The disk is uniformly charged with total charge Q. Find an expression for the on-axis electric potential at distance z from the center of the disk. Verify that your expression has the correct behavior when $R_{in} \to 0$.

In Problems 71 through 73 you are given the equation(s) used to solve a problem. For each of these,
   a. Write a realistic problem for which this is the correct equation(s).
   b. Finish the solution of the problem.

71. $$\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)q_1 q_2}{0.030 \text{ m}} = 90 \times 10^{-6} \text{ J}$$
$$q_1 + q_2 = 40 \text{ nC}$$

72. $$\tfrac{1}{2}(1.67 \times 10^{-27} \text{ kg})(2.5 \times 10^6 \text{ m/s})^2 + 0 =$$
$$\tfrac{1}{2}(1.67 \times 10^{-27} \text{ kg})v_i^2 + \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(2.0 \times 10^{-9} \text{ C})(1.60 \times 10^{-19} \text{ C})}{0.0010 \text{ m}}$$

73. $$\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(3.0 \times 10^{-9} \text{ C})}{0.030 \text{ m}} + \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(3.0 \times 10^{-9} \text{ C})}{(0.030 \text{ m}) + d} = 1200 \text{ V}$$

Challenge Problems

74. A proton and an alpha particle ($q = +2e$, $m = 4$ u) are fired directly toward each other from far away, each with an initial speed of $0.010c$. What is their distance of closest approach, as measured between their centers?

75. Bead A has a mass of 15 g and a charge of −5.0 nC. Bead B has a mass of 25 g and a charge of −10.0 nC. The beads are held 12 cm apart (measured between their centers) and released. What maximum speed is achieved by each bead?

76. Two 2.0-mm-diameter beads, C and D, are 10 mm apart, measured between their centers. Bead C has mass 1.0 g and charge 2.0 nC. Bead D has mass 2.0 g and charge −1.0 nC. If the beads are released from rest, what are the speeds v_C and v_D at the instant the beads collide?

77. An electric dipole has dipole moment p. If $r \gg s$, where s is the separation between the charges, show that the electric potential of the dipole can be written
$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$
where r is the distance from the center of the dipole and θ is the angle from the dipole axis.

78. Electrodes of area A are spaced distance d apart to form a parallel-plate capacitor. The electrodes are charged to $\pm q$.
   a. What is the infinitesimal increase in electric potential energy dU if an infinitesimal amount of charge dq is moved from the negative electrode to the positive electrode?
   b. An uncharged capacitor can be charged to $\pm Q$ by transferring charge dq over and over and over. Use your answer to part a to show that the potential energy of a capacitor charged to $\pm Q$ is $U_{cap} = \frac{1}{2}Q\,\Delta V_C$.

79. A sphere of radius R has charge q.
   a. What is the infinitesimal increase in electric potential energy dU if an infinitesimal amount of charge dq is brought from infinity to the surface of the sphere?
   b. An uncharged sphere can acquire total charge Q by the transfer of charge dq over and over and over. Use your answer to part a to find an expression for the potential energy of a sphere of radius R with total charge Q.
   c. Your answer to part b is the amount of energy needed to assemble a charged sphere. It is often called the *self-energy* of the sphere. What is the self-energy of a proton, assuming it to be a charged sphere with a diameter of 1.0×10^{-15} m?

80. The wire in **FIGURE CP28.80** has linear charge density λ. What is the electric potential at the center of the semicircle?

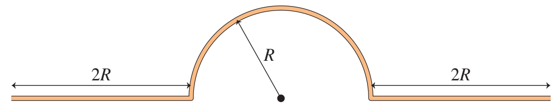


FIGURE CP28.80

81. A circular disk of radius R and total charge Q has the charge distributed with surface charge density $\eta = cr$, where c is a constant. Find an expression for the electric potential at distance z on the axis of the disk. Your expression should include R and Q, but not c.

82. A hollow cylindrical shell of length L and radius R has charge Q uniformly distributed along its length. What is the electric potential at the center of the cylinder?

STOP TO THINK ANSWERS

Stop to Think 28.1: Zero. The motion is always perpendicular to the electric force.

Stop to Think 28.2: $U_b = U_d > U_a = U_c$. The potential energy depends inversely on r. The effects of doubling the charge and doubling the distance cancel each other.

Stop to Think 28.3: c. The proton gains speed by losing potential energy. It loses potential energy by moving in the direction of decreasing electric potential.

Stop to Think 28.4: $V_a = V_b > V_c > V_d = V_e$. The potential decreases steadily from the positive to the negative plate. It depends only on the distance from the positive plate.

Stop to Think 28.5: $\Delta V_{ac} = \Delta V_{bc} > \Delta V_{ab}$. The potential depends only on the *distance* from the charge, not the direction. $\Delta V_{ab} = 0$ because these points are at the same distance.

SUMMARY

The goal of Chapter 29 has been to understand how the electric potential is related to the electric field.

General Principles

Connecting V and $\vec{E}$

The electric potential and the electric field are two different perspectives of how source charges alter the space around them. V and $\vec{E}$ are related by

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds$$

where s is measured from point i to point f and E_s is the component of $\vec{E}$ parallel to the line of integration.

Graphically

$\Delta V =$ the negative of the area under the E_s graph

and

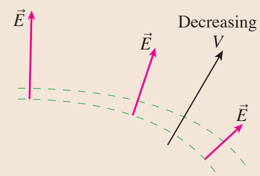
$$E_s = -\frac{dV}{ds}$$

$=$ the negative of the slope of the potential graph

The Geometry of Potential and Field

The electric field

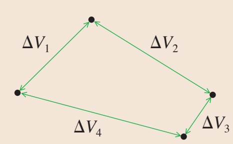
- Is perpendicular to the equipotential surfaces.
- Points "downhill" in the direction of decreasing V.
- Is inversely proportional to the spacing Δs between the equipotential surfaces.



Conservation of Energy

The sum of all potential differences around a closed path is zero.

$$\sum(\Delta V)_i = 0$$

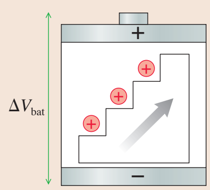


Important Concepts

A **battery** is a source of potential. The charge escalator in a battery uses chemical reactions to move charges from the negative terminal to the positive terminal:

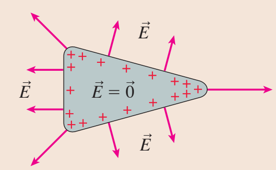
$$\Delta V_{\text{bat}} = \mathcal{E}$$

where the emf $\mathcal{E}$ is the work per charge done by the charge escalator.



For a conductor in electrostatic equilibrium

- The interior electric field is zero.
- The exterior electric field is perpendicular to the surface.
- The surface is an equipotential.
- The interior is at the same potential as the surface.

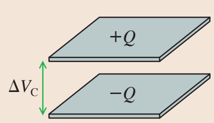


Applications

Capacitors

The **capacitance** of two conductors charged to $\pm Q$ is

$$C = \frac{Q}{\Delta V_C}$$



A parallel-plate capacitor has

$$C = \frac{\epsilon_0 A}{d}$$

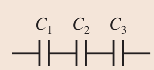
Filling the space between the plates with a dielectric of dielectric constant κ increases the capacitance to $C = \kappa C_0$.

The energy stored in a capacitor is $u_C = \frac{1}{2}C(\Delta V_C)^2$.

This energy is stored in the electric field at density $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$.

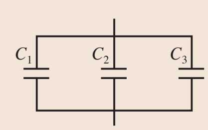
Combinations of capacitors

Series capacitors



$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

Parallel capacitors



$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$

Terms and Notation

Van de Graaff generator
battery
charge escalator model
ideal battery
emf, $\mathcal{E}$
terminal voltage, ΔV_{bat}
Kirchhoff's loop law
capacitance, C
farad, F
parallel capacitors
series capacitors
equivalent capacitance, C_{eq}
energy density, u_E
dielectric
induced electric field
dielectric constant, κ
dielectric strength

CONCEPTUAL QUESTIONS

1. **FIGURE Q29.1** shows the x-component of $\vec{E}$ as a function of x. Draw a graph of V versus x in this same region of space. Let $V = 0$ V at $x = 0$ m and include an appropriate vertical scale.

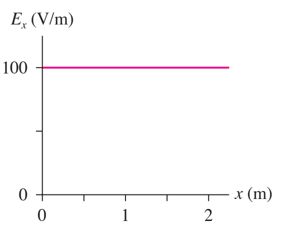


FIGURE Q29.1

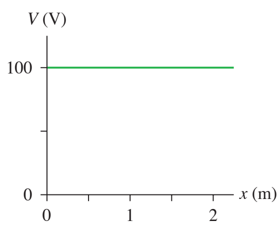


FIGURE Q29.2

2. **FIGURE Q29.2** shows the electric potential as a function of x. Draw a graph of E_x versus x in this same region of space.
3. a. Suppose that $\vec{E} = \vec{0}$ V/m throughout some region of space. Can you conclude that $V = 0$ V in this region? Explain.
   b. Suppose that $V = 0$ V throughout some region of space. Can you conclude that $\vec{E} = \vec{0}$ V/m in this region? Explain.
4. For each contour map in **FIGURE Q29.4**, estimate the electric fields $\vec{E}_1$ and $\vec{E}_2$ at points 1 and 2. Don't forget that $\vec{E}$ is a vector.

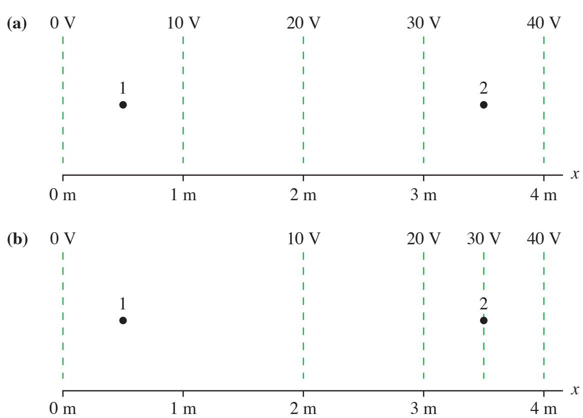


FIGURE Q29.4

5. An electron is released from rest at $x = 2$ m in the potential shown in **FIGURE Q29.5**. Does it move? If so, to the left or to the right? Explain.

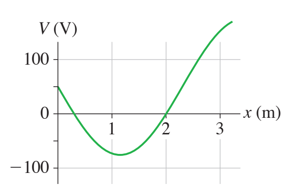


FIGURE Q29.5

6. **FIGURE Q29.6** shows an electric field diagram. Dashed lines 1 and 2 are two surfaces in space, not physical objects.
   a. Is the electric potential at point a higher than, lower than, or equal to the electric potential at point b? Explain.
   b. Rank in order, from largest to smallest, the magnitudes of the potential differences ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}.
   c. Is surface 1 an equipotential surface? What about surface 2? Explain why or why not.

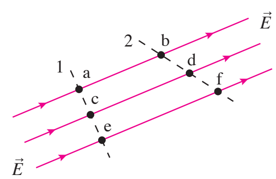


FIGURE Q29.6

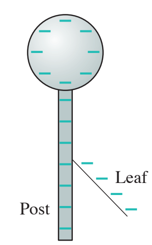


FIGURE Q29.7

7. **FIGURE Q29.7** shows a negatively charged electroscope. The gold leaf stands away from the rigid metal post. Is the electric potential of the leaf higher than, lower than, or equal to the potential of the post? Explain.
8. The two metal spheres in **FIGURE Q29.8** are connected by a metal wire with a switch in the middle. Initially the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge Q_1, is at potential V_1, and the electric field strength at its surface is E_1. The values for sphere 2 are Q_2, V_2, and E_2.
   a. Is V_1 larger than, smaller than, or equal to V_2? Explain.
   b. Is Q_1 larger than, smaller than, or equal to Q_2? Explain.
   c. Is E_1 larger than, smaller than, or equal to E_2? Explain.

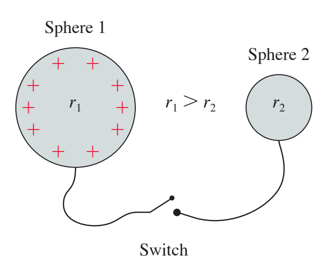


FIGURE Q29.8

9. FIGURE Q29.9 shows a 3 V battery with metal wires attached to each end. What are the potential differences ΔV_{12}, ΔV_{23}, ΔV_{34}, and ΔV_{14}?

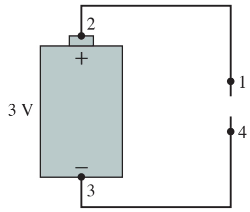


FIGURE Q29.9

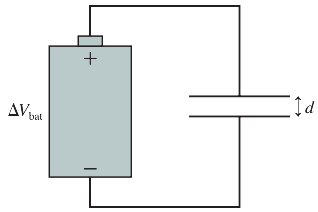


FIGURE Q29.10

10. The parallel-plate capacitor in FIGURE Q29.10 is connected to a battery having potential difference ΔV_{bat}. Without breaking any of the connections, insulating handles are used to increase the plate separation to $2d$.

a. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?
b. Does the capacitance change? If so, by what factor? If not, why not?
c. Does the capacitor charge Q change? If so, by what factor? If not, why not?

11. Rank in order, from largest to smallest, the potential differences $(\Delta V_C)_1$ to $(\Delta V_C)_4$ of the four capacitors in FIGURE Q29.11. Explain.

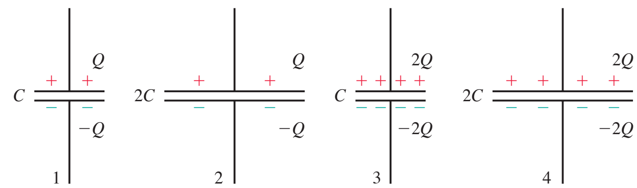


FIGURE Q29.11

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 29.1 Connecting Potential and Field

1. | What is the potential difference between $x_i = 10$ cm and $x_f = 30$ cm in the uniform electric field $E_x = 1000$ V/m?
2. | What is the potential difference between $y_i = -5$ cm and $y_f = 5$ cm in the uniform electric field $\vec{E} = (20{,}000\hat{\imath} - 50{,}000\hat{\jmath})$ V/m?
3. | FIGURE EX29.3 is a graph of E_x. What is the potential difference between $x_i = 1.0$ m and $x_f = 3.0$ m?

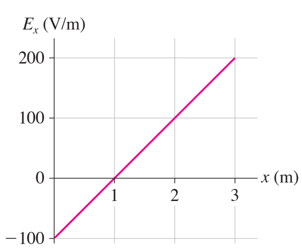


FIGURE EX29.3

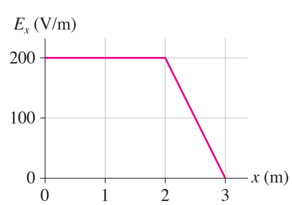


FIGURE EX29.4

4. | FIGURE EX29.4 is a graph of E_x. The potential at the origin is -50 V. What is the potential at $x = 3.0$ m?

Section 29.2 Sources of Electric Potential

5. | How much work does the charge escalator do to move 1.0 μC of charge from the negative terminal to the positive terminal of a 1.5 V battery?
6. | How much work does the electric motor of a Van de Graaff generator do to lift a positive ion ($q = e$) if the potential of the spherical electrode is 1.0 MV?
7. | How much charge does a 9.0 V battery transfer from the negative to the positive terminal while doing 27 J of work?
8. | Light from the sun allows a solar cell to move electrons from the positive to the negative terminal, doing 2.4×10^{-19} J of work per electron. What is the emf of this solar cell?

Section 29.3 Finding the Electric Field from the Potential

9. | What are the magnitude and direction of the electric field at the dot in FIGURE EX29.9?

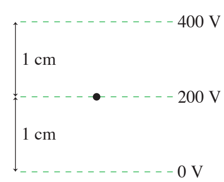


FIGURE EX29.9

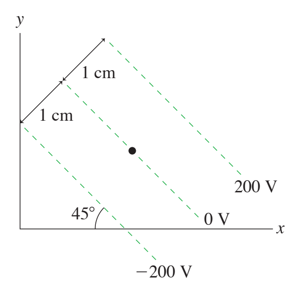


FIGURE EX29.10

10. | What are the magnitude and direction of the electric field at the dot in FIGURE EX29.10?
11. | FIGURE EX29.11 is a graph of V versus x. Draw the corresponding graph of E_x versus x.

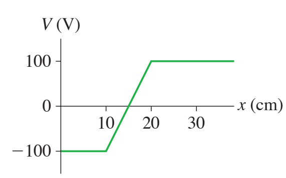


FIGURE EX29.11

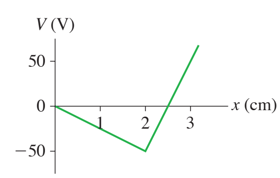


FIGURE EX29.12

12. | FIGURE EX29.12 is a graph of V versus x. Draw the corresponding graph of E_x versus x.

13. | | The electric potential in a region of uniform electric field is -1000 V at $x = -1.0$ m and $+1000$ V at $x = +1.0$ m. What is E_x?

14. | | The electric potential along the x-axis is $V = 100x^2$ V, where x is in meters. What is E_x at (a) $x = 0$ m and (b) $x = 1$ m?

15. | | The electric potential along the x-axis is $V = 100e^{-2x}$ V, where x is in meters. What is E_x at (a) $x = 1.0$ m and (b) $x = 2.0$ m?

16. | What is the potential difference ΔV_{34} in **FIGURE EX29.16**?

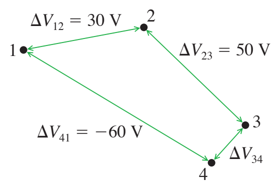


FIGURE EX29.16

Section 29.5 Capacitance and Capacitors

17. | Two 3.0-cm-diameter aluminum electrodes are spaced 0.50 mm apart. The electrodes are connected to a 100 V battery.
    a. What is the capacitance?
    b. What is the magnitude of the charge on each electrode?

18. | | You need to construct a 100 pF capacitor for a science project. You plan to cut two $L \times L$ metal squares and insert small spacers between their corners. The thinnest spacers you have are 0.20 mm thick. What is the proper value of L?

19. | A switch that connects a battery to a 10 μF capacitor is closed. Several seconds later you find that the capacitor plates are charged to ± 30 μC. What is the emf of the battery?

20. | A 6 μF capacitor, a 10 μF capacitor, and a 16 μF capacitor are connected in series. What is their equivalent capacitance?

21. | A 6 μF capacitor, a 10 μF capacitor, and a 16 μF capacitor are connected in parallel. What is their equivalent capacitance?

22. | You need a capacitance of 50 μF, but you don't happen to have a 50 μF capacitor. You do have a 30 μF capacitor. What additional capacitor do you need to produce a total capacitance of 50 μF? Should you join the two capacitors in parallel or in series?

23. | You need a capacitance of 50 μF, but you don't happen to have a 50 μF capacitor. You do have a 75 μF capacitor. What additional capacitor do you need to produce a total capacitance of 50 μF? Should you join the two capacitors in parallel or in series?

24. | | What is the capacitance of the two metal spheres shown in **FIGURE EX29.24**?

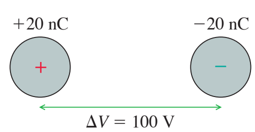


FIGURE EX29.24

Section 29.6 The Energy Stored in a Capacitor

25. | | To what potential should you charge a 1.0 μF capacitor to store 1.0 J of energy?

26. | | **FIGURE EX29.26** shows Q versus t for a 2.0 μF capacitor. Draw a graph showing U_C versus t.

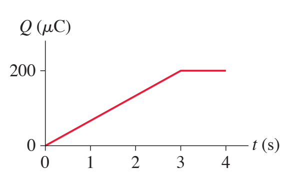


FIGURE EX29.26

27. | Capacitor 2 has half the capacitance and twice the potential difference as capacitor 1. What is the ratio U_{C1}/U_{C2}?

28. | | 50 pJ of energy is stored in a $2.0 \text{ cm} \times 2.0 \text{ cm} \times 2.0 \text{ cm}$ region of uniform electric field. What is the electric field strength?

29. | | A 2.0-cm-diameter parallel-plate capacitor with a spacing of 0.50 mm is charged to 200 V. What are (a) the total energy stored in the electric field and (b) the energy density?

Section 29.7 Dielectrics

30. | | Two $4.0 \text{ cm} \times 4.0 \text{ cm}$ metal plates are separated by a 0.20-mm-thick piece of Teflon.
    a. What is the capacitance?
    b. What is the maximum potential difference between the plates?

31. | | Two $5.0 \text{ mm} \times 5.0 \text{ mm}$ electrodes with a 0.10-mm-thick sheet of Mylar between them are attached to a 9.0 V battery. Without disconnecting the battery, the Mylar is withdrawn. (Very small spacers keep the electrode separation unchanged.) What are the charge, potential difference, and electric field (a) before and (b) after the Mylar is withdrawn?

32. | | BIO A typical cell has a layer of negative charge on the inner surface of the cell wall and a layer of positive charge on the outside surface, thus making the cell wall a capacitor. What is the capacitance of a 50-μm-diameter cell with a 7.0-nm-thick cell wall whose dielectric constant is 9.0? Because the cell's diameter is much larger than the wall thickness, it is reasonable to ignore the curvature of the cell and think of it as a parallel-plate capacitor.

Problems

33. | | a. Which point in **FIGURE P29.33**, A or B, has a larger electric potential?
    b. What is the potential difference between A and B?

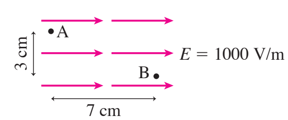


FIGURE P29.33

34. | | | The electric field in a region of space is $E_x = -1000x$ V/m, where x is in meters.
    a. Graph E_x versus x over the region $-1 \text{ m} \le x \le 1$ m.
    b. What is the potential difference between $x_i = -20$ cm and $x_f = 30$ cm?

35. | | The electric field in a region of space is $E_x = 5000x$ V/m, where x is in meters.
    a. Graph E_x versus x over the region $-1 \text{ m} \le x \le 1$ m.
    b. Find an expression for the potential V at position x. As a reference, let $V = 0$ V at the origin.
    c. Graph V versus x over the region $-1 \text{ m} \le x \le 1$ m.

36. | | An infinitely long cylinder of radius R has linear charge density λ. The potential on the surface of the cylinder is V_0, and the electric field outside the cylinder is $E_r = \lambda/2\pi\epsilon_0 r$. Find the potential relative to the surface at a point that is distance r from the axis, assuming $r > R$.

37. ‖ FIGURE P29.37 is an edge view of three charged metal electrodes. Let the left electrode be the zero point of the electric potential. What are V and $\vec{E}$ at (a) $x = 0.5$ cm, (b) $x = 1.5$ cm, and (c) $x = 2.5$ cm?

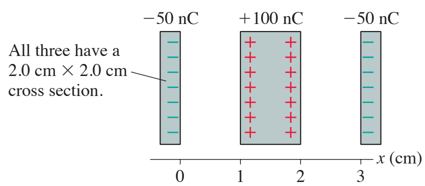


FIGURE P29.37

38. ‖ FIGURE P29.38 shows a graph of V versus x in a region of space. The potential is independent of y and z. What is E_x at (a) $x = -2$ cm, (b) $x = 0$ cm, and (c) $x = 2$ cm?

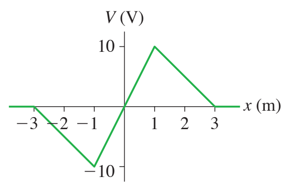


FIGURE P29.38

39. ‖ Use the on-axis potential of a charged disk from Chapter 28 to find the on-axis electric field of a charged disk.

40. ‖ a. Use the methods of Chapter 28 to find the potential at distance x on the axis of the charged rod shown in FIGURE P29.40.
b. Use the result of part a to find the electric field at distance x on the axis of a rod.

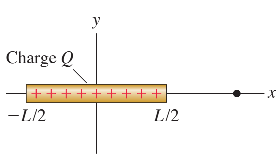


FIGURE P29.40

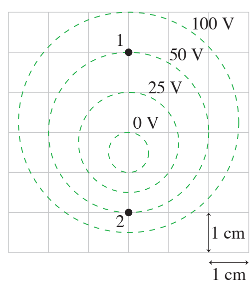


FIGURE P29.41

41. | Determine the magnitude and direction of the electric field at points 1 and 2 in FIGURE P29.41.

42. ‖ It is postulated that the radial electric field of a group of charges falls off as $E_r = C/r^n$, where r is the distance from the center of the group and n is an unknown exponent. To test this hypothesis, you make a *field probe* consisting of two needle tips spaced 1.00 mm apart. You orient the needles so that a line between the tips points to the center of the charges, then use a voltmeter to read the potential difference between the tips. After you take measurements at several distances from the center of the group, your data are as follows:

Distance (cm)	Potential difference (mV)
2.0	34.7
4.0	6.6
6.0	2.1
8.0	1.2
10.0	0.6

Use an appropriate graph of the data to determine the constants C and n.

43. ‖ The electric potential in a region of space is $V = (150x^2 - 200y^2)$ V, where x and y are in meters. What are the strength and direction of the electric field at $(x, y) = (2.0\text{ m}, 2.0\text{ m})$? Give the direction as an angle cw or ccw (specify which) from the positive x-axis.

44. ‖ The electric potential in a region of space is $V = 200/\sqrt{x^2 + y^2}$, where x and y are in meters. What are the strength and direction of the electric field at $(x, y) = (2.0\text{ m}, 1.0\text{ m})$? Give the direction as an angle cw or ccw (specify which) from the positive x-axis.

45. ‖ Metal sphere 1 has a positive charge of 6.0 nC. Metal sphere 2, which is twice the diameter of sphere 1, is initially uncharged. The spheres are then connected together by a long, thin metal wire. What are the final charges on each sphere?

46. ‖ The metal spheres in FIGURE P29.46 are charged to ±300 V. Draw this figure on your paper, then draw a plausible contour map of the potential, showing and labeling the −300 V, −200 V, −100 V, . . . , 300 V equipotential surfaces.

FIGURE P29.46

47. ‖ The potential at the center of a 4.0-cm-diameter copper sphere is 500 V, relative to $V = 0$ V at infinity. How much excess charge is on the sphere?

48. ‖ Two 2.0 cm × 2.0 cm metal electrodes are spaced 1.0 mm apart and connected by wires to the terminals of a 9.0 V battery.
a. What are the charge on each electrode and the potential difference between them?
The wires are disconnected, and insulated handles are used to pull the plates apart to a new spacing of 2.0 mm.
b. What are the charge on each electrode and the potential difference between them?

49. | Two 2.0 cm × 2.0 cm metal electrodes are spaced 1.0 mm apart and connected by wires to the terminals of a 9.0 V battery.
a. What are the charge on each electrode and the potential difference between them?
While the plates are still connected to the battery, insulated handles are used to pull them apart to a new spacing of 2.0 mm.
b. What are the charge on each electrode and the potential difference between them?

50. | Find expressions for the equivalent capacitance of (a) N identical capacitors C in parallel and (b) N identical capacitors C in series.

51. | What is the equivalent capacitance of the three capacitors in FIGURE P29.51?

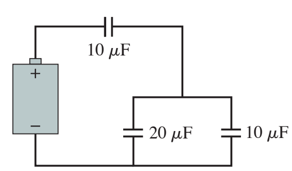


FIGURE P29.51

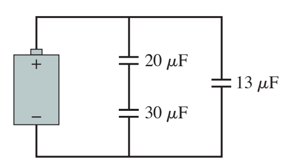


FIGURE P29.52

52. | What is the equivalent capacitance of the three capacitors in FIGURE P29.52?

53. | What are the charge on and the potential difference across each capacitor in FIGURE P29.53?

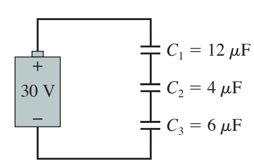


FIGURE P29.53

54. || What are the charge on and the potential difference across each capacitor in **FIGURE P29.54**?

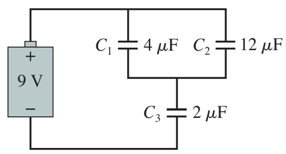


FIGURE P29.54

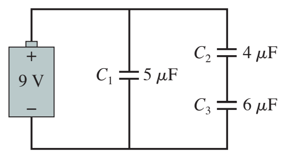


FIGURE P29.55

55. || What are the charge on and the potential difference across each capacitor in **FIGURE P29.55**?

56. | You have three 12 μF capacitors. Draw diagrams showing how you could arrange all three so that their equivalent capacitance is (a) 4.0 μF, (b) 8.0 μF, (c) 18 μF, and (d) 36 μF.

57. | Six identical capacitors with capacitance C are connected as shown in **FIGURE P29.57**.
   a. What is the equivalent capacitance of these six capacitors?
   b. What is the potential difference between points a and b?

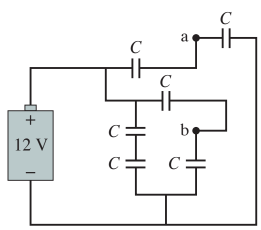


FIGURE P29.57

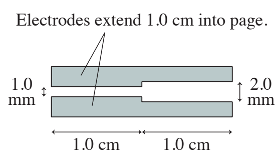


FIGURE P29.58

58. || What is the capacitance of the two electrodes in **FIGURE P29.58**? **Hint:** Can you think of this as a combination of capacitors?

59. || Initially, the switch in **FIGURE P29.59** is in position A and capacitors C_2 and C_3 are uncharged. Then the switch is flipped to position B. Afterward, what are the charge on and the potential difference across each capacitor?

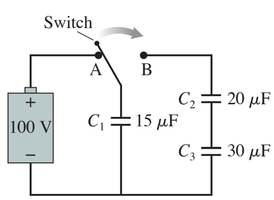


FIGURE P29.59

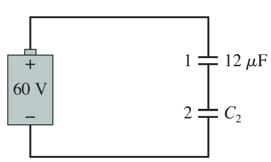


FIGURE P29.60

60. || A battery with an emf of 60 V is connected to the two capacitors shown in **FIGURE P29.60**. Afterward, the charge on capacitor 2 is 450 μC. What is the capacitance of capacitor 2?

61. || Capacitors $C_1 = 10\ \mu$F and $C_2 = 20\ \mu$F are each charged to 10 V, then disconnected from the battery without changing the charge on the capacitor plates. The two capacitors are then connected in parallel, with the positive plate of C_1 connected to the negative plate of C_2 and vice versa. Afterward, what are the charge on and the potential difference across each capacitor?

62. || An isolated 5.0 μF parallel-plate capacitor has 4.0 mC of charge. An external force changes the distance between the electrodes until the capacitance is 2.0 μF. How much work is done by the external force?

63. || A parallel-plate capacitor is constructed from two 10 cm $\times$ 10 cm electrodes spaced 1.0 mm apart. The capacitor plates are charged to ± 10 nC, then disconnected from the battery.
   a. How much energy is stored in the capacitor?
   b. Insulating handles are used to pull the capacitor plates apart until the spacing is 2.0 mm. Now how much energy is stored in the capacitor?
   c. Energy must be conserved. How do you account for the difference between a and b?

64. || What is the energy density in the electric field at the surface of a 1.0-cm-diameter sphere charged to a potential of 1000 V?

65. || The 90 μF capacitor in a defibrillator unit supplies an average
BIO of 6500 W of power to the chest of the patient during a discharge lasting 5.0 ms. To what voltage is the capacitor charged?

66. | The flash unit in a camera uses a 3.0 V battery to charge a capacitor. The capacitor is then discharged through a flashlamp. The discharge takes 10 μs, and the average power dissipated in the flashlamp is 10 W. What is the capacitance of the capacitor?

67. || You need to use a motor and lightweight cable to lift a 2.0 kg copper weight to a height of 3.0 m. To do so, you've decided to use a 1000 V power supply to charge a capacitor, then run the motor by letting the capacitor discharge through it. If the motor is 90% efficient (that is, 10% of the energy supplied to the motor is dissipated as heat), what minimum capacitance do you need?

68. || Two 5.0-cm-diameter metal disks separated by a 0.50-mm-thick piece of Pyrex glass are charged to a potential difference of 1000 V. What are (a) the surface charge density on the disks and (b) the surface charge density on the glass?

69. || A typical cell has a membrane potential of -70 mV, mean-
BIO ing that the potential inside the cell is 70 mV less than the potential outside due to a layer of negative charge on the inner surface of the cell wall and a layer of positive charge on the outer surface. This effectively makes the cell wall a charged capacitor. Because a cell's diameter is much larger than the wall thickness, it is reasonable to ignore the curvature of the cell and think of it as a parallel-plate capacitor. How much energy is stored in the electric field of a 50-μm-diameter cell with a 7.0-nm-thick cell wall whose dielectric constant is 9.0?

70. ||| A nerve cell in its resting state has a membrane potential of
BIO -70 mV, meaning that the potential inside the cell is 70 mV less than the potential outside due to a layer of negative charge on the inner surface of the cell wall and a layer of positive charge on the outer surface. This effectively makes the cell wall a charged capacitor. When the nerve cell fires, sodium ions, Na^+, flood through the cell wall to briefly switch the membrane potential to $+40$ mV. Model the central body of a nerve cell—the *soma*—as a 50-μm-diameter sphere with a 7.0-nm-thick cell wall whose dielectric constant is 9.0. Because a cell's diameter is much larger than the wall thickness, it is reasonable to ignore the curvature of the cell and think of it as a parallel-plate capacitor. How many sodium ions enter the cell as it fires?

71. || Derive Equation 29.34 for the induced surface charge density on the dielectric in a capacitor.

72. || A vacuum-insulated parallel-plate capacitor with plate separation d has capacitance C_0. What is the capacitance if an insulator with dielectric constant κ and thickness is $d/2$ slipped between the electrodes?

In Problems 73 through 75 you are given the equation(s) used to solve a problem. For each of these, you are to

a. Write a realistic problem for which this is the correct equation(s).
b. Finish the solution of the problem.

73. $2az\text{ V/m} = -\dfrac{dV}{dz}$, where a is a constant with units of V/m^2

$V(z = 0) = 10\text{ V}$

74. $400\text{ nC} = (100\text{ V})C$

$$C = \frac{(8.85 \times 10^{-12}\text{ C}^2/\text{N m}^2)(0.10\text{ m} \times 0.10\text{ m})}{d}$$

75. $\left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1} + C = 4\ \mu\text{F}$

Challenge Problems

76. The electric potential in a region of space is $V = 100(x^2 - y^2)$ V, where x and y are in meters.
   a. Draw a contour map of the potential, showing and labeling the −400 V, −100 V, 0 V, +100 V, and +400 V equipotential surfaces.
   b. Find an expression for the electric field $\vec{E}$ at position (x, y).
   c. Draw the electric field lines on your diagram of part a.
77. An electric dipole at the origin consists of two charges $\pm q$ spaced distance s apart along the y-axis.
   a. Find an expression for the potential $V(x, y)$ at an arbitrary point in the xy-plane. Your answer will be in terms of q, s, x, and y.
   b. Use the binomial approximation to simplify your result of part a when $s \ll x$ and $s \ll y$.
   c. Assuming $s \ll x$ and y, find expressions for E_x and E_y, the components of $\vec{E}$ for a dipole.
   d. What is the on-axis field $\vec{E}$? Does your result agree with Equation 26.11?
   e. What is the field $\vec{E}$ on the bisecting axis? Does your result agree with Equation 26.12?
78. Charge is uniformly distributed with charge density ρ inside a very long cylinder of radius R. Find the potential difference between the surface and the axis of the cylinder.
79. Consider a uniformly charged sphere of radius R and total charge Q. The electric field E_{out} *outside* the sphere $(r \geq R)$ is simply that of a point charge Q. In Chapter 27, we used Gauss's law to find that the electric field E_{in} *inside* the sphere $(r \leq R)$ is radially outward with field strength

$$E_{in} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r$$

   a. The electric potential V_{out} *outside* the sphere is that of a point charge Q. Find an expression for the electric potential V_{in} at position r *inside* the sphere. As a reference, let $V_{in} = V_{out}$ at the surface of the sphere.
   b. What is the ratio $V_{center}/V_{surface}$?
   c. Graph V versus r for $0 \leq r \leq 3R$.
80. a. Find an expression for the capacitance of a *spherical capacitor*, consisting of concentric spherical shells of radii R_1 (inner shell) and R_2 (outer shell).
   b. A spherical capacitor with a 1.0 mm gap between the shells has a capacitance of 100 pF. What are the diameters of the two spheres?
81. High-frequency signals are often transmitted along a *coaxial cable*, such as the one shown in **FIGURE CP29.81**. For example, the cable TV hookup coming into your home is a coaxial cable. The signal is carried on a wire of radius R_1 while the outer conductor of radius R_2 is grounded (i.e., at $V = 0$ V). An insulating material fills the space between them, and an insulating plastic coating goes around the outside.

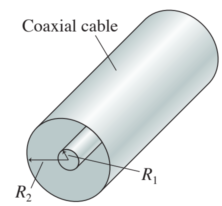


FIGURE CP29.81

   a. Find an expression for the capacitance per meter of a coaxial cable. Assume that the insulating material between the cylinders is air.
   b. Evaluate the capacitance per meter of a cable having $R_1 = 0.50$ mm and $R_2 = 3.0$ mm.
82. Each capacitor in **FIGURE CP29.82** has capacitance C. What is the equivalent capacitance between points a and b?

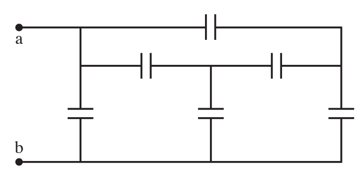


FIGURE CP29.82

STOP TO THINK ANSWERS

Stop to Think 29.1: 5.0 V. The potentials add, but $\Delta V_2 = -1.0$ V because the charge escalator goes *down* by 1.0 V.

Stop to Think 29.2: c. E_y is the negative of the slope of the V-versus-y graph. E_y is positive because $\vec{E}$ points up, so the graph has a negative slope. E_y has constant magnitude, so the slope has a constant value.

Stop to Think 29.3: c. $\vec{E}$ points "downhill," so V must decrease from right to left. E is larger on the left than on the right, so the contour lines must be closer together on the left.

Stop to Think 29.4: b. Because of the connecting wire, the three spheres form a single conductor in electrostatic equilibrium. Thus all points are at the same potential. The electric field of a sphere is related to the sphere's potential by $E = V/R$, so a smaller-radius sphere has a larger E.

Stop to Think 29.5: $(C_{eq})_b > (C_{eq})_a = (C_{eq})_d > (C_{eq})_c$. $(C_{eq})_b = 3\ \mu\text{F} + 3\ \mu\text{F} = 6\ \mu\text{F}$. The equivalent capacitance of series capacitors is less than any capacitor in the group, so $(C_{eq})_c < 3\ \mu\text{F}$. Only d requires any real calculation. The two 4 μF capacitors are in series and are equivalent to a single 2 μF capacitor. The 2 μF equivalent capacitor is in parallel with 3 μF, so $(C_{eq})_d = 5\ \mu\text{F}$.