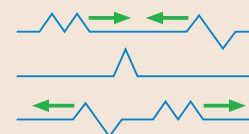# SUMMARY

The goal of Chapter 21 has been to understand and use the idea of superposition.
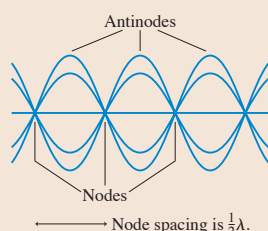
## General Principles

### Principle of Superposition

The displacement of a medium when more than one wave is present is the sum of the displacements due to each individual wave.

## Important Concepts

**Standing waves** are due to the superposition of two traveling waves moving in opposite directions.

Antinodes

Nodes

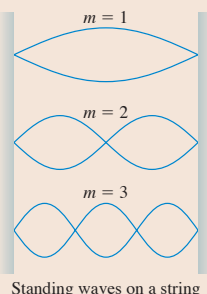Node spacing is $\frac{1}{2}\lambda$.

The amplitude at position $x$ is

$$A(x) = 2a \sin kx$$

where $a$ is the amplitude of each wave.

The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are **modes** of the system.

$m = 1$

$m = 2$

$m = 3$

Standing waves on a string

### Interference

In general, the superposition of two or more waves into a single wave is called interference.

**Maximum constructive interference** occurs where crests are aligned with crests and troughs with troughs. These waves are in phase. The maximum displacement is $A = 2a$.

**Perfect destructive interference** occurs where crests are aligned with troughs. These waves are out of phase. The amplitude is $A = 0$.

Interference depends on the **phase difference** $\Delta\phi$ between the two waves.

Constructive: $\Delta\phi = 2\pi \dfrac{\Delta r}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$

Destructive: $\Delta\phi = 2\pi \dfrac{\Delta r}{\lambda} + \Delta\phi_0 = \left(m + \dfrac{1}{2}\right) \cdot 2\pi$

Antinodal lines, constructive interference. $A = 2a$

Nodal lines, destructive interference. $A = 0$

$\Delta r$ is the path-length difference of the two waves, and $\Delta\phi_0$ is any phase difference between the sources. For identical sources (in phase, $\Delta\phi_0 = 0$):

Interference is constructive if the path-length difference $\Delta r = m\lambda$.

Interference is destructive if the path-length difference $\Delta r = \left(m + \frac{1}{2}\right)\lambda$.

The amplitude at a point where the phase difference is $\Delta\phi$ is $A = \left| 2a \cos\left(\dfrac{\Delta\phi}{2}\right) \right|$.
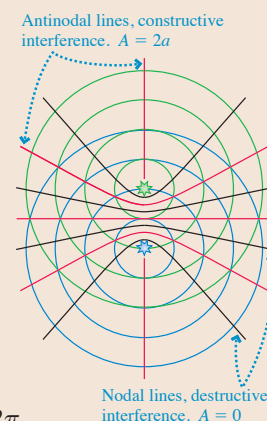
## Applications

### Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends:

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\frac{v}{2L} = mf_1$$

where $m = 1, 2, 3, \ldots$.

The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\frac{v}{4L} = mf_1$$

where $m = 1, 3, 5, 7, \ldots$.

**Beats** (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.

$D$

Soft  Loud  Soft  Loud  Soft
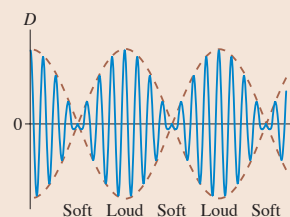
The beat frequency between waves of frequencies $f_1$ and $f_2$ is

$$f_{beat} = f_1 - f_2$$

# Terms and Notation

| | | |
|---|---|---|
| principle of superposition | mode | path-length difference, $\Delta x$ or $\Delta r$ |
| standing wave | interference | thin-film optical coating |
| node | in phase | antinodal line |
| antinode | constructive interference | nodal line |
| amplitude function, $A(x)$ | out of phase | beats |
| boundary condition | destructive interference | modulation |
| fundamental frequency, $f_1$ | phase difference, $\Delta\phi$ | beat frequency, $f_{beat}$ |
| harmonic | | |

# CONCEPTUAL QUESTIONS

1. **FIGURE Q21.1** shows a standing wave oscillating on a string at frequency $f_0$.

   
   **FIGURE Q21.1**

   a. What mode ($m$-value) is this?
   b. How many antinodes will there be if the frequency is doubled to $2f_0$?

2. If you take snapshots of a standing wave on a string, there are certain instants when the string is totally flat. What has happened to the energy of the wave at those instants?

3. **FIGURE Q21.3** shows the displacement of a standing sound wave in a 32-cm-long horizontal tube of air open at both ends.

   
   **FIGURE Q21.3**

   a. What mode ($m$-value) is this?
   b. Are the air molecules moving horizontally or vertically? Explain.
   c. At what distances from the left end of the tube do the molecules oscillate with maximum amplitude?
   d. At what distances from the left end of the tube does the air pressure oscillate with maximum amplitude?

4. An organ pipe is tuned to exactly 384 Hz when the room temperature is 20°C. If the room temperature later increases to 22°C, does the pipe's frequency increase, decrease, or stay the same? Explain.

5. If you pour liquid into a tall, narrow glass, you may hear sound with a steadily rising pitch. What is the source of the sound? And why does the pitch rise as the glass fills?

6. A flute filled with helium will, until the helium escapes, play notes at a much higher pitch than normal. Why?
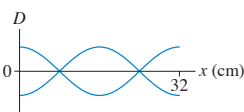
7. In music, two notes are said to be an *octave* apart when one note is exactly twice the frequency of the other. Suppose you have a guitar string playing frequency $f_0$. To increase the frequency by an octave, to $2f_0$, by what factor would you have to (a) increase the tension or (b) decrease the length?

8. **FIGURE Q21.8** is a snapshot graph of two plane waves passing through a region of space. Each wave has a 2.0 mm amplitude and the same wavelength. What is the net displacement of the medium at points a, b, and c?
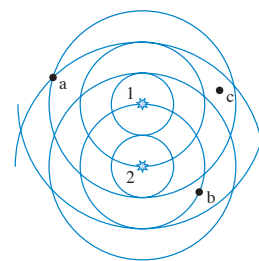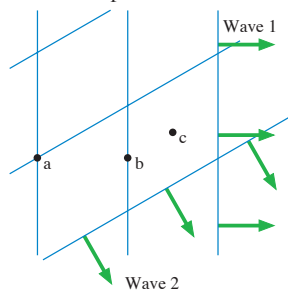
   
   **FIGURE Q21.8**    **FIGURE Q21.9**

9. **FIGURE Q21.9** shows the circular waves emitted by two in-phase sources. Are points a, b, and c points of maximum constructive interference or perfect destructive interference? Explain.

10. A trumpet player hears 5 beats per second when she plays a note and simultaneously sounds a 440 Hz tuning fork. After pulling her tuning valve out to slightly increase the length of her trumpet, she hears 3 beats per second against the tuning fork. Was her initial frequency 435 Hz or 445 Hz? Explain.
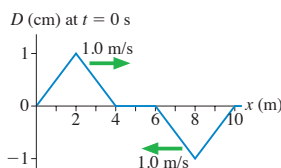
# EXERCISES AND PROBLEMS

Problems labeled ▨ integrate material from earlier chapters.

## Exercises

### Section 21.1 The Principle of Superposition

1. | **FIGURE EX21.1** is a snapshot graph at $t = 0$ s of two waves approaching each other at 1.0 m/s. Draw six snapshot graphs, stacked vertically, showing the string at 1 s intervals from $t = 1$ s to $t = 6$ s.

   
   **FIGURE EX21.1**

2. | **FIGURE EX21.2** is a snapshot graph at $t = 0$ s of two waves approaching each other at 1.0 m/s. Draw six snapshot graphs, stacked vertically, showing the string at 1 s intervals from $t = 1$ s to $t = 6$ s.

   
   **FIGURE EX21.2**
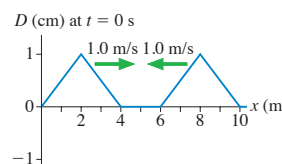
3. ‖ **FIGURE EX21.3** is a snapshot graph at $t = 0$ s of two waves approaching each other at 1.0 m/s. Draw four snapshot graphs, stacked vertically, showing the string at $t = 2, 4, 6,$ and 8 s.
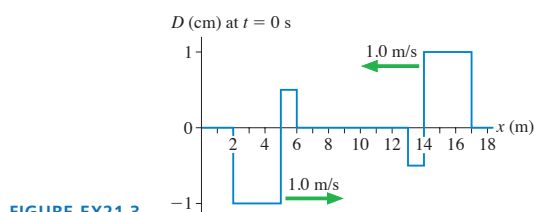


**FIGURE EX21.3**

4. ‖ **FIGURE EX21.4a** is a snapshot graph at $t = 0$ s of two waves approaching each other at 1.0 m/s.
   a. At what time was the snapshot graph in **FIGURE EX21.4b** taken?
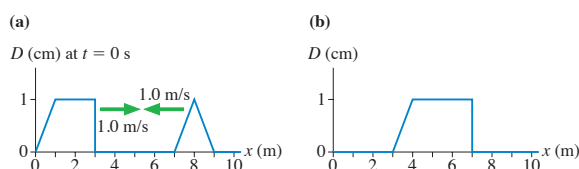   b. Draw a history graph of the string at $x = 5.0$ m from $t = 0$ s to $t = 6$ s.



**FIGURE EX21.4**

## Section 21.2 Standing Waves

## Section 21.3 Standing Waves on a String

5. ‖ **FIGURE EX21.5** is a snapshot graph at $t = 0$ s of two waves moving to the right at 1.0 m/s. The string is fixed at $x = 8.0$ m. Draw four snapshot graphs, stacked vertically, showing the string at $t = 2, 4, 6,$ and 8 s.
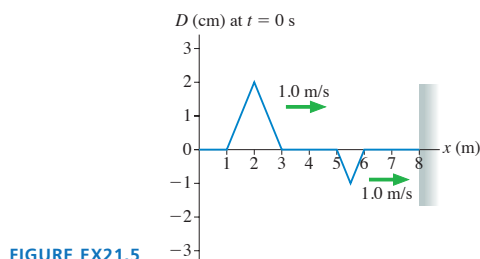


**FIGURE EX21.5**

6. | **FIGURE EX21.6** shows a standing wave oscillating at 100 Hz on a string. What is the wave speed?
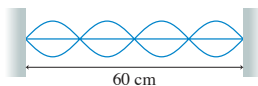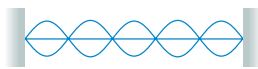


**FIGURE EX21.6**    **FIGURE EX21.7**

7. ‖ **FIGURE EX21.7** shows a standing wave on a 2.0-m-long string that has been fixed at both ends and tightened until the wave speed is 40 m/s. What is the frequency?

8. ‖ **FIGURE EX21.8** shows a standing wave that is oscillating at frequency $f_0$.
   a. How many antinodes will there be if the frequency is doubled to $2f_0$? Explain.



**FIGURE EX21.8**

b. If the tension in the string is increased by a factor of four, for what frequency, in terms of $f_0$, will the string continue to oscillate as a standing wave with four antinodes?

9. | a. What are the three longest wavelengths for standing waves on a 240-cm-long string that is fixed at both ends?
   b. If the frequency of the second-longest wavelength is 50 Hz, what is the frequency of the third-longest wavelength?

10. | Standing waves on a 1.0-m-long string that is fixed at both ends are seen at successive frequencies of 36 Hz and 48 Hz.
   a. What are the fundamental frequency and the wave speed?
   b. Draw the standing-wave pattern when the string oscillates at 48 Hz.

11. ‖ A heavy piece of hanging sculpture is suspended by a 90-cm-long, 5.0 g steel wire. When the wind blows hard, the wire hums at its fundamental frequency of 80 Hz. What is the mass of the sculpture?

12. | A carbon dioxide laser is an infrared laser. A $CO_2$ laser with a cavity length of 53.00 cm oscillates in the $m = 100,000$ mode. What are the wavelength and frequency of the laser beam?

## Section 21.4 Standing Sound Waves and Musical Acoustics

13. | What are the three longest wavelengths for standing sound waves in a 121-cm-long tube that is (a) open at both ends and (b) open at one end, closed at the other?

14. | **FIGURE EX21.14** shows a standing sound wave in an 80-cm-long tube. The tube is filled with an unknown gas. What is the speed of sound in this gas?
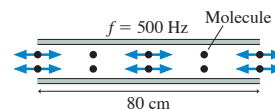


**FIGURE EX21.14**    80 cm

15. ‖ The fundamental frequency of an open-open tube is 1500 Hz when the tube is filled with 0°C helium. What is its frequency when filled with 0°C air?

16. | We can make a simple model of the human vocal tract as an
BIO open-closed tube extending from the opening of the mouth to the diaphragm. What is the length of this tube if its fundamental frequency equals a typical speech frequency of 250 Hz? The speed of sound in the warm air is 350 m/s.

17. ‖ The lowest note on a grand piano has a frequency of 27.5 Hz. The entire string is 2.00 m long and has a mass of 400 g. The vibrating section of the string is 1.90 m long. What tension is needed to tune this string properly?

18. ‖ A violin string is 30 cm long. It sounds the musical note A (440 Hz) when played without fingering. How far from the end of the string should you place your finger to play the note C (523 Hz)?

## Section 21.5 Interference in One Dimension

## Section 21.6 The Mathematics of Interference

19. ‖ Two loudspeakers emit sound waves along the $x$-axis. The sound has maximum intensity when the speakers are 20 cm apart. The sound intensity decreases as the distance between the speakers is increased, reaching zero at a separation of 60 cm.
   a. What is the wavelength of the sound?
   b. If the distance between the speakers continues to increase, at what separation will the sound intensity again be a maximum?

20. ‖ Two loudspeakers in a 20°C room emit 686 Hz sound waves along the $x$-axis.
    a. If the speakers are in phase, what is the smallest distance between the speakers for which the interference of the sound waves is perfectly destructive?
    b. If the speakers are out of phase, what is the smallest distance between the speakers for which the interference of the sound waves is maximum constructive?

21. | What is the thinnest film of $MgF_2$ ($n = 1.39$) on glass that produces a strong reflection for orange light with a wavelength of 600 nm?

22. ‖ A very thin oil film ($n = 1.25$) floats on water ($n = 1.33$). What is the thinnest film that produces a strong reflection for green light with a wavelength of 500 nm?

### Section 21.7 Interference in Two and Three Dimensions

23. ‖ **FIGURE EX21.23** shows the circular wave fronts emitted by two wave sources.
    a. Are these sources in phase or out of phase? Explain.
    b. Make a table with rows labeled P, Q, and R and columns labeled $r_1$, $r_2$, $\Delta r$, and C/D. Fill in the table for points P, Q, and R, giving the distances as multiples of $\lambda$ and indicating, with a C or a D, whether the interference at that point is constructive or destructive.
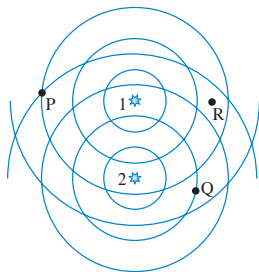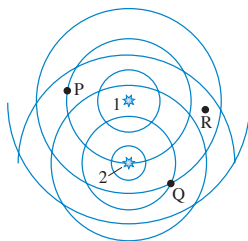


**FIGURE EX21.23**            **FIGURE EX21.24**

24. ‖ **FIGURE EX21.24** shows the circular wave fronts emitted by two wave sources.
    a. Are these sources in phase or out of phase? Explain.
    b. Make a table with rows labeled P, Q, and R and columns labeled $r_1$, $r_2$, $\Delta r$, and C/D. Fill in the table for points P, Q, and R, giving the distances as multiples of $\lambda$ and indicating, with a C or a D, whether the interference at that point is constructive or destructive.

25. ‖ Two in-phase speakers 2.0 m apart in a plane are emitting 1800 Hz sound waves into a room where the speed of sound is 340 m/s. Is the point 4.0 m in front of one of the speakers, perpendicular to the plane of the speakers, a point of maximum constructive interference, perfect destructive interference, or something in between?

26. ‖ Two out-of-phase radio antennas at $x = \pm 300$ m on the $x$-axis are emitting 3.0 MHz radio waves. Is the point $(x, y) = (300 \text{ m}, 800 \text{ m})$ a point of maximum constructive interference, perfect destructive interference, or something in between?

### Section 21.8 Beats

27. | Two strings are adjusted to vibrate at exactly 200 Hz. Then the tension in one string is increased slightly. Afterward, three beats per second are heard when the strings vibrate at the same time. What is the new frequency of the string that was tightened?

28. | A flute player hears four beats per second when she compares her note to a 523 Hz tuning fork (the note C). She can match the frequency of the tuning fork by pulling out the "tuning joint" to lengthen her flute slightly. What was her initial frequency?

29. | Two microwave signals of nearly equal wavelengths can generate a beat frequency if both are directed onto the same microwave detector. In an experiment, the beat frequency is 100 MHz. One microwave generator is set to emit microwaves with a wavelength of 1.250 cm. If the second generator emits the longer wavelength, what is that wavelength?

## Problems

30. ‖ Two waves on a string travel in opposite directions at 100 m/s. **FIGURE P21.30** shows a snapshot graph of the string at $t = 0$ s, when the two waves are overlapped, and a snapshot graph of the left-traveling wave at $t = 0.050$ s. Draw a snapshot graph of the right-traveling wave at $t = 0.050$ s.
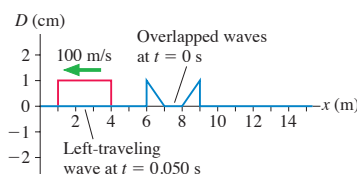


**FIGURE P21.30**

31. | A 2.0-m-long string vibrates at its second-harmonic frequency with a maximum amplitude of 2.0 cm. One end of the string is at $x = 0$ cm. Find the oscillation amplitude at $x = 10, 20, 30, 40,$ and 50 cm.

32. ‖ A string vibrates at its third-harmonic frequency. The amplitude at a point 30 cm from one end is half the maximum amplitude. How long is the string?

33. ‖ A string of length $L$ vibrates at its fundamental frequency. The amplitude at a point $\frac{1}{4}L$ from one end is 2.0 cm. What is the amplitude of each of the traveling waves that form this standing wave?

34. ‖ Two sinusoidal waves with equal wavelengths travel along a string in opposite directions at 3.0 m/s. The time between two successive instants when the antinodes are at maximum height is 0.25 s. What is the wavelength?

35. ‖ BIO Tendons are, essentially, elastic cords stretched between two fixed ends. As such, they can support standing waves. A woman has a 20-cm-long Achilles tendon—connecting the heel to a muscle in the calf—with a cross-section area of 90 mm². The density of tendon tissue is 1100 kg/m³. For a reasonable tension of 500 N, what will be the fundamental frequency of her Achilles tendon?

36. ‖ BIO Biologists think that some spiders "tune" strands of their web to give enhanced response at frequencies corresponding to those at which desirable prey might struggle. Orb spider web silk has a typical diameter of 20 $\mu$m, and spider silk has a density of 1300 kg/m³. To have a fundamental frequency at 100 Hz, to what tension must a spider adjust a 12-cm-long strand of silk?

37. ‖ A particularly beautiful note reaching your ear from a rare Stradivarius violin has a wavelength of 39.1 cm. The room is slightly warm, so the speed of sound is 344 m/s. If the string's linear density is 0.600 g/m and the tension is 150 N, how long is the vibrating section of the violin string?

38. ‖ A violinist places her finger so that the vibrating section of a 1.0 g/m string has a length of 30 cm, then she draws her bow across it. A listener nearby in a 20°C room hears a note with a wavelength of 40 cm. What is the tension in the string?

39. ‖ A steel wire is used to stretch the spring of **FIGURE P21.39**. An oscillating magnetic field drives the steel wire back and forth. A standing wave with three antinodes is created when the spring is stretched 8.0 cm. What stretch of the spring produces a standing wave with two antinodes?

Steel wire   Pull

**FIGURE P21.39**   Spring

40. ‖ Astronauts visiting Planet X have a 250-cm-long string whose mass is 5.00 g. They tie the string to a support, stretch it horizontally over a pulley 2.00 m away, and hang a 4.00 kg mass on the free end. Then the astronauts begin to excite standing waves on the horizontal portion of the string. Their data are as follows:

| $m$ | Frequency (Hz) |
|---|---|
| 1 | 31 |
| 2 | 66 |
| 3 | 95 |
| 4 | 130 |
| 5 | 162 |

Use the best-fit line of an appropriate graph to determine the value of $g$, the free-fall acceleration on Planet X.

41. ‖ A 75 g bungee cord has an equilibrium length of 1.20 m. The cord is stretched to a length of 1.80 m, then vibrated at 20 Hz. This produces a standing wave with two antinodes. What is the spring constant of the bungee cord?

42. ‖ A metal wire under tension $T_0$ vibrates at its fundamental frequency. For what tension will the second-harmonic frequency be the same as the fundamental frequency at tension $T_0$?

43. ‖‖ In a laboratory experiment, one end of a horizontal string is tied to a support while the other end passes over a frictionless pulley and is tied to a 1.5 kg sphere. Students determine the frequencies of standing waves on the horizontal segment of the string, then they raise a beaker of water until the hanging 1.5 kg sphere is completely submerged. The frequency of the fifth harmonic with the sphere submerged exactly matches the frequency of the third harmonic before the sphere was submerged. What is the diameter of the sphere?

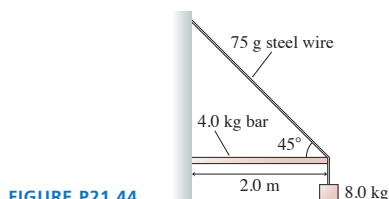44. ‖‖ What is the fundamental frequency of the steel wire in **FIGURE P21.44**?

75 g steel wire

4.0 kg bar

45°

**FIGURE P21.44**   2.0 m   8.0 kg

45. ‖ The two strings in **FIGURE P21.45** are of equal length and are being driven at equal frequencies. The linear density of the left string is $\mu_0$. What is the linear density of the right string?
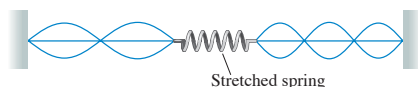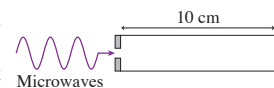
Stretched spring

**FIGURE P21.45**

46. | Microwaves pass through a small hole into the "microwave cavity" of **FIGURE P21.46**. What frequencies between 10 GHz and 20 GHz will create standing waves in the cavity?

10 cm

Microwaves

**FIGURE P21.46**

47. ‖ An open-open organ pipe is 78.0 cm long. An open-closed pipe has a fundamental frequency equal to the third harmonic of the open-open pipe. How long is the open-closed pipe?

48. | A narrow column of 20°C air is found to have standing waves at frequencies of 390 Hz, 520 Hz, and 650 Hz and at no frequencies in between these. The behavior of the tube at frequencies less than 390 Hz or greater than 650 Hz is not known.
   a. Is this an open-open tube or an open-closed tube? Explain.
   b. How long is the tube?

49. ‖ Deep-sea divers often breathe a mixture of helium and oxygen
BIO to avoid getting the "bends" from breathing high-pressure nitrogen. The helium has the side effect of making the divers' voices sound odd. Although your vocal tract can be roughly described as an open-closed tube, the way you hold your mouth and position your lips greatly affects the standing-wave frequencies of the vocal tract. This is what allows different vowels to sound different. The "ee" sound is made by shaping your vocal tract to have standing-wave frequencies at, normally, 270 Hz and 2300 Hz. What will these frequencies be for a helium-oxygen mixture in which the speed of sound at body temperature is 750 m/s? The speed of sound in air at body temperature is 350 m/s.

50. ‖ In 1866, the German scientist Adolph Kundt developed a technique for accurately measuring the speed of sound in various gases. A long glass tube, known today as a Kundt's tube, has a vibrating piston at one end and is closed at the other. Very finely ground particles of cork are sprinkled in the bottom of the tube before the piston is inserted. As the vibrating piston is slowly moved forward, there are a few positions that cause the cork particles to collect in small, regularly spaced piles along the bottom. **FIGURE P21.50** shows an experiment in which the tube is filled with pure oxygen and the piston is driven at 400 Hz. What is the speed of sound in oxygen?
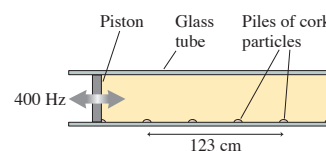
Piston   Glass tube   Piles of cork particles

400 Hz

123 cm

**FIGURE P21.50**

51. ‖ The 40-cm-long tube of **FIGURE P21.51** has a 40-cm-long insert that can be pulled in and out. A vibrating tuning fork is held next to the tube. As the insert is slowly pulled out, the sound from the tuning fork creates standing waves in the tube when the total length $L$ is 42.5 cm, 56.7 cm, and 70.9 cm. What is the frequency of the tuning fork? Assume $v_{sound} = 343$ m/s.
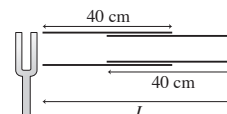
40 cm

40 cm

**FIGURE P21.51**   $L$

52. ‖ A 1.0-m-tall vertical tube is filled with 20°C water. A tuning fork vibrating at 580 Hz is held just over the top of the tube as the water is slowly drained from the bottom. At what water heights, measured from the bottom of the tube, will there be a standing wave in the tube above the water?

53. ‖ A 25-cm-long wire with a linear density of 20 g/m passes across the open end of an 85-cm-long open-closed tube of air. If the wire, which is fixed at both ends, vibrates at its fundamental frequency, the sound wave it generates excites the second vibrational mode of the tube of air. What is the tension in the wire? Assume $v_{sound} = 340$ m/s.

54. ‖ A longitudinal standing wave can be created in a long, thin aluminum rod by stroking the rod with very dry fingers. This is often done as a physics demonstration, creating a high-pitched, very annoying whine. From a wave perspective, the standing wave is equivalent to a sound standing wave in an open-open tube. As FIGURE P21.54 shows, both ends of the rod are anti-nodes. What is the fundamental frequency of a 2.0-m-long aluminum rod?

Aluminum rod

**FIGURE P21.54**

55. ‖ An old mining tunnel disappears into a hillside. You would like to know how long the tunnel is, but it's too dangerous to go inside. Recalling your recent physics class, you decide to try setting up standing-wave resonances inside the tunnel. Using your subsonic amplifier and loudspeaker, you find resonances at 4.5 Hz and 6.3 Hz, and at no frequencies between these. It's rather chilly inside the tunnel, so you estimate the sound speed to be 335 m/s. Based on your measurements, how far is it to the end of the tunnel?

56. ‖ Analyze the standing sound waves in an open-closed tube to show that the possible wavelengths and frequencies are given by Equation 21.18.

57. ‖‖ Two in-phase loudspeakers emit identical 1000 Hz sound waves along the x-axis. What distance should one speaker be placed behind the other for the sound to have an amplitude 1.5 times that of each speaker alone?

58. ‖ Two loudspeakers emit sound waves of the same frequency along the x-axis. The amplitude of each wave is a. The sound intensity is minimum when speaker 2 is 10 cm behind speaker 1. The intensity increases as speaker 2 is moved forward and first reaches maximum, with amplitude 2a, when it is 30 cm in front of speaker 1. What is
   a. The wavelength of the sound?
   b. The phase difference between the two loudspeakers?
   c. The amplitude of the sound (as a multiple of a) if the speakers are placed side by side?

59. ‖‖ Two loudspeakers emit sound waves along the x-axis. A listener in front of both speakers hears a maximum sound intensity when speaker 2 is at the origin and speaker 1 is at $x = 0.50$ m. If speaker 1 is slowly moved forward, the sound intensity decreases and then increases, reaching another maximum when speaker 1 is at $x = 0.90$ m.
   a. What is the frequency of the sound? Assume $v_{sound} = 340$ m/s.
   b. What is the phase difference between the speakers?

60. | A sheet of glass is coated with a 500-nm-thick layer of oil ($n = 1.42$).
   a. For what *visible* wavelengths of light do the reflected waves interfere constructively?
   b. For what *visible* wavelengths of light do the reflected waves interfere destructively?
   c. What is the color of reflected light? What is the color of transmitted light?

61. ‖ A manufacturing firm has hired your company, Acoustical Consulting, to help with a problem. Their employees are complaining about the annoying hum from a piece of machinery. Using a frequency meter, you quickly determine that the machine emits a rather loud sound at 1200 Hz. After investigating, you tell the owner that you cannot solve the problem entirely, but you can at least improve the situation by eliminating reflections of this sound from the walls. You propose to do this by installing mesh screens in front of the walls. A portion of the sound will reflect from the mesh; the rest will pass through the mesh and reflect from the wall. How far should the mesh be placed in front of the wall for this scheme to work?

62. ‖ A soap bubble is essentially a very thin film of water ($n = 1.33$) surrounded by air. The colors that you see in soap bubbles are produced by interference.
   a. Derive an expression for the wavelengths $\lambda_C$ for which constructive interference causes a strong reflection from a soap bubble of thickness d.
   **Hint:** Think about the reflection phase shifts at both boundaries.
   b. What visible wavelengths of light are strongly reflected from a 390-nm-thick soap bubble? What color would such a soap bubble appear to be?

63. ‖ Two radio antennas are separated by 2.0 m. Both broadcast identical 750 MHz waves. If you walk around the antennas in a circle of radius 10 m, how many maxima will you detect?

64. ‖ You are standing 2.5 m directly in front of one of the two loudspeakers shown in FIGURE P21.64. They are 3.0 m apart and both are playing a 686 Hz tone in phase. As you begin to walk directly away from the speaker, at what distances from the speaker do you hear a *minimum* sound intensity? The room temperature is 20°C.
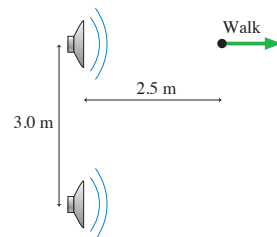
Walk

2.5 m

3.0 m

**FIGURE P21.64**

65. ‖ Two loudspeakers in a plane, 5.0 apart, are playing the same frequency. If you stand 12.0 m in front of the plane of the speakers, centered between them, you hear a sound of maximum intensity. As you walk parallel to the plane of the speakers, staying 12.0 m in front of them, you first hear a minimum of sound intensity when you are directly in front of one of the speakers. What is the frequency of the sound? Assume a sound speed of 340 m/s.

66. ‖ Two in-phase loudspeakers are located at $(x, y)$ coordinates $(-3.0$ m, $+2.0$ m) and $(-3.0$ m, $-2.0$ m). They emit identical sound waves with a 2.0 m wavelength and amplitude a. Determine the amplitude of the sound at the five positions on the y-axis ($x = 0$) with $y = 0.0$ m, 0.5 m, 1.0 m, 1.5 m, and 2.0 m.

67. ‖ Two identical loudspeakers separated by distance $\Delta x$ each emit sound waves of wavelength $\lambda$ and amplitude a along the x-axis. What is the minimum value of the ratio $\Delta x/\lambda$ for which the amplitude of their superposition is also a?

68. ‖ Two radio antennas are 100 m apart along a north-south line. They broadcast identical radio waves at a frequency of 3.0 MHz. Your job is to monitor the signal strength with a hand-held receiver. To get to your first measuring point, you walk 800 m east from the midpoint between the antennas, then 600 m north.
    a. What is the phase difference between the waves at this point?
    b. Is the interference at this point maximum constructive, perfect destructive, or somewhere in between? Explain.
    c. If you now begin to walk farther north, does the signal strength increase, decrease, or stay the same? Explain.

69. ‖ The three identical loudspeakers in **FIGURE P21.69** play a 170 Hz tone in a room where the speed of sound is 340 m/s. You are standing 4.0 m in front of the middle speaker. At this point, the amplitude of the wave from each speaker is $a$.
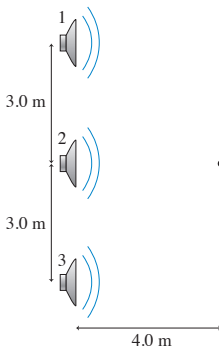    a. What is the amplitude at this point?
    b. How far must speaker 2 be moved to the left to produce a maximum amplitude at the point where you are standing?
    c. When the amplitude is maximum, by what factor is the sound intensity greater than the sound intensity from a single speaker?

**FIGURE P21.69**

70. | Piano tuners tune pianos by listening to the beats between the *harmonics* of two different strings. When properly tuned, the note A should have a frequency of 440 Hz and the note E should be at 659 Hz.
    a. What is the frequency difference between the third harmonic of the A and the second harmonic of the E?
    b. A tuner first tunes the A string very precisely by matching it to a 440 Hz tuning fork. She then strikes the A and E strings simultaneously and listens for beats between the harmonics. What beat frequency indicates that the E string is properly tuned?
    c. The tuner starts with the tension in the E string a little low, then tightens it. What is the frequency of the E string when she hears four beats per second?

71. ‖ A flutist assembles her flute in a room where the speed of sound is 342 m/s. When she plays the note A, it is in perfect tune with a 440 Hz tuning fork. After a few minutes, the air inside her flute has warmed to where the speed of sound is 346 m/s.
    a. How many beats per second will she hear if she now plays the note A as the tuning fork is sounded?
    b. How far does she need to extend the "tuning joint" of her flute to be in tune with the tuning fork?

72. ‖ Two loudspeakers face each other from opposite walls of a room. Both are playing exactly the same frequency, thus setting up a standing wave with distance $\lambda/2$ between antinodes. Assume that $\lambda$ is much less than the room width, so there are many antinodes.
    a. Yvette starts at one speaker and runs toward the other at speed $v_Y$. As the does so, she hears a loud-soft-loud modulation of the sound intensity. From your perspective, as you sit at rest in the room, Yvette is running through the nodes and antinodes of the standing wave. Find an expression for the number of sound maxima she hears per second.

    b. From Yvette's perspective, the two sound waves are Doppler shifted. They're not the same frequency, so they don't create a standing wave. Instead, she hears a loud-soft-loud modulation of the sound intensity because of beats. Find an expression for the beat frequency that Yvette hears.
    c. Are your answers to parts a and b the same or different? *Should* they be the same or different?

73. ‖ Two loudspeakers emit 400 Hz notes. One speaker sits on the ground. The other speaker is in the back of a pickup truck. You hear eight beats per second as the truck drives away from you. What is the truck's speed?

## Challenge Problems

74. a. The frequency of a standing wave on a string is $f$ when the string's tension is $T_s$. If the tension is changed by the *small* amount $\Delta T_s$, without changing the length, show that the frequency changes by an amount $\Delta f$ such that

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T_s}{T_s}$$

    b. Two identical strings vibrate at 500 Hz when stretched with the same tension. What percentage increase in the tension of one of the strings will cause five beats per second when both strings vibrate simultaneously?

75. A 280 Hz sound wave is directed into one end of the trombone slide seen in **FIGURE CP21.75**. A microphone is placed at the other end to record the intensity of sound waves that are transmitted through the tube. The straight sides of the slide are 80 cm in length and 10 cm apart with a semicircular bend at the end. For what slide extensions $s$ will the microphone detect a maximum of sound intensity?
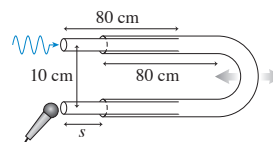
**FIGURE CP21.75**

76. As the captain of the scientific team sent to Planet Physics, one of your tasks is to measure $g$. You have a long, thin wire labeled 1.00 g/m and a 1.25 kg weight. You have your accurate space cadet chronometer but, unfortunately, you seem to have forgotten a meter stick. Undeterred, you first find the midpoint of the wire by folding it in half. You then attach one end of the wire to the wall of your laboratory, stretch it horizontally to pass over a pulley at the midpoint of the wire, then tie the 1.25 kg weight to the end hanging over the pulley. By vibrating the wire, and measuring time with your chronometer, you find that the wire's second-harmonic frequency is 100 Hz. Next, with the 1.25 kg weight still tied to one end of the wire, you attach the other end to the ceiling to make a pendulum. You find that the pendulum requires 314 s to complete 100 oscillations. Pulling out your trusty calculator, you get to work. What value of $g$ will you report back to headquarters?

77. When mass $M$ is tied to the bottom of a long, thin wire suspended from the ceiling, the wire's second-harmonic frequency is 200 Hz. Adding an additional 1.0 kg to the hanging mass increases the second-harmonic frequency to 245 Hz. What is $M$?

78. Ultrasound has many medical applications, one of which is to
BIO monitor fetal heartbeats by reflecting ultrasound off a fetus in the womb.
    a. Consider an object moving at speed $v_o$ toward an at-rest source that is emitting sound waves of frequency $f_0$. Show that the reflected wave (i.e., the echo) that returns to the source has a Doppler-shifted frequency

$$f_{echo} = \left(\frac{v + v_o}{v - v_o}\right) f_0$$

where $v$ is the speed of sound in the medium.
    b. Suppose the object's speed is much less than the wave speed: $v_o \ll v$. Then $f_{echo} \approx f_0$, and a microphone that is sensitive to these frequencies will detect a beat frequency if it listens to $f_0$ and $f_{echo}$ simultaneously. Use the binomial approximation and other appropriate approximations to show that the beat frequency is $f_{beat} \approx (2v_o/v)f_0$.
    c. The reflection of 2.40 MHz ultrasound waves from the surface of a fetus's beating heart is combined with the 2.40 MHz wave to produce a beat frequency that reaches a maximum of 65 Hz. What is the maximum speed of the surface of the heart? The speed of ultrasound waves within the body is 1540 m/s.
    d. Suppose the surface of the heart moves in simple harmonic motion at 90 beats/min. What is the amplitude in mm of the heartbeat?

79. A water wave is called a *deep-water wave* if the water's depth is more than one-quarter of the wavelength. Unlike the waves we've considered in this chapter, the speed of a deep-water wave depends on its wavelength:

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

Longer wavelengths travel faster. Let's apply this to standing waves. Consider a diving pool that is 5.0 m deep and 10.0 m wide. Standing water waves can set up across the width of the pool. Because water sloshes up and down at the sides of the pool, the boundary conditions require antinodes at $x = 0$ and $x = L$. Thus a standing water wave resembles a standing sound wave in an open-open tube.
    a. What are the wavelengths of the first three standing-wave modes for water in the pool? Do they satisfy the condition for being deep-water waves? Draw a graph of each.
    b. What are the wave speeds for each of these waves?
    c. Derive a general expression for the frequencies $f_m$ of the possible standing waves. Your expression should be in terms of $m$, $g$, and $L$.
    d. What are the oscillation *periods* of the first three standing-wave modes?

80. The broadcast antenna of an AM radio station is located at the edge of town. The station owners would like to beam all of the energy into town and none into the countryside, but a single antenna radiates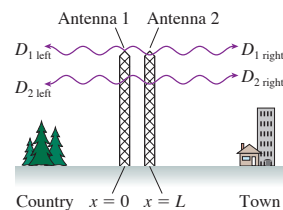 energy equally in all directions. **FIGURE CP21.80** shows two parallel antennas separated by distance $L$. Both antennas broadcast a signal at wavelength $\lambda$, but antenna 2 can delay its broadcast relative to antenna 1 by a time interval $\Delta t$ in order to create a phase difference $\Delta\phi_0$ between the sources. Your task is to find values of $L$ and $\Delta t$ such that the waves interfere constructively on the town side and destructively on the country side.



**FIGURE CP21.80**

Let antenna 1 be at $x = 0$. The wave that travels to the right is $a\sin[2\pi(x/\lambda - t/T)]$. The left wave is $a\sin[2\pi(-x/\lambda - t/T)]$. (It must be this, rather than $a\sin[2\pi(x/\lambda + t/T)]$, so that the two waves match at $x = 0$.) Antenna 2 is at $x = L$. It broadcasts wave $a\sin[2\pi((x - L)/\lambda - t/T) + \phi_{20}]$ to the right and wave $a\sin[2\pi(-(x - L)/\lambda - t/T) + \phi_{20}]$ to the left.
    a. What is the smallest value of $L$ for which you can create perfect constructive interference on the town side and perfect destructive interference on the country side? Your answer will be a multiple or fraction of the wavelength $\lambda$.
    b. What phase constant $\phi_{20}$ of antenna 2 is needed?
    c. What fraction of the oscillation period $T$ must $\Delta t$ be to produce the proper value of $\phi_{20}$?
    d. Evaluate both $L$ and $\Delta t$ for the realistic AM radio frequency of 1000 KHz.

**Comment:** This is a simple example of what is called a *phased array*, where phase differences between identical emitters are used to "steer" the radiation in a particular direction. Phased arrays are widely used in radar technology.

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### STOP TO THINK ANSWERS

**Stop to Think 21.1: c.** The figure shows the two waves at $t = 6$ s and their superposition. The superposition is the *point-by-point* addition of the displacements of the two individual waves.
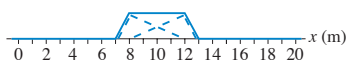


**Stop to Think 21.2: a.** The allowed standing-wave frequencies are $f_m = m(v/2L)$, so the mode number of a standing wave of frequency $f$ is $m = 2Lf/v$. Quadrupling $T_s$ increases the wave speed $v$ by a factor of 2. The initial mode number was 2, so the new mode number is 1.

**Stop to Think 21.3: b.** 300 Hz and 400 Hz are allowed standing waves, but they are not $f_1$ and $f_2$ because 400 Hz $\neq 2 \times 300$ Hz. Because there's a 100 Hz difference between them, these must be $f_3 = 3 \times 100$ Hz and $f_4 = 4 \times 100$ Hz, with a fundamental frequency $f_1 = 100$ Hz. Thus the second harmonic is $f_2 = 2 \times 100$ Hz $= 200$ Hz.

**Stop to Think 21.4: c.** Shifting the top wave 0.5 m to the left aligns crest with crest and trough with trough.

**Stop to Think 21.5: a.** $r_1 = 0.5\lambda$ and $r_2 = 2.5\lambda$, so $\Delta r = 2.0\lambda$. This is the condition for maximum constructive interference.

**Stop to Think 21.6: Maximum constructive.** The path-length difference is $\Delta r = 1.0$ m $= \lambda$. For identical sources, interference is constructive when $\Delta r$ is an integer multiple of $\lambda$.

**Stop to Think 21.7: f.** The beat frequency is the difference between the two frequencies.

# SUMMARY

The goal of Chapter 22 has been to understand and apply the wave model of light.

## General Principles

Huygens' principle says that each point on a wave front is the source of a spherical wavelet. The wave front at a later time is tangent to all the wavelets.

Diffraction is the spreading of a wave after it passes through an opening.

Constructive and destructive interference are due to the overlap of two or more waves as they spread behind openings.

## Important Concepts

The wave model of light considers light to be a wave propagating through space. Diffraction and interference are important.

The ray model of light considers light to travel in straight lines like little particles. Diffraction and interference are not important.

Diffraction is important when the width of the diffraction pattern of an aperture equals or exceeds the size of the aperture. For a circular aperture, the crossover between the ray and wave models occurs for an opening of diameter $D_c \approx \sqrt{2\lambda L}$.

In practice, $D_c \approx 1$ mm for visible light. Thus

- Use the wave model when light passes through openings $< 1$ mm in size. Diffraction effects are usually important.

- Use the ray model when light passes through openings $> 1$ mm in size. Diffraction is usually not important.

## Applications

Single slit of width $a$.
A bright **central maximum** of width

$$w = \frac{2\lambda L}{a}$$

is flanked by weaker **secondary maxima.**
Dark fringes are located at angles such that

$$a\sin\theta_p = p\lambda \qquad p = 1, 2, 3, \ldots$$

If $\lambda/a \ll 1$, then from the small-angle approximation

$$\theta_p = \frac{p\lambda}{a} \qquad y_p = \frac{p\lambda L}{a}$$

**Interference due to wave-front division**

Waves overlap as they spread out behind slits. Constructive interference occurs along antinodal lines. Bright fringes are seen where the antinodal lines intersect the viewing screen.

Double slit with separation $d$.
Equally spaced bright fringes are located at

$$\theta_m = \frac{m\lambda}{d} \qquad y_m = \frac{m\lambda L}{d} \qquad m = 0, 1, 2, \ldots$$

The **fringe spacing** is $\Delta y = \dfrac{\lambda L}{d}$

Diffraction grating with slit spacing $d$.
Very bright and narrow fringes are located at angles and positions

$$d\sin\theta_m = m\lambda \qquad y_m = L\tan\theta_m$$

Circular aperture of diameter $D$.
A bright central maximum of diameter

$$w = \frac{2.44\lambda L}{D}$$

is surrounded by circular secondary maxima.
The first dark fringe is located at

$$\theta_1 = \frac{1.22\lambda}{D} \qquad y_1 = \frac{1.22\lambda L}{D}$$

For an aperture of any shape, a smaller opening causes a more rapid spreading of the wave behind the opening.

**Interference due to amplitude division**

An interferometer divides a wave, lets the two waves travel different paths, then recombines them. Interference is constructive if one wave travels an integer number of wavelengths more or less than the other wave. The difference can be due to an actual path-length difference or to a different index of refraction.

Michelson interferometer

The number of bright-dark-bright fringe shifts as mirror $M_2$ moves distance $\Delta L_2$ is

$$\Delta m = \frac{\Delta L_2}{\lambda/2}$$

# Terms and Notation

| | | | |
|---|---|---|---|
| optics | photon model | diffraction grating | Huygens' principle |
| diffraction | double slit | order, $m$ | circular aperture |
| models of light | interference fringes | spectroscopy | interferometer |
| wave model | central maximum | single-slit diffraction | beam splitter |
| ray model | fringe spacing, $\Delta y$ | secondary maxima | hologram |

# CONCEPTUAL QUESTIONS

1. **FIGURE Q22.1** shows light waves passing through two closely spaced, narrow slits. The graph shows the intensity of light on a screen behind the slits. Reproduce these graph axes, including the zero and the tick marks locating the double-slit fringes, then draw a graph to show how the light-intensity pattern will appear if the right slit is blocked, allowing light to go through only the left slit. Explain your reasoning.
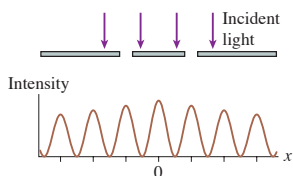


FIGURE Q22.1

2. In a double-slit interference experiment, which of the following actions (perhaps more than one) would cause the fringe spacing to increase? (a) Increasing the wavelength of the light. (b) Increasing the slit spacing. (c) Increasing the distance to the viewing screen. (d) Submerging the entire experiment in water.

3. **FIGURE Q22.3** shows the viewing screen in a double-slit experiment. Fringe C is the central maximum. What will happen to the fringe spacing if
   a. The wavelength of the light is decreased?
   b. The spacing between the slits is decreased?
   c. The distance to the screen is decreased?
   d. Suppose the wavelength of the light is 500 nm. How much farther is it from the dot on the screen in the center of fringe E to the left slit than it is from the dot to the right slit?
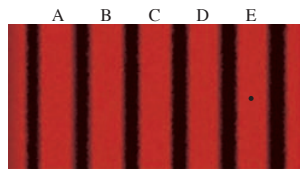


FIGURE Q22.3

4. **FIGURE Q22.3** is the interference pattern seen on a viewing screen behind 2 slits. Suppose the 2 slits were replaced by 20 slits having the same spacing $d$ between adjacent slits.
   a. Would the number of fringes on the screen increase, decrease, or stay the same?
   b. Would the fringe spacing increase, decrease, or stay the same?
   c. Would the width of each fringe increase, decrease, or stay the same?
   d. Would the brightness of each fringe increase, decrease, or stay the same?

5. **FIGURE Q22.5** shows the light intensity on a viewing screen behind a single slit of width $a$. The light's wavelength is $\lambda$. Is $\lambda < a, \lambda = a, \lambda > a$, or is it not possible to tell? Explain.



FIGURE Q22.5          FIGURE Q22.6

6. **FIGURE Q22.6** shows the light intensity on a viewing screen behind a circular aperture. What happens to the width of the central maximum if
   a. The wavelength of the light is increased?
   b. The diameter of the aperture is increased?
   c. How will the screen appear if the aperture diameter is less than the light wavelength?

7. Narrow, bright fringes are observed on a screen behind a diffraction grating. The entire experiment is then immersed in water. Do the fringes on the screen get closer together, get farther apart, remain the same, or disappear? Explain.

8. a. Green light shines through a 100-mm-diameter hole and is observed on a screen. If the hole diameter is increased by 20%, does the circular spot of light on the screen decrease in diameter, increase in diameter, or stay the same? Explain.
   b. Green light shines through a 100-$\mu$m-diameter hole and is observed on a screen. If the hole diameter is increased by 20%, does the circular spot of light on the screen decrease in diameter, increase in diameter, or stay the same? Explain.

9. A Michelson interferometer using 800 nm light is adjusted to have a bright central spot. One mirror is then moved 200 nm forward, the other 200 nm back. Afterward, is the central spot bright, dark, or in between? Explain.

10. A Michelson interferometer is set up to display constructive interference (a bright central spot in the fringe pattern of Figure 22.21) using light of wavelength $\lambda$. If the wavelength is changed to $\lambda/2$, does the central spot remain bright, does the central spot become dark, or do the fringes disappear? Explain. Assume the fringes are viewed by a detector sensitive to both wavelengths.

# EXERCISES AND PROBLEMS

Problems labeled [    ] integrate material from earlier chapters.

## Exercises

### Section 22.2 The Interference of Light

1. | Two narrow slits 80 $\mu$m apart are illuminated with light of wavelength 600 nm. What is the angle of the $m = 3$ bright fringe in radians? In degrees?

2. | A double slit is illuminated simultaneously with orange light of wavelength 600 nm and light of an unknown wavelength. The $m = 4$ bright fringe of the unknown wavelength overlaps the $m = 3$ bright orange fringe. What is the unknown wavelength?

3. | Light of wavelength 500 nm illuminates a double slit, and the interference pattern is observed on a screen. At the position of the $m = 2$ bright fringe, how much farther is it to the more distant slit than to the nearer slit?

4. | A double-slit experiment is performed with light of wavelength 600 nm. The bright interference fringes are spaced 1.8 mm apart on the viewing screen. What will the fringe spacing be if the light is changed to a wavelength of 400 nm?

5. ‖ Light of 600 nm wavelength illuminates a double slit. The intensity pattern shown in **FIGURE EX22.5** is seen on a screen 2.0 m behind the slits. What is the spacing (in mm) between the slits?
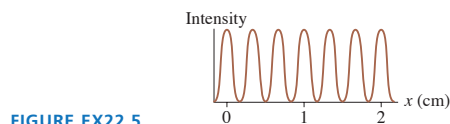
**FIGURE EX22.5**

6. ‖ Light from a sodium lamp ($\lambda = 589$ nm) illuminates two narrow slits. The fringe spacing on a screen 150 cm behind the slits is 4.0 mm. What is the spacing (in mm) between the two slits?

7. ‖ In a double-slit experiment, the slit separation is 200 times the wavelength of the light. What is the angular separation (in degrees) between two adjacent bright fringes?

8. ‖ A double-slit interference pattern is created by two narrow slits spaced 0.20 mm apart. The distance between the first and the fifth minimum on a screen 60 cm behind the slits is 6.0 mm. What is the wavelength (in nm) of the light used in this experiment?

### Section 22.3 The Diffraction Grating

9. | A 4.0-cm-wide diffraction grating has 2000 slits. It is illuminated by light of wavelength 550 nm. What are the angles (in degrees) of the first two diffraction orders?

10. ‖ A diffraction grating produces a first-order maximum at an angle of 20.0°. What is the angle of the second-order maximum?

11. ‖ Light of wavelength 600 nm illuminates a diffraction grating. The second-order maximum is at angle 39.5°. How many lines per millimeter does this grating have?

12. ‖ The two most prominent wavelengths in the light emitted by a hydrogen discharge lamp are 656 nm (red) and 486 nm (blue). Light from a hydrogen lamp illuminates a diffraction grating with 500 lines/mm, and the light is observed on a screen 1.5 m behind the grating. What is the distance between the first-order red and blue fringes?

13. ‖ A helium-neon laser ($\lambda = 633$ nm) illuminates a diffraction grating. The distance between the two $m = 1$ bright fringes is 32 cm on a screen 2.0 m behind the grating. What is the spacing between slits of the grating?

14. | A diffraction grating is illuminated simultaneously with red light of wavelength 660 nm and light of an unknown wavelength. The fifth-order maximum of the unknown wavelength exactly overlaps the third-order maximum of the red light. What is the unknown wavelength?

### Section 22.4 Single-Slit Diffraction

15. | A helium-neon laser ($\lambda = 633$ nm) illuminates a single slit and is observed on a screen 1.5 m behind the slit. The distance between the first and second minima in the diffraction pattern is 4.75 mm. What is the width (in mm) of the slit?

16. | In a single-slit experiment, the slit width is 200 times the wavelength of the light. What is the width (in mm) of the central maximum on a screen 2.0 m behind the slit?

17. | The central maximum of a single slit has width $4000\lambda$ when viewed on a screen 1.0 m behind the slit. How wide (in mm) is the slit?

18. ‖ Light of 600 nm wavelength illuminates a single slit. The intensity pattern shown in **FIGURE EX22.18** is seen on a screen 2.0 m behind the slits. What is the width (in mm) of the slit?
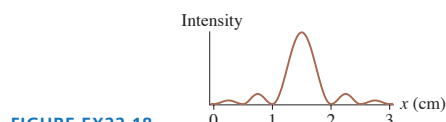
**FIGURE EX22.18**

19. ‖ A 0.50-mm-wide slit is illuminated by light of wavelength 500 nm. What is the width (in mm) of the central maximum on a screen 2.0 m behind the slit?

20. ‖ You need to use your cell phone, which broadcasts an 800 MHz signal, but you're behind two massive, radio-wave-absorbing buildings that have only a 15 m space between them. What is the angular width, in degrees, of the electromagnetic wave after it emerges from between the buildings?

21. | The opening to a cave is a tall, 30-cm-wide crack. A bat that is preparing to leave the cave emits a 30 kHz ultrasonic chirp. How wide is the "sound beam" 100 m outside the cave opening? Use $v_{\text{sound}} = 340$ m/s.

### Section 22.5 Circular-Aperture Diffraction

22. ‖ A 0.50-mm-diameter hole is illuminated by light of wavelength 500 nm. What is the width (in mm) of the central maximum on a screen 2.0 m behind the slit?

23. | Infrared light of wavelength 2.5 $\mu$m illuminates a 0.20-mm-diameter hole. What is the angle of the first dark fringe in radians? In degrees?

24. | You want to photograph a circular diffraction pattern whose central maximum has a diameter of 1.0 cm. You have a helium-neon laser ($\lambda = 633$ nm) and a 0.12-mm-diameter pinhole. How far behind the pinhole should you place the screen that's to be photographed?

25. ‖ Light from a helium-neon laser ($\lambda = 633$ nm) passes through a circular aperture and is observed on a screen 4.0 m behind the aperture. The width of the central maximum is 2.5 cm. What is the diameter (in mm) of the hole?

### Section 22.6 Interferometers

26. | A Michelson interferometer uses red light with a wavelength of 656.45 nm from a hydrogen discharge lamp. How many bright-dark-bright fringe shifts are observed if mirror $M_2$ is moved exactly 1 cm?

27. | Moving mirror $M_2$ of a Michelson interferometer a distance of 100 $\mu$m causes 500 bright-dark-bright fringe shifts. What is the wavelength of the light?

28. ‖ A Michelson interferometer uses light whose wavelength is known to be 602.446 nm. Mirror $M_2$ is slowly moved while exactly 33,198 bright-dark-bright fringe shifts are observed. What distance has $M_2$ moved? Be sure to give your answer to an appropriate number of significant figures.

29. | A Michelson interferometer uses light from a sodium lamp. Sodium atoms emit light having wavelengths 589.0 nm and 589.6 nm. The interferometer is initially set up with both arms of equal length ($L_1 = L_2$), producing a bright spot at the center of the interference pattern. How far must mirror $M_2$ be moved so that one wavelength has produced one more new maximum than the other wavelength?

### Problems

30. | **FIGURE P22.30** shows the light intensity on a screen 2.5 m behind an aperture. The aperture is illuminated with light of wavelength 600 nm.
    a. Is the aperture a single slit or a double slit? Explain.
    b. If the aperture is a single slit, what is its width? If it is a double slit, what is the spacing between the slits?

Intensity



**FIGURE P22.30**   0   1   2   3   4   5   $x$ (cm)

31. | **FIGURE P22.31** shows the light intensity on a screen 2.5 m behind an aperture. The aperture is illuminated with light of wavelength 600 nm.
    a. Is the aperture a single slit or a double slit? Explain.
    b. If the aperture is a single slit, what is its width? If it is a double slit, what is the spacing between the slits?

Intensity



**FIGURE P22.31**   0   1   2   3   4   5   6   $x$ (cm)
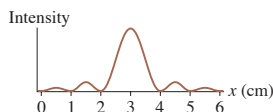
32. ‖ Light from a helium-neon laser ($\lambda = 633$ nm) is used to illuminate two narrow slits. The interference pattern is observed on a screen 3.0 m behind the slits. Twelve bright fringes are seen, spanning a distance of 52 mm. What is the spacing (in mm) between the slits?

33. ‖ **FIGURE P22.33** shows the light intensity on a screen behind a double slit. The slit spacing is 0.20 mm and the wavelength of the light is 600 nm. What is the distance from the slits to the screen?

Intensity (mW/m$^2$)



**FIGURE P22.33**   2.0 cm

34. ‖ **FIGURE P22.33** shows the light intensity on a screen behind a double slit. The slit spacing is 0.20 mm and the screen is 2.0 m behind the slits. What is the wavelength (in nm) of the light?
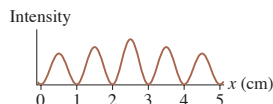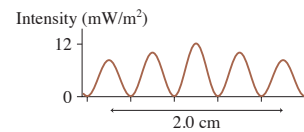
35. | **FIGURE P22.33** shows the light intensity on a screen behind a double slit. Suppose one slit is covered. What will be the light intensity at the center of the screen due to the remaining slit?

36. ‖ A laser beam with a wavelength of 524 nm is exactly perpendicular to a screen having two narrow slits spaced 0.150 mm apart. Interference fringes, including a central maximum, are observed on a viewing screen 1.00 m away. The direction of the laser beam is then slowly rotated by 1.0° around an axis parallel to the slits until it makes an 89.0° angle with the screen. How far does the central maximum move on the viewing screen?

37. ‖ A double-slit experiment is set up using a helium-neon laser ($\lambda = 633$ nm). Then a very thin piece of glass ($n = 1.50$) is placed over one of the slits. Afterward, the central point on the screen is occupied by what had been the $m = 10$ dark fringe. How thick is the glass?

38. ‖ A diffraction grating having 500 lines/mm diffracts visible light at 30°. What is the light's wavelength?

39. ‖ Helium atoms emit light at several wavelengths. Light from a helium lamp illuminates a diffraction grating and is observed on a screen 50.0 cm behind the grating. The emission at wavelength 501.5 nm creates a first-order bright fringe 21.90 cm from the central maximum. What is the wavelength of the bright fringe that is 31.60 cm from the central maximum?

40. ‖ A triple-slit experiment consists of three narrow slits, equally spaced by distance $d$ and illuminated by light of wavelength $\lambda$. Each slit alone produces intensity $I_1$ on the viewing screen at distance $L$.
    a. Consider a point on the distant viewing screen such that the path-length difference between any two adjacent slits is $\lambda$. What is the intensity at this point?
    b. What is the intensity at a point where the path-length difference between any two adjacent slits is $\lambda/2$?

41. ‖ Because sound is a wave, it's possible to make a diffraction grating for sound from a large board of sound-absorbing material with several parallel slits cut for sound to go through. When 10 kHz sound waves pass through such a grating, listeners 10 m from the grating report "loud spots" 1.4 m on both sides of center. What is the spacing between the slits? Use 340 m/s for the speed of sound.

42. ‖ A diffraction grating with 600 lines/mm is illuminated with light of wavelength 500 nm. A very wide viewing screen is 2.0 m behind the grating.
    a. What is the distance between the two $m = 1$ bright fringes?
    b. How many bright fringes can be seen on the screen?

43. ‖ A 500 line/mm diffraction grating is illuminated by light of wavelength 510 nm. How many bright fringes are seen on a 2.0-m-wide screen located 2.0 m behind the grating?

44. ‖ White light (400–700 nm) incident on a 600 line/mm diffraction grating produces rainbows of diffracted light. What is the width of the first-order rainbow on a screen 2.0 m behind the grating?

45. ‖ For your science fair project you need to design a diffraction grating that will disperse the visible spectrum (400–700 nm) over 30.0° in first order.
   a. How many lines per millimeter does your grating need?
   b. What is the first-order diffraction angle of light from a sodium lamp ($\lambda = 589$ nm)?

46. ‖ FIGURE P22.46 shows the interference pattern on a screen 1.0 m behind an 800 line/mm diffraction grating. What is the wavelength (in nm) of the light?
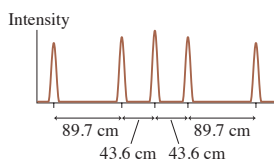
Intensity

89.7 cm    89.7 cm
        43.6 cm   43.6 cm

FIGURE P22.46

47. ‖ FIGURE P22.46 shows the interference pattern on a screen 1.0 m behind a diffraction grating. The wavelength of the light is 600 nm. How many lines per millimeter does the grating have?

48. ‖ Light from a sodium lamp ($\lambda = 589$ nm) illuminates a narrow slit and is observed on a screen 75 cm behind the slit. The distance between the first and third dark fringes is 7.5 mm. What is the width (in mm) of the slit?

49. | The wings of some beetles have closely spaced parallel lines of melanin, causing the wing to act as a reflection grating. Sup-
BIO pose sunlight shines straight onto a beetle wing. If the melanin lines on the wing are spaced 2.0 $\mu$m apart, what is the first-order diffraction angle for green light ($\lambda = 550$ nm)?

50. | If sunlight shines straight onto a peacock feather, the feather
BIO appears bright blue when viewed from 15° on either side of the incident beam of light. The blue color is due to diffraction from parallel rods of melanin in the feather barbules, as was shown in the photograph on page 636. Other wavelengths in the incident light are diffracted at different angles, leaving only the blue light to be seen. The average wavelength of blue light is 470 nm. Assuming this to be the first-order diffraction, what is the spacing of the melanin rods in the feather?

51. ‖ You've found an unlabeled diffraction grating. Before you can use it, you need to know how many lines per mm it has. To find out, you illuminate the grating with light of several different wavelengths and then measure the distance between the two first-order bright fringes on a viewing screen 150 cm behind the grating. Your data are as follows:

| Wavelength (nm) | Distance (cm) |
|---|---|
| 430 | 109.6 |
| 480 | 125.4 |
| 530 | 139.8 |
| 580 | 157.2 |
| 630 | 174.4 |
| 680 | 194.8 |

Use the best-fit line of an appropriate graph to determine the number of lines per mm.

52. ‖ A diffraction grating has slit spacing $d$. Fringes are viewed on a screen at distance $L$. What wavelength of light produces a first-order fringe on the viewing screen at distance $L$ from the center of the screen?

53. | For what slit-width-to-wavelength ratio does the first minimum of a single-slit diffraction pattern appear at (a) 30°, (b) 60°, and (c) 90°?

54. ‖ Light from a helium-neon laser ($\lambda = 633$ nm) is incident on a single slit. What is the largest slit width for which there are no minima in the diffraction pattern?

55. ‖ FIGURE P22.55 shows the light intensity on a screen behind a single slit. The slit width is 0.20 mm and the screen is 1.5 m behind the slit. What is the wavelength (in nm) of the light?
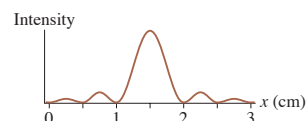
Intensity

FIGURE P22.55      $x$ (cm)
        0    1    2    3

56. ‖ FIGURE P22.55 shows the light intensity on a screen behind a single slit. The wavelength of the light is 600 nm and the slit width is 0.15 mm. What is the distance from the slit to the screen?

57. ‖ FIGURE P22.55 shows the light intensity on a screen behind a circular aperture. The wavelength of the light is 500 nm and the screen is 1.0 m behind the slit. What is the diameter (in mm) of the aperture?

58. ‖ Light from a helium-neon laser ($\lambda = 633$ nm) illuminates a circular aperture. It is noted that the diameter of the central maximum on a screen 50 cm behind the aperture matches the diameter of the geometric image. What is the aperture's diameter (in mm)?

59. ‖ One day, after pulling down your window shade, you notice that sunlight is passing through a pinhole in the shade and making a small patch of light on the far wall. Having recently studied optics in your physics class, you're not too surprised to see that the patch of light seems to be a circular diffraction pattern. It appears that the central maximum is about 1 cm across, and you estimate that the distance from the window shade to the wall is about 3 m. Estimate (a) the average wavelength of the sunlight (in nm) and (b) the diameter of the pinhole (in mm).

60. | A radar for tracking aircraft broadcasts a 12 GHz microwave beam from a 2.0-m-diameter circular radar antenna. From a wave perspective, the antenna is a circular aperture through which the microwaves diffract.
   a. What is the diameter of the radar beam at a distance of 30 km?
   b. If the antenna emits 100 kW of power, what is the average microwave intensity at 30 km?

61. ‖ Scientists use *laser range-finding* to measure the distance to the moon with great accuracy. A brief laser pulse is fired at the moon, then the time interval is measured until the "echo" is seen by a telescope. A laser beam spreads out as it travels because it diffracts through a circular exit as it leaves the laser. In order for the reflected light to be bright enough to detect, the laser spot on the moon must be no more than 1.0 km in diameter. Staying within this diameter is accomplished by using a special large-diameter laser. If $\lambda = 532$ nm, what is the minimum diameter of the circular opening from which the laser beam emerges? The earth-moon distance is 384,000 km.

62. ‖ Light of wavelength 600 nm passes though two slits separated by 0.20 mm and is observed on a screen 1.0 m behind the slits. The location of the central maximum is marked on the screen and labeled $y = 0$.
    a. At what distance, on either side of $y = 0$, are the $m = 1$ bright fringes?
    b. A very thin piece of glass is then placed in one slit. Because light travels slower in glass than in air, the wave passing through the glass is delayed by $5.0 \times 10^{-16}$ s in comparison to the wave going through the other slit. What fraction of the period of the light wave is this delay?
    c. With the glass in place, what is the phase difference $\Delta\phi_0$ between the two waves as they leave the slits?
    d. The glass causes the interference fringe pattern on the screen to shift sideways. Which way does the central maximum move (toward or away from the slit with the glass) and by how far?

63. ‖ A 600 line/mm diffraction grating is in an empty aquarium tank. The index of refraction of the glass walls is $n_{glass} = 1.50$. A helium-neon laser ($\lambda = 633$ nm) is outside the aquarium. The laser beam passes through the glass wall and illuminates the diffraction grating.
    a. What is the first-order diffraction angle of the laser beam?
    b. What is the first-order diffraction angle of the laser beam after the aquarium is filled with water ($n_{water} = 1.33$)?

64. | You've set up a Michelson interferometer with a helium-neon laser ($\lambda = 632.8$ nm). After adjusting mirror $M_2$ to produce a bright spot at the center of the pattern, you carefully move $M_2$ away from the beam splitter while counting 1200 new bright spots at the center. Then you put the laser away. Later another student wants to restore the interferometer to its starting condition, but he mistakenly sets up a hydrogen discharge lamp and uses the 656.5 nm emission from hydrogen atoms. He then counts 1200 new bright spots while slowly moving $M_2$ back toward the beam splitter. What is the net displacement of $M_2$ when he is done? Is $M_2$ now closer to or farther from the beam splitter?

65. ‖ A Michelson interferometer operating at a 600 nm wavelength has a 2.00-cm-long glass cell in one arm. To begin, the air is pumped out of the cell and mirror $M_2$ is adjusted to produce a bright spot at the center of the interference pattern. Then a valve is opened and air is slowly admitted into the cell. The index of refraction of air at 1.00 atm pressure is 1.00028. How many bright-dark-bright fringe shifts are observed as the cell fills with air?

66. | A 0.10-mm-thick piece of glass is inserted into one arm of a Michelson interferometer that is using light of wavelength 500 nm. This causes the fringe pattern to shift by 200 fringes. What is the index of refraction of this piece of glass?

67. ‖ Optical computers require microscopic optical switches to turn signals on and off. One device for doing so, which can be implemented in an integrated circuit, is the *Mach-Zender interferometer* seen in **FIGURE P22.67**. Light from an on-chip infrared laser ($\lambda = 1.000$ μm) is split into two waves that travel equal distances around the arms of the interferometer. One arm passes through an *electro-optic crystal,* a transparent material that can change its index of refraction in response to an applied voltage. Suppose both arms are exactly the same length and the crystal's index of refraction with no applied voltage is 1.522.
    a. With no voltage applied, is the output bright (switch closed, optical signal passing through) or dark (switch open, no signal)? Explain.

b. What is the first index of refraction of the electro-optic crystal larger than 1.522 that changes the optical switch to the state opposite the state you found in part a?
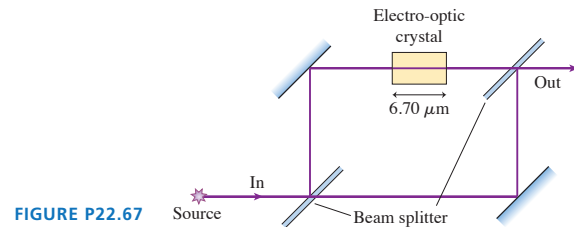


**FIGURE P22.67**

68. ‖ To illustrate one of the ideas of holography in a simple way, consider a diffraction grating with slit spacing $d$. The small-angle approximation is usually not valid for diffraction gratings, because $d$ is only slightly larger than $\lambda$, but assume that the $\lambda/d$ ratio of this grating is small enough to make the small-angle approximation valid.
    a. Use the small-angle approximation to find an expression for the fringe spacing on a screen at distance $L$ behind the grating.
    b. Rather than a screen, suppose you place a piece of film at distance $L$ behind the grating. The bright fringes will expose the film, but the dark spaces in between will leave the film unexposed. After being developed, the film will be a series of alternating light and dark stripes. What if you were to now "play" the film by using it as a diffraction grating? In other words, what happens if you shine the same laser through the film and look at the film's diffraction pattern on a screen at the same distance $L$? Demonstrate that the film's diffraction pattern is a reproduction of the original diffraction grating.

## Challenge Problems

69. A helium-neon laser ($\lambda = 633$ nm) is built with a glass tube of inside diameter 1.0 mm, as shown in **FIGURE CP22.69**. One mirror is partially transmitting to allow the laser beam out. An electrical discharge in the tube causes it to glow like a neon light. From an optical perspective, the laser beam is a light wave that diffracts out through a 1.0-mm-diameter circular opening.
    a. Can a laser beam be *perfectly* parallel, with no spreading? Why or why not?
    b. The angle $\theta_1$ to the first minimum is called the *divergence angle* of a laser beam. What is the divergence angle of this laser beam?
    c. What is the diameter (in mm) of the laser beam after it travels 3.0 m?
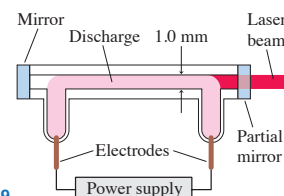    d. What is the diameter of the laser beam after it travels 1.0 km?



**FIGURE CP22.69**

70. The intensity at the central maximum of a double-slit interference pattern is $4I_1$. The intensity at the first minimum is zero. At what fraction of the distance from the central maximum to the first minimum is the intensity $I_1$?

71. Light consisting of two nearly equal wavelengths $\lambda + \Delta\lambda$ and $\lambda$, where $\Delta\lambda \ll \lambda$, is incident on a diffraction grating. The slit separation of the grating is $d$.
   a. Show that the angular separation of these two wavelengths in the $m$th order is

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{(d/m)^2 - \lambda^2}}$$

   b. Sodium atoms emit light at 589.0 nm and 589.6 nm. What are the first-order and second-order angular separations (in degrees) of these two wavelengths for a 600 line/mm grating?

72. **FIGURE CP22.72** shows two nearly overlapped intensity peaks of the sort you might produce with a diffraction grating (see Figure 22.8b). As a practical matter, two peaks can just barely be resolved if their spacing $\Delta y$ equals the width $w$ of each peak, where $w$ is measured at half of the peak's height. Two peaks closer together than $w$ will merge into a single peak. We can use this idea to understand the *resolution* of a diffraction grating.
   a. In the small-angle approximation, the position of the $m = 1$ peak of a diffraction grating falls at the same location as the $m = 1$ fringe of a double slit: $y_1 = \lambda L/d$. Suppose two wavelengths differing by $\Delta\lambda$ pass through a grating at the same time. Find an expression for $\Delta y$, the separation of their first-order peaks.
   b. We noted that the widths of the bright fringes are proportional to $1/N$, where $N$ is the number of slits in the grating. Let's hypothesize that the fringe width is $w = y_1 /N$. Show that this is true for the double-slit pattern. We'll then assume it to be true as $N$ increases.
   c. Use your results from parts a and b together with the idea that $\Delta y_{min} = w$ to find an expression for $\Delta\lambda_{min}$, the minimum wavelength separation (in first order) for which the diffraction fringes can barely be resolved.
   d. Ordinary hydrogen atoms emit red light with a wavelength of 656.45 nm. In deuterium, which is a "heavy" isotope of hydrogen, the wavelength is 656.27 nm. What is the minimum number of slits in a diffraction grating that can barely resolve these two wavelengths in the first-order diffraction pattern?
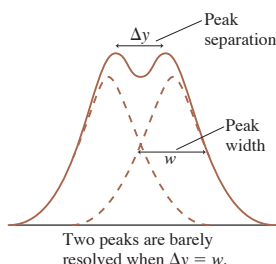


FIGURE CP22.72

Peak separation
$\Delta y$
Peak width
$w$
Two peaks are barely resolved when $\Delta y = w$.

73. The diffraction grating analysis in this chapter assumed that the incident light is normal to the grating. **FIGURE CP22.73** shows a plane wave approaching a diffraction grating at angle $\phi$.
   a. Show that the angles $\theta_m$ for constructive interference are given by the grating equation

$$d(\sin\theta_m + \sin\phi) = m\lambda$$

   where $m = 0, \pm 1, \pm 2, \dots$. Angles are considered positive if they are above the horizontal line, negative if below it.
   b. The two first-order maxima, $m = +1$ and $m = -1$, are no longer symmetrical about the center. Find $\theta_1$ and $\theta_{-1}$ for 500 nm light incident on a 600 line/mm grating at $\phi = 30°$.
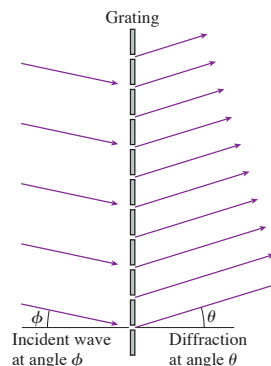


Grating

FIGURE CP22.73   Incident wave at angle $\phi$     Diffraction at angle $\theta$

74. **FIGURE CP22.74** shows light of wavelength $\lambda$ incident at angle $\phi$ on a *reflection* grating of spacing $d$. We want to find the angles $\theta_m$ at which constructive interference occurs.
   a. The figure shows paths 1 and 2 along which two waves travel and interfere. Find an expression for the path-length difference $\Delta r = r_2 - r_1$.
   b. Using your result from part a, find an equation (analogous to Equation 22.15) for the angles $\theta_m$ at which diffraction occurs when the light is incident at angle $\phi$. Notice that $m$ can be a negative integer in your expression, indicating that path 2 is shorter than path 1.
   c. Show that the zeroth-order diffraction is simply a "reflection." That is, $\theta_0 = \phi$.
   d. Light of wavelength 500 nm is incident at $\phi = 40°$ on a reflection grating having 700 reflection lines/mm. Find all angles $\theta_m$ at which light is diffracted. Negative values of $\theta_m$ are interpreted as an angle left of the vertical.
   e. Draw a picture showing a *single* 500 nm light ray incident at $\phi = 40°$ and showing all the diffracted waves at the correct angles.



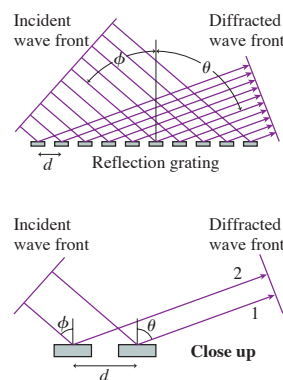FIGURE CP22.74

75. The pinhole camera of **FIGURE CP22.75** images distant objects by allowing only a narrow bundle of light rays to pass through the hole and strike the film. If light consist-



FIGURE CP22.75

ed of particles, you could make the image sharper and sharper (at the expense of getting dimmer and dimmer) by making the aperture smaller and smaller. In practice, diffraction of light
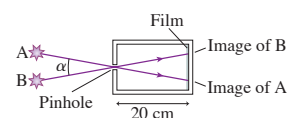
by the circular aperture limits the maximum sharpness that can be obtained. Consider two distant points of light, such as two distant streetlights. Each will produce a circular diffraction pattern on the film. The two images can just barely be resolved if the central maximum of one image falls on the first dark fringe of the other image. (This is called Rayleigh's criterion, and we will explore its implication for optical instruments in Chapter 24.)

a. Optimum sharpness of one image occurs when the diameter of the central maximum equals the diameter of the pinhole. What is the optimum hole size for a pinhole camera in which the film is 20 cm behind the hole? Assume $\lambda = 550$ nm, an average value for visible light.
b. For this hole size, what is the angle $\alpha$ (in degrees) between two distant sources that can barely be resolved?
c. What is the distance between two street lights 1 km away that can barely be resolved?

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**STOP TO THINK ANSWERS**

**Stop to Think 22.1: b.** The antinodal lines seen in Figure 22.3b are diverging.

**Stop to Think 22.2: Smaller.** Shorter-wavelength light doesn't spread as rapidly as longer-wavelength light. The fringe spacing $\Delta y$ is directly proportional to the wavelength $\lambda$.

**Stop to Think 22.3: d.** Larger wavelengths have larger diffraction angles. Red light has a larger wavelength than violet light, so red light is diffracted farther from the center.

**Stop to Think 22.4: b or c.** The width of the central maximum, which is proportional to $\lambda/a$, has increased. This could occur either because the wavelength has increased or because the slit width has decreased.

**Stop to Think 22.5: d.** Moving $M_1$ in by $\lambda$ decreases $r_1$ by $2\lambda$. Moving $M_2$ out by $\lambda$ increases $r_2$ by $2\lambda$. These two actions together change the path length by $\Delta r = 4\lambda$.
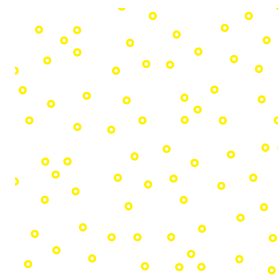
# SUMMARY

The goals of Chapter 23 have been to understand and apply the ray model of light.

## General Principles

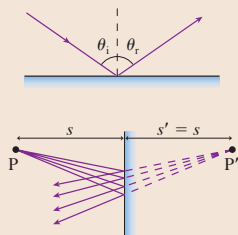### Reflection

Law of reflection: $\theta_r = \theta_i$

Reflection can be **specular** (mirror-like) or **diffuse** (from rough surfaces).

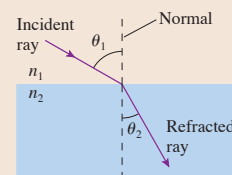Plane mirrors: A virtual image is formed at $P'$ with $s' = s$.

### Refraction

Snell's law of refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

**Index of refraction** is $n = c/v$. The ray is closer to the normal on the side with the larger index of refraction.

If $n_2 < n_1$, **total internal reflection** (TIR) occurs when the angle of incidence $\theta_1 \geq \theta_c = \sin^{-1}(n_2/n_1)$.

## Important Concepts

### The ray model of light

Light travels along straight lines, called **light rays,** at speed $v = c/n$.

A light ray continues forever unless an interaction with matter causes it to reflect, refract, scatter, or be absorbed.

Light rays come from **objects.** Each point on the object sends rays in all directions.

The eye sees an object (or an image) when diverging rays are collected by the pupil and focused on the retina.

▶ Ray optics is valid when lenses, mirrors, and apertures are larger than $\approx 1$ mm.

### Image formation

If rays diverge from P and interact with a lens or mirror so that the refracted rays *converge* at $P'$, then $P'$ is a real image of P.
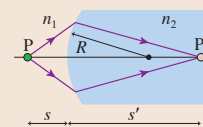
If rays diverge from P and interact with a lens or mirror so that the refracted/reflected rays *diverge* from $P'$ and appear to come from $P'$, then $P'$ is a virtual image of P.

**Spherical surface:** Object and image distances are related by

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

**Plane surface:** $R \to \infty$, so $|s'/s| = n_2/n_1$.

## Applications

### Ray tracing

3 special rays in 3 basic situations:

Converging lens Real image

Converging lens Virtual image

Diverging lens Virtual image
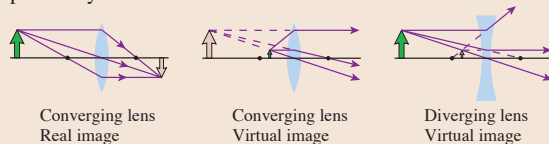
**Magnification** $m = -\dfrac{s'}{s}$

$m$ is $+$ for an upright image, $-$ for inverted.
The height ratio is $h'/h = |m|$.

### Thin lenses

The image and object distances are related by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where the focal length is given by the lens maker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$R$ $+$ for surface convex toward object $-$ for concave
$f$ $+$ for a converging lens $-$ for diverging
$s'$ $+$ for a real image $-$ for virtual
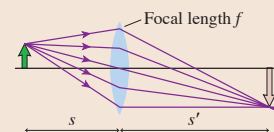
### Spherical mirrors

The image and object distances are related by

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$R, f$ $+$ for concave mirror $-$ for convex
$s'$ $+$ for a real image $-$ for virtual

Focal length $f = R/2$

## Terms and Notation

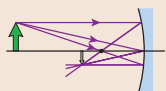| | | | |
|---|---|---|---|
| light ray | diffuse reflection | dispersion | object plane |
| object | virtual image | Rayleigh scattering | image plane |
| point source | refraction | lens | inverted image |
| parallel bundle | angle of refraction | ray tracing | lateral magnification, $m$ |
| ray diagram | Snell's law | converging lens | upright image |
| camera obscura | total internal reflection (TIR) | focal point | spherical mirror |
| aperture | critical angle, $\theta_c$ | focal length, $f$ | concave mirror |
| specular reflection | object distance, $s$ | diverging lens | convex mirror |
| angle of incidence | image distance, $s'$ | thin lens | |
| angle of reflection | optical axis | lens plane | |
| law of reflection | paraxial rays | real image | |

# CONCEPTUAL QUESTIONS

1. If you turn on your car headlights during the day, the road ahead of you doesn't appear to get brighter. Why not?

2. Suppose you have two pinhole cameras. The first has a small round hole in the front. The second is identical except it has a square hole of the same area as the round hole in the first camera. Would the pictures taken by these two cameras, under the same conditions, be different in any obvious way? Explain.

3. You are looking at the image of a pencil in a mirror, as shown in **FIGURE Q23.3**.
   a. What happens to the image if the top half of the mirror, down to the midpoint, is covered with a piece of cardboard? Explain.
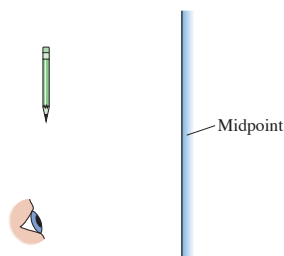   b. What happens to the image if the bottom half of the mirror is covered with a piece of cardboard? Explain.



—Midpoint

**FIGURE Q23.3**

4. One problem with using optical fibers for communication is that a light ray passing directly down the center of the fiber takes less time to travel from one end to the other than a ray taking a longer, zig-zag path. Thus light rays starting at the same time but traveling in slightly different directions reach the end of the fiber at different times. This problem can be solved by making the refractive index of the glass change gradually from a higher value in the center to a lower value near the edges of the fiber. Explain how this reduces the difference in travel times.

5. Suppose you looked at the sky on a clear day through pieces of red and blue plastic oriented as shown in **FIGURE Q23.5**. Describe the color and brightness of the light coming through sections 1, 2, and 3.
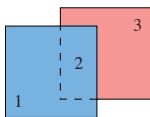


**FIGURE Q23.5**

6. A red card is illuminated by red light. What color will the card appear? What if it's illuminated by blue light?

7. The center of the galaxy is filled with low-density hydrogen gas. An astronomer wants to take a picture of the center of the galaxy. Will the view be better using ultraviolet light, visible light, or infrared light? (High-quality telescopes are available in all three spectral regions.) Explain.

8. Consider *one* point on an object near a lens.
   a. What is the minimum number of rays needed to locate its image point? Explain.
   b. How many rays from this point actually strike the lens and refract to the image point?

9. The object and lens in **FIGURE Q23.9** are positioned to form a well-focused, inverted image on a viewing screen. Then a piece of cardboard is lowered just in front of the lens to cover the top half of the lens. Describe what you see on the screen when the cardboard is in place.



Screen
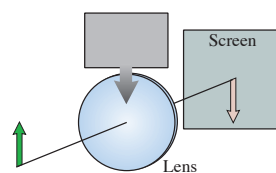
**FIGURE Q23.9**    Lens

10. **FIGURE Q23.10** shows an object near a lens. The focal points are marked. Is there an image? If so, is the image real or virtual? Is it upright or inverted? If not, why not? Explain.
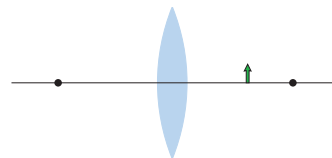


**FIGURE Q23.10**

11. A concave mirror brings the sun's rays to a focus in front of the mirror. Suppose the mirror is submerged in a swimming pool but still pointed up at the sun. Will the sun's rays be focused nearer to, farther from, or at the same distance from the mirror? Explain.

12. When you look at your reflection in the bowl of a spoon, it is upside down. Why?

# EXERCISES AND PROBLEMS

## Exercises

### Section 23.1 The Ray Model of Light

1. ‖ a. How long (in ns) does it take light to travel 1.0 m in vacuum?
   b. What distance does light travel in water, glass, and cubic zirconia during the time that it travels 1.0 m in vacuum?

2. ‖ A point source of light illuminates an aperture 2.0 m away. A 12.0-cm-wide bright patch of light appears on a screen 1.0 m behind the aperture. How wide is the aperture?

3. ‖ A 5.0-cm-thick layer of oil is sandwiched between a 1.0-cm-thick sheet of glass and a 2.0-cm-thick sheet of polystyrene plastic. How long (in ns) does it take light incident perpendicular to the glass to pass through this 8.0-cm-thick sandwich?

4. ‖ A student has built a 15-cm-long pinhole camera for a science fair project. She wants to photograph her 180-cm-tall friend and have the image on the film be 5.0 cm high. How far should the front of the camera be from her friend?

### Section 23.2 Reflection

5. | The mirror in **FIGURE EX23.5** deflects a horizontal laser beam by 60°. What is the angle $\phi$?
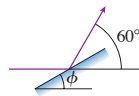


**FIGURE EX23.5**

6. | A light ray leaves point A in **FIGURE EX23.6**, reflects from the mirror, and reaches point B. How far below the top edge does the ray strike the mirror?
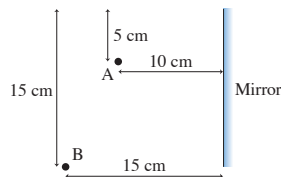


**FIGURE EX23.6**

7. ‖ The laser beam in **FIGURE EX23.7** is aimed at the center of a rotating hexagonal mirror. How long is the streak of laser light as the reflected laser beam sweeps across the wall behind the laser?
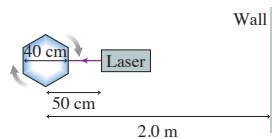


**FIGURE EX23.7**

8. ‖ At what angle $\phi$ should the laser beam in **FIGURE EX23.8** be aimed at the mirrored ceiling in order to hit the midpoint of the far wall?
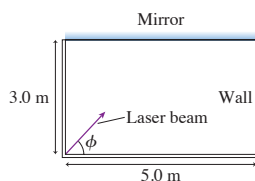


**FIGURE EX23.8**

9. ‖ It is 165 cm from your eyes to your toes. You're standing 200 cm in front of a tall mirror. How far is it from your eyes to the image of your toes?

### Section 23.3 Refraction

10. ‖ A 1.0-cm-thick layer of water stands on a horizontal slab of glass. A light ray in the air is incident on the water 60° from the normal. What is the ray's direction of travel in the glass?

11. ‖ A costume jewelry pendant made of cubic zirconia is submerged in oil. A light ray strikes one face of the zirconia crystal at an angle of incidence of 25°. Once inside, what is the ray's angle with respect to the face of the crystal?

12. ‖ An underwater diver sees the sun 50° above horizontal. How high is the sun above the horizon to a fisherman in a boat above the diver?

13. | A laser beam in air is incident on a liquid at an angle of 53° with respect to the normal. The laser beam's angle in the liquid is 35°. What is the liquid's index of refraction?

14. ‖ The glass core of an optical fiber has an index of refraction 1.60. The index of refraction of the cladding is 1.48. What is the maximum angle a light ray can make with the wall of the core if it is to remain inside the fiber?

15. ‖ A thin glass rod is submerged in oil. What is the critical angle for light traveling inside the rod?

### Section 23.4 Image Formation by Refraction

16. ‖ A fish in a flat-sided aquarium sees a can of fish food on the counter. To the fish's eye, the can looks to be 30 cm outside the aquarium. What is the actual distance between the can and the aquarium? (You can ignore the thin glass wall of the aquarium.)

17. | A biologist keeps a specimen of his favorite beetle embedded in a cube of polystyrene plastic. The hapless bug appears to be 2.0 cm within the plastic. What is the beetle's actual distance beneath the surface?

18. | A 150-cm-tall diver is standing completely submerged on the bottom of a swimming pool full of water. You are sitting on the end of the diving board, almost directly over her. How tall does the diver appear to be?

19. ‖ To a fish in an aquarium, the 4.00-mm-thick walls appear to be only 3.50 mm thick. What is the index of refraction of the walls?

### Section 23.5 Color and Dispersion

20. ‖ A sheet of glass has $n_{red} = 1.52$ and $n_{violet} = 1.55$. A narrow beam of white light is incident on the glass at 30°. What is the angular spread of the light inside the glass?

21. | A narrow beam of white light is incident on a sheet of quartz. The beam disperses in the quartz, with red light ($\lambda \approx 700$ nm) traveling at an angle of 26.3° with respect to the normal and violet light ($\lambda \approx 400$ nm) traveling at 25.7°. The index of refraction of quartz for red light is 1.45. What is the index of refraction of quartz for violet light?

22. ‖ A hydrogen discharge lamp emits light with two prominent wavelengths: 656 nm (red) and 486 nm (blue). The light enters a flint-glass prism perpendicular to one face and then refracts through the hypotenuse back into the air. The angle between these two faces is 35°.
    a. Use Figure 23.28 to estimate to ±0.002 the index of refraction of flint glass at these two wavelengths.
    b. What is the angle (in degrees) between the red and blue light as it leaves the prism?

23. ‖ Infrared telescopes, which use special infrared detectors, are able to peer farther into star-forming regions of the galaxy because infrared light is not scattered as strongly as is visible light by the tenuous clouds of hydrogen gas from which new stars are created. For what wavelength of light is the scattering only 1% that of light with a visible wavelength of 500 nm?

### Section 23.6 Thin Lenses: Ray Tracing

24. ‖ An object is 20 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted?

25. ‖ An object is 30 cm in front of a converging lens with a focal length of 5 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted?

26. ‖ An object is 6 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted?

27. ‖ An object is 15 cm in front of a diverging lens with a focal length of −15 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted?

### Section 23.7 Thin Lenses: Refraction Theory

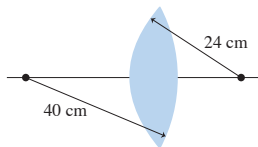28. | Find the focal length of the glass lens in **FIGURE EX23.28**.



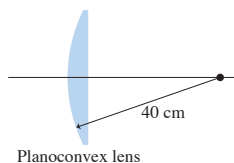**FIGURE EX23.28**



Planoconvex lens
**FIGURE EX23.29**

29. | Find the focal length of the planoconvex polystyrene plastic lens in **FIGURE EX23.29**.

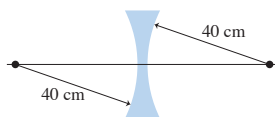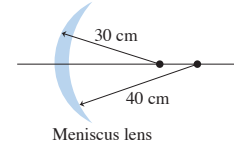30. ‖ Find the focal length of the glass lens in **FIGURE EX23.30**.



**FIGURE EX23.30**



Meniscus lens
**FIGURE EX23.31**

31. ‖ Find the focal length of the meniscus polystyrene plastic lens in **FIGURE EX23.31**.

32. ‖ An air bubble inside an 8.0-cm-diameter plastic ball is 2.0 cm from the surface. As you look at the ball with the bubble turned toward you, how far beneath the surface does the bubble appear to be?

33. ‖ A goldfish lives in a 50-cm-diameter spherical fish bowl. The fish sees a cat watching it. If the cat's face is 20 cm from the edge of the bowl, how far from the edge does the fish see it as being? (You can ignore the thin glass wall of the bowl.)

34. | A 1.0-cm-tall candle flame is 60 cm from a lens with a focal length of 20 cm. What are the image distance and the height of the flame's image?

### Section 23.8 Image Formation with Spherical Mirrors

35. ‖ An object is 40 cm in front of a concave mirror with a focal length of 20 cm. Use ray tracing to locate the image. Is the image upright or inverted?

36. ‖ An object is 12 cm in front of a concave mirror with a focal length of 20 cm. Use ray tracing to locate the image. Is the image upright or inverted?

37. ‖ An object is 30 cm in front of a convex mirror with a focal length of −20 cm. Use ray tracing to locate the image. Is the image upright or inverted?

## Problems

38. ‖ An advanced computer sends information to its various parts via infrared light pulses traveling through silicon fibers. To acquire data from memory, the central processing unit sends a light-pulse request to the memory unit. The memory unit processes the request, then sends a data pulse back to the central processing unit. The memory unit takes 0.5 ns to process a request. If the information has to be obtained from memory in 2.0 ns, what is the maximum distance the memory unit can be from the central processing unit?

39. ‖ A red ball is placed at point A in **FIGURE P23.39**.
    a. How many images are seen by an observer at point O?
    b. What are the (x, y) coordinates of each image?
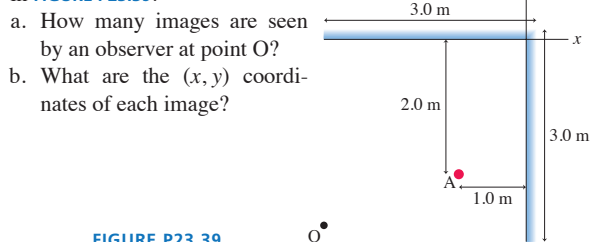


**FIGURE P23.39**

40. | A laser beam is incident on the left mirror in **FIGURE P23.40**. Its initial direction is parallel to a line that bisects the mirrors. What is the angle φ of the reflected laser beam?
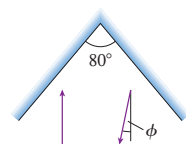


**FIGURE P23.40**

41. ‖ The place you get your hair cut has two nearly parallel mirrors 5.0 m apart. As you sit in the chair, your head is 2.0 m from the nearer mirror. Looking toward this mirror, you first see your face and then, farther away, the back of your head. (The mirrors need to be slightly nonparallel for you to be able to see the back of your head, but you can treat them as parallel in this problem.) How far away does the back of your head appear to be? Neglect the thickness of your head.

42. ‖ You're helping with an experiment in which a vertical cylinder will rotate about its axis by a very small angle. You need to devise a way to measure this angle. You decide to use what is called an *optical lever*. You begin by mounting a small mirror

on top of the cylinder. A laser 5.0 m away shoots a laser beam at the mirror. Before the experiment starts, the mirror is adjusted to reflect the laser beam directly back to the laser. Later, you measure that the reflected laser beam, when it returns to the laser, has been deflected sideways by 2.0 mm. Through how many degrees has the cylinder rotated?

43. ‖ A microscope is focused on a black dot. When a 1.00-cm-thick piece of plastic is placed over the dot, the microscope objective has to be raised 0.40 cm to bring the dot back into focus. What is the index of refraction of the plastic?

44. ‖ A light ray in air is incident on a transparent material whose index of refraction is $n$.
   a. Find an expression for the (non-zero) angle of incidence whose angle of refraction is half the angle of incidence.
   b. Evaluate your expression for light incident on glass.

45. ‖ The meter stick in **FIGURE P23.45** lies on the bottom of a 100-cm-long tank with its zero mark against the left edge. You look into the tank at a 30° angle, with your line of sight just grazing the upper left edge of the tank. What mark do you see on the meter stick if the tank is (a) empty, (b) half full of water, and (c) completely full of water?
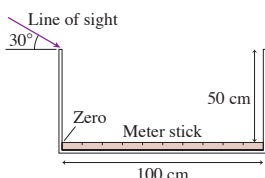


**FIGURE P23.45**

46. ‖ The 80-cm-tall, 65-cm-wide tank shown in **FIGURE P23.46** is completely filled with water. The tank has marks every 10 cm along one wall, and the 0 cm mark is barely submerged. As you stand beside the opposite wall, your eye is level with the top of the water.
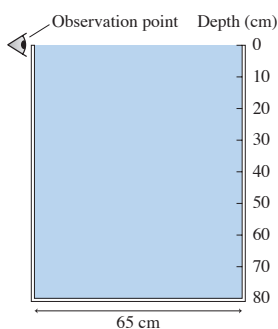   a. Can you see the marks from the top of the tank (the 0 cm mark) going down, or from the bottom of the tank (the 80 cm mark) coming up? Explain.
   b. Which is the lowest or highest mark, depending on your answer to part a, that you can see?



**FIGURE P23.46**

47. ‖ A 4.0-m-wide swimming pool is filled to the top. The bottom of the pool becomes completely shaded in the afternoon when the sun is 20° above the horizon. How deep is the pool?

48. ‖ It's nighttime, and you've dropped your goggles into a 3.0-m-deep swimming pool. If you hold a laser pointer 1.0 m above the edge of the pool, you can illuminate the goggles if the laser beam enters the water 2.0 m from the edge. How far are the goggles from the edge of the pool?

49. ‖ Shown from above in **FIGURE P23.49** is one corner of a rectangular box filled with water. A laser beam starts 10 cm from side A of the container and enters the water at position $x$. You can ignore the thin walls of the container.
   a. If $x = 15$ cm, does the laser beam refract back into the air through side B or reflect from side B back into the water? Determine the angle of refraction or reflection.

   b. Repeat part a for $x = 25$ cm.
   c. Find the minimum value of $x$ for which the laser beam passes through side B and emerges into the air.
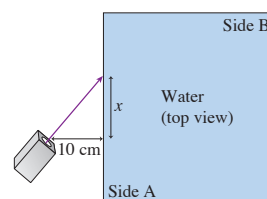


**FIGURE P23.49**

50. ‖ A fish is 20 m from the shore of a lake. A bonfire is burning on the edge of the lake nearest the fish.
   a. Does the fish need to be shallow (just below the surface) or very deep to see the light from the bonfire? Explain.
   b. What is the deepest or shallowest, depending on your answer to part a, that the fish can be and still see light from the fire?

51. ‖ Your supervisor asks you to measure the index of refraction of a piece of plastic. You notice that, because of scattering of the light, you can see the path of a laser beam through the plastic. You decide to shoot a laser beam toward the plastic at several different incident angles and measure the refraction angle in the plastic. Your data are as follows:

| Incident angle | Refraction angle |
|:---:|:---:|
| 15° | 9° |
| 30° | 19° |
| 45° | 26° |
| 60° | 34° |
| 75° | 37° |

Use the best-fit line of an appropriate graph to determine the plastic's index of refraction.

52. ‖ One of the contests at the school carnival is to throw a spear at an underwater target lying flat on the bottom of a pool. The water is 1.0 m deep. You're standing on a small stool that places your eyes 3.0 m above the bottom of the pool. As you look at the target, your gaze is 30° below horizontal. At what angle below horizontal should you throw the spear in order to hit the target? Your raised arm brings the spear point to the level of your eyes as you throw it, and over this short distance you can assume that the spear travels in a straight line rather than a parabolic trajectory.

53. ‖ White light is incident onto a 30° prism at the 40° angle shown in **FIGURE P23.53**. Violet light emerges perpendicular to the rear face of the prism. The index of refraction of violet light in this glass is 2.0% larger than the index of refraction of red light. At what angle $\phi$ does red light emerge from the rear face?



**FIGURE P23.53**

54. ‖ There's one angle of incidence $\beta$ onto a prism for which the light inside an isosceles prism travels parallel to the base and emerges at angle $\beta$.
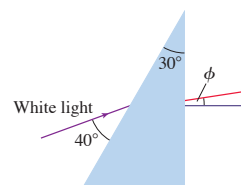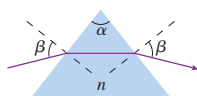


**FIGURE P23.54**

   a. Find an expression for $\beta$ in terms of the prism's apex angle $\alpha$ and index of refraction $n$.

   b. A laboratory measurement finds that $\beta = 52.2°$ for a prism shaped like an equilateral triangle. What is the prism's index of refraction?

55. ‖ Paraxial light rays approach a transparent sphere parallel to an optical axis passing through the center of the sphere. The rays come to a focus on the far surface of the sphere. What is the sphere's index of refraction?

56. ‖ A 6.0-cm-diameter cubic zirconia sphere has an air bubble exactly in the center. As you look into the sphere, how far beneath the surface does the bubble appear to be?

57. ‖ A 1.0-cm-tall object is 10 cm in front of a converging lens that has a 30 cm focal length.

   a. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.

   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

58. ‖ A 2.0-cm-tall object is 40 cm in front of a converging lens that has a 20 cm focal length.

   a. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.

   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

59. ‖ A 1.0-cm-tall object is 75 cm in front of a converging lens that has a 30 cm focal length.

   a. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.

   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

60. ‖ A 2.0-cm-tall object is 15 cm in front of a converging lens that has a 20 cm focal length.

   a. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.

   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

61. ‖ A 1.0-cm-tall object is 60 cm in front of a diverging lens that has a $-30$ cm focal length.

   a. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.

   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

62. ‖ A 2.0-cm-tall object is 15 cm in front of a diverging lens that has a $-20$ cm focal length.

   a. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid.

Determine the image distance and image height by making measurements on your diagram.

   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

63. ‖ To determine the focal length of a lens, you place the lens in front of a small lightbulb and then adjust a viewing screen to get a sharply focused image. Varying the lens position produces the following data:

| Bulb to lens (cm) | Lens to screen (cm) |
| --- | --- |
| 20 | 61 |
| 22 | 47 |
| 24 | 39 |
| 26 | 37 |
| 28 | 32 |

Use the best-fit line of an appropriate graph to determine the focal length of the lens.

64. | A 1.0-cm-tall object is 20 cm in front of a concave mirror that has a 60 cm focal length. Calculate the position and height of the image. State whether the image is in front of or behind the mirror, and whether the image is upright or inverted.

65. | A 1.0-cm-tall object is 20 cm in front of a convex mirror that has a $-60$ cm focal length. Calculate the position and height of the image. State whether the image is in front of or behind the mirror, and whether the image is upright or inverted.

66. ‖ BIO The illumination lights in an operating room use a concave mirror to focus an image of a bright lamp onto the surgical site. One such light uses a mirror with a 30 cm radius of curvature. If the mirror is 1.2 m from the patient, how far should the lamp be from the mirror?

67. ‖ BIO A dentist uses a curved mirror to view the back side of teeth in the upper jaw. Suppose she wants an upright image with a magnification of 1.5 when the mirror is 1.2 cm from a tooth. Should she use a convex or a concave mirror? What focal length should it have?

68. ‖ A 2.0-cm-tall candle flame is 2.0 m from a wall. You happen to have a lens with a focal length of 32 cm. How many places can you put the lens to form a well-focused image of the candle flame on the wall? For each location, what are the height and orientation of the image?

69. ‖ A lightbulb is 3.0 m from a wall. What are the focal length and the position (measured from the bulb) of a lens that will form an image on the wall that is twice the size of the lightbulb?

70. ‖ BIO a. Estimate the diameter of your eyeball.

   b. Bring this page up to the closest distance at which the text is sharp—not the closest at which you can still read it, but the closest at which the letters remain sharp. If you wear glasses or contact lenses, leave them on. This distance is called the *near point* of your (possibly corrected) eye. Measure it.

   c. Estimate the effective focal length of your eye. The effective focal length includes the focusing due to the lens, the curvature of the cornea, and any corrections you wear. Ignore the effects of the fluid in your eye.

71. ‖ A slide projector needs to create a 98-cm-high image of a 2.0-cm-tall slide. The screen is 300 cm from the slide.

   a. What focal length does the lens need? Assume that it is a thin lens.

   b. How far should you place the lens from the slide?

72. ‖ A lens placed 10 cm in front of an object creates an upright image twice the height of the object. The lens is then moved along the optical axis until it creates an inverted image twice the height of the object. How far did the lens move?

73. ‖ An object is 60 cm from a screen. What are the radii of a symmetric converging plastic lens (i.e., two equally curved surfaces) that will form an image on the screen twice the height of the object?

74. ‖ A sports photographer has a 150-mm-focal-length lens on his camera. The photographer wants to photograph a sprinter running straight away from him at 5.0 m/s. What is the speed (in mm/s) of the sprinter's image at the instant the sprinter is 10 m in front of the lens?

75. ‖ A concave mirror has a 40 cm radius of curvature. How far from the mirror must an object be placed to create an upright image three times the height of the object?

76. ‖‖ A 2.0-cm-tall object is placed in front of a mirror. A 1.0-cm-tall upright image is formed behind the mirror, 150 cm from the object. What is the focal length of the mirror?

77. ‖ A spherical mirror of radius $R$ has its center at C, as shown in **FIGURE P23.77**. A ray parallel to the axis reflects through F, the focal point. Prove that $f = R/2$ if $\phi \ll 1$ rad.
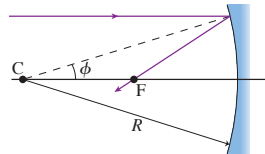


**FIGURE P23.77**

## Challenge Problems

78. Consider a lens having index of refraction $n_2$ and surfaces with radii $R_1$ and $R_2$. The lens is immersed in a fluid that has index of refraction $n_1$.
    a. Derive a generalized lens maker's equation to replace Equation 23.27 when the lens is surrounded by a medium other than air. That is, when $n_1 \neq 1$.
    b. A symmetric converging glass lens (i.e., two equally curved surfaces) has two surfaces with radii of 40 cm. Find the focal length of this lens in air and the focal length of this lens in water.

79. **FIGURE CP23.79** shows a light ray that travels from point A to point B. The ray crosses the boundary at position $x$, making angles $\theta_1$ and $\theta_2$ in the two media. Suppose that you did *not* know Snell's law.
    a. Write an expression for the *time t* it takes the light ray to travel from A to B. Your expression should be in terms of the distances $a$, $b$, and $w$; the variable $x$; and the indices of refraction $n_1$ and $n_2$.

b. The time depends on $x$. There's one value of $x$ for which the light travels from A to B in the shortest possible time. We'll call it $x_{min}$. Write an expression (but don't try to solve it!) from which $x_{min}$ could be found.
    c. Now, by using the geometry of the figure, derive Snell's law from your answer to part b.
    You've proven that Snell's law is equivalent to the statement that "light traveling between two points follows the path that requires the shortest time." This interesting way of thinking about refraction is called *Fermat's principle*.
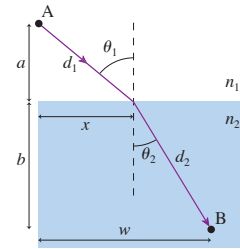


**FIGURE P23.79**

80. A fortune teller's "crystal ball" (actually just glass) is 10 cm in diameter. Her secret ring is placed 6.0 cm from the edge of the ball.
    a. An image of the ring appears on the opposite side of the crystal ball. How far is the image from the center of the ball?
    b. Draw a ray diagram showing the formation of the image.
    c. The crystal ball is removed and a thin lens is placed where the center of the ball had been. If the image is still in the same position, what is the focal length of the lens?

81. A beam of white light enters a transparent material. Wavelengths for which the index of refraction is $n$ are refracted at angle $\theta_2$. Wavelengths for which the index of refraction is $n + \delta n$, where $\delta n \ll n$, are refracted at angle $\theta_2 + \delta\theta$.
    a. Show that the angular separation in radians is $\delta\theta = -(\delta n/n)\tan\theta_2$.
    b. A beam of white light is incident on a piece of glass at 30.0°. Deep violet light is refracted 0.28° more than deep red light. The index of refraction for deep red light is known to be 1.552. What is the index of refraction for deep violet light?

82. Consider an object of thickness $ds$ (parallel to the axis) in front of a lens or mirror. The image of the object has thickness $ds'$. Define the *longitudinal magnification* as $M = ds'/ds$. Prove that $M = -m^2$, where $m$ is the lateral magnification.

---

### STOP TO THINK ANSWERS

**Stop to Think 23.1: c.** The light spreads vertically as it goes through the vertical aperture. The light spreads horizontally due to different points on the horizontal lightbulb.

**Stop to Think 23.2: c.** There's one image behind the vertical mirror and a second behind the horizontal mirror. A third image in the corner arises from rays that reflect twice, once off each mirror.

**Stop to Think 23.3: a.** The ray travels closer to the normal in both media 1 and 3 than in medium 2, so $n_1$ and $n_3$ are both larger than $n_2$. The angle is smaller in medium 3 than in medium 1, so $n_3 > n_1$.

**Stop to Think 23.4: e.** The rays from the object are diverging. Without a lens, the rays cannot converge to form any kind of image on the screen.

**Stop to Think 23.5: a, e, or f.** Any of these will increase the angle of refraction $\theta_2$.

**Stop to Think 23.6: Away from.** You need to decrease $s'$ to bring the image plane onto the screen. $s'$ is decreased by increasing $s$.
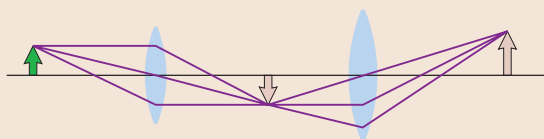
**Stop to Think 23.7: c.** A concave mirror forms a real image in front of the mirror. Because the object distance is $s \approx \infty$, the image distance is $s' \approx f$.

# SUMMARY

The goal of Chapter 24 has been to understand some common optical instruments and their limitations.
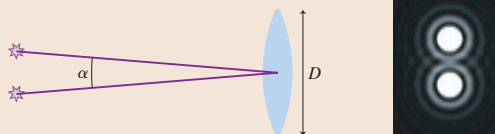
## Important Concepts

### Lens Combinations



**The image of the first lens acts as the object for the second lens.**

Lens **power**: $P = \dfrac{1}{f}$ diopters, $1 \text{ D} = 1 \text{ m}^{-1}$

### Resolution



The angular resolution of a lens of diameter $D$ is

$$\theta_{min} = 1.22\lambda/D$$

Rayleigh's criterion states that two objects separated by an angle $\alpha$ are marginally resolvable if $\alpha = \theta_{min}$.

## Applications

### Cameras

Forms a real, inverted image on a detector. The lens's **f-number** is

$$f\text{-number} = \frac{f}{D}$$

The light intensity on the detector is

$$I \propto \frac{1}{(f\text{-number})^2}$$

### Magnifiers

For relaxed-eye viewing, the angular magnification is

$$M = \frac{25 \text{ cm}}{f}$$

For microscopes and telescopes, angular magnification, not lateral magnification, is the important characteristic. The eyepiece acts as a magnifier to view the image formed by the objective lens.

### Vision

Refraction at the cornea is responsible for most of the focusing. The lens provides fine-tuning by changing its shape **(accommodation).**



In normal vision, the eye can focus from a far point (FP) at $\infty$ (relaxed eye) to a near point (NP) at $\approx 25$ cm (maximum accommodation).

- **Hyperopia** (farsightedness) is corrected with a converging lens.
- **Myopia** (nearsightedness) is corrected with a diverging lens.

### Microscopes

The object is very close to the focal point of the objective. The total angular magnification is

$$M = -\frac{L}{f_{obj}} \frac{25 \text{ cm}}{f_{eye}}$$

The best possible spatial resolution of a microscope, limited by diffraction, is about one wavelength of light.
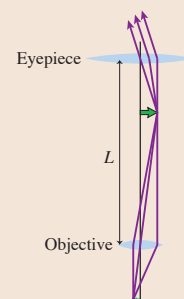


### Focusing and spatial resolution

The minimum spot size to which a lens of diameter $D$ can focus light is limited by diffraction to

$$w_{min} = \frac{2.44\lambda f}{D}$$

With the best lenses that can be manufactured, $w_{min} \approx \lambda$.

### Telescopes

The object is very far from the objective.



The total angular magnification is $M = -\dfrac{f_{obj}}{f_{eye}}$.

# Terms and Notation

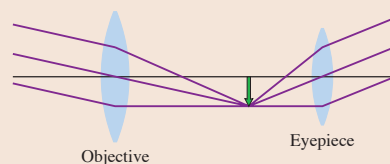| | | | |
|---|---|---|---|
| camera | iris | hyperopia | reflecting telescope |
| effective focal length, $f$ | retina | myopia | chromatic aberration |
| aperture | accommodation | angular size | spherical aberration |
| $f$-number | far point | magnifier | minimum spot size, $w_{min}$ |
| CCD | near point | angular magnification, $M$ | Rayleigh's criterion |
| pixel | presbyopia | objective | angular resolution |
| cornea | power, $P$ | eyepiece | |
| pupil | diopter, D | refracting telescope | |

# CONCEPTUAL QUESTIONS

1. Suppose a camera's exposure is correct when the lens has a focal length of 8.0 mm. Will the picture be overexposed, underexposed, or still correct if the focal length is "zoomed" to 16.0 mm without changing the diameter of the lens aperture? Explain.

2. A camera has a circular aperture immediately behind the lens. Reducing the aperture diameter to half its initial value will
   A. Make the image blurry.
   B. Cut off the outer half of the image and leave the inner half unchanged.
   C. Make the image less bright.
   D. All the above.
   Explain your choice.

3. Suppose you wanted special glasses designed to let you see underwater without a face mask. Should the glasses use a converging or diverging lens? Explain.

4. A friend lends you the eyepiece of his microscope to use on your own microscope. He claims the spatial resolution of your microscope will be halved, since his eyepiece has the same diameter as yours but twice the magnification. Is his claim valid? Explain.

5. A diffraction-limited lens can focus light to a 10-$\mu$m-diameter spot on a screen. Do the following actions make the spot diameter larger, make it smaller, or leave it unchanged?
   A. Decreasing the wavelength of the light.
   B. Decreasing the lens diameter.
   C. Decreasing the lens focal length.
   D. Decreasing the lens-to-screen distance.

6. To focus parallel light rays to the smallest possible spot, should you use a lens with a small $f$-number or a large $f$-number? Explain.

7. An astronomer is trying to observe two distant stars. The stars are marginally resolved when she looks at them through a filter that passes green light with a wavelength near 550 nm. Which of the following actions would improve the resolution? Assume that the resolution is not limited by the atmosphere.
   A. Changing the filter to a different wavelength. If so, should she use a shorter or a longer wavelength?
   B. Using a telescope with an objective lens of the same diameter but a different focal length. If so, should she select a shorter or a longer focal length?
   C. Using a telescope with an objective lens of the same focal length but a different diameter. If so, should she select a larger or a smaller diameter?
   D. Using an eyepiece with a different magnification. If so, should she select an eyepiece with more or less magnification?

# EXERCISES AND PROBLEMS

## Exercises

### Section 24.1 Lenses in Combination

1. ‖ Two converging lenses with focal lengths of 40 cm and 20 cm are 10 cm apart. A 2.0-cm-tall object is 15 cm in front of the 40-cm-focal-length lens.
   a. Use ray tracing to find the position and height of the image. Do this accurately with a ruler or paper with a grid. Estimate the image distance and image height by making measurements on your diagram.
   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

2. ‖ A converging lens with a focal length of 40 cm and a diverging lens with a focal length of −40 cm are 160 cm apart. A 2.0-cm-tall object is 60 cm in front of the converging lens.
   a. Use ray tracing to find the position and height of the image. Do this accurately with a ruler or paper with a grid. Estimate the image distance and image height by making measurements on your diagram.
   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

3. ‖ A 2.0-cm-tall object is 20 cm to the left of a lens with a focal length of 10 cm. A second lens with a focal length of 15 cm is 30 cm to the right of the first lens.
   a. Use ray tracing to find the position and height of the image. Do this accurately with a ruler or paper with a grid. Estimate the image distance and image height by making measurements on your diagram.
   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

4. ‖ A 2.0-cm-tall object is 20 cm to the left of a lens with a focal length of 10 cm. A second lens with a focal length of 5 cm is 30 cm to the right of the first lens.
   a. Use ray tracing to find the position and height of the image. Do this accurately with a ruler or paper with a grid. Estimate the image distance and image height by making measurements on your diagram.
   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

5. ‖‖ A 2.0-cm-tall object is 20 cm to the left of a lens with a focal length of 10 cm. A second lens with a focal length of −5 cm is 30 cm to the right of the first lens.
   a. Use ray tracing to find the position and height of the image. Do this accurately with a ruler or paper with a grid. Estimate the image distance and image height by making measurements on your diagram.
   b. Calculate the image position and height. Compare with your ray-tracing answers in part a.

## Section 24.2 The Camera

6. | A 2.0-m-tall man is 10 m in front of a camera with a 15-mm-focal-length lens. How tall is his image on the detector?

7. | What is the f-number of a lens with a 35 mm focal length and a 7.0-mm-diameter aperture?

8. | A 12-mm-focal-length lens has a 4.0-mm-diameter aperture. What is the aperture diameter of an 18-mm-focal-length lens with the same f-number?

9. | What is the aperture diameter of a 12-mm-focal-length lens set to f/4.0?

10. | A camera takes a properly exposed photo at f/5.6 and 1/125 s. What shutter speed should be used if the lens is changed to f/4.0?

11. ‖‖ A camera takes a properly exposed photo with a 3.0-mm-diameter aperture and a shutter speed of 1/125 s. What is the appropriate aperture diameter for a 1/500 s shutter speed?

## Section 24.3 Vision

12. ‖ Ramon has contact lenses with the prescription +2.0 D.
   BIO
   a. What eye condition does Ramon have?
   b. What is his near point without the lenses?

13. | Ellen wears eyeglasses with the prescription −1.0 D.
   BIO
   a. What eye condition does Ellen have?
   b. What is her far point without the glasses?

14. | What is the f-number of a relaxed eye with the pupil fully
   BIO dilated to 8.0 mm? Model the eye as a single lens 2.4 cm in front of the retina.

## Section 24.4 Optical Systems That Magnify

15. | A magnifier has a magnification of 5×. How far from the lens should an object be held so that its image is at the near-point distance of 25 cm?

16. ‖ A microscope has a 20 cm tube length. What focal-length objective will give total magnification 500× when used with a eyepiece having a focal length of 5.0 cm?

17. ‖ A standardized biological microscope has an 8.0-mm-focal-length objective. What focal-length eyepiece should be used to achieve a total magnification of 100×?

18. ‖ A 6.0-mm-diameter microscope objective has a focal length of 9.0 mm. What object distance gives a lateral magnification of −40?

19. | A 20× telescope has a 12-cm-diameter objective lens. What minimum diameter must the eyepiece lens have to collect all the light rays from an on-axis distant source?

20. ‖ A reflecting telescope is built with a 20-cm-diameter mirror having a 1.00 m focal length. It is used with a 10× eyepiece. What are (a) the magnification and (b) the f-number of the telescope?

## Section 24.5 The Resolution of Optical Instruments

21. ‖ A scientist needs to focus a helium-neon laser beam ($\lambda = 633$ nm) to a 10-$\mu$m-diameter spot 8.0 cm behind a lens.
   a. What focal-length lens should she use?
   b. What minimum diameter must the lens have?

22. ‖ Two lightbulbs are 1.0 m apart. From what distance can these lightbulbs be marginally resolved by a small telescope with a 4.0-cm-diameter objective lens? Assume that the lens is diffraction limited and $\lambda = 600$ nm.

## Problems

23. ‖ A 1.0-cm-tall object is located 4.0 cm to the left of a converging lens with a focal length of 5.0 cm. A diverging lens, of focal length −8.0 cm, is 12 cm to the right of the first lens. Find the position, size, and orientation of the final image.

24. | In **FIGURE P24.24**, are parallel rays from the left focused to a point? If so, on which side of the lens and at what distance?
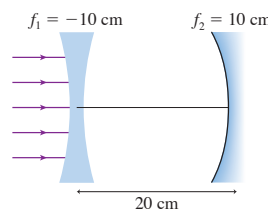


$f_1 = -10$ cm    $f_2 = 10$ cm    $f = 10$ cm    $f = 10$ cm

20 cm

5.0 cm    5.0 cm

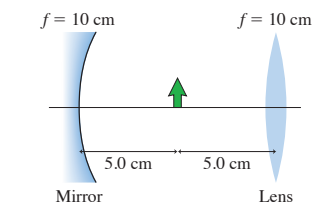Mirror    Lens

**FIGURE P24.24**    **FIGURE P24.25**

25. ‖ The rays leaving the two-component optical system of **FIGURE P24.25** produce two distinct images of the 1.0-cm-tall object.
   a. What are the position (relative to the lens), orientation, and height of each image?
   b. Draw two ray diagrams, one for each image, showing how the images are formed.

26. | A common optical instrument in a laser laboratory is a *beam expander.* One type of beam expander is shown in **FIGURE P24.26**. The parallel rays of a laser beam of width $w_1$ enter from the left.
   a. For what lens spacing d does a parallel laser beam exit from the right?
   b. What is the width $w_2$ of the exiting laser beam?



$f_2 > f_1$

$f_1$

$w_1$

$w_2$

$d$

**FIGURE P24.26**

27. |  A common optical instrument in a laser laboratory is a *beam expander*. One type of beam expander is shown in FIGURE P24.27. The parallel rays of a laser beam of width $w_1$ enter from the left.
   a.  For what lens spacing $d$ does a parallel laser beam exit from the right?
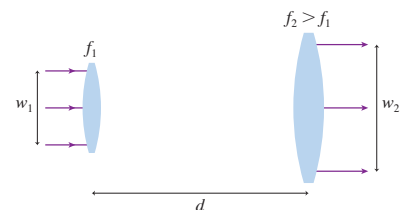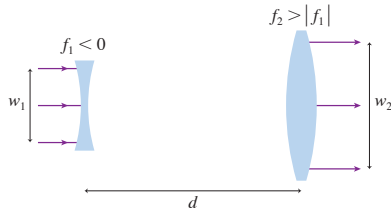   b.  What is the width $w_2$ of the exiting laser beam?



FIGURE P24.27

28. ‖  A 15-cm-focal-length converging lens is 20 cm to the right of a 7.0-cm-focal-length converging lens. A 1.0-cm-tall object is distance $L$ to the left of the first lens.
   a.  For what value of $L$ is the final image of this two-lens system halfway between the two lenses?
   b.  What are the height and orientation of the final image?

29. |  A 1.0-cm-tall object is 110 cm from a screen. A diverging lens with focal length $-20$ cm is 20 cm in front of the object. What are the focal length and distance from the screen of a second lens that will produce a well-focused, 2.0-cm-tall image on the screen?

30. ‖  You use your 8✕ binoculars to focus on a 14-cm-long bird in a tree 18 m away from you. What angle (in degrees) does the image of the warbler subtend on your retina?

31. ‖ BIO  Yang can focus on objects 150 cm away with a relaxed eye. With full accommodation, she can focus on objects 20 cm away. After her eyesight is corrected for distance vision, what will her near point be while wearing her glasses?

32. ‖ BIO  The cornea, a boundary between the air and the aqueous humor, has a 3.0 cm focal length when acting alone. What is its radius of curvature?

33. |  The objective lens of a telescope is a symmetric glass lens with 100 cm radii of curvature. The eyepiece lens is also a symmetric glass lens. What are the radii of curvature of the eyepiece lens if the telescope's magnification is 20✕?

34. ‖  You've been asked to build a telescope from a 2.0✕ magnifying lens and a 5.0✕ magnifying lens.
   a.  What is the maximum magnification you can achieve?
   b.  Which lens should be used as the objective? Explain.
   c.  What will be the length of your telescope?

35. |  Marooned on a desert island and with a lot of time on your hands, you decide to disassemble your glasses to make a crude telescope with which you can scan the horizon for rescuers. Luckily you're farsighted, and, like most people, your two eyes have different lens prescriptions. Your left eye uses a lens of power $+4.5$ D, and your right eye's lens is $+3.0$ D.
   a.  Which lens should you use for the objective and which for the eyepiece? Explain.
   b.  What will be the magnification of your telescope?
   c.  How far apart should the two lenses be when you focus on distant objects?

36. ‖  You've been asked to build a 12✕ microscope from a 2.0✕ magnifying lens and a 4.0✕ magnifying lens.
   a.  Which lens should be used as the objective?
   b.  What will be the tube length of your microscope?

37. ‖  A microscope with a tube length of 180 mm achieves a total magnification of 800✕ with a 40✕ objective and a 20✕ eyepiece. The microscope is focused for viewing with a relaxed eye. How far is the sample from the objective lens?

38. |  High-power lasers are used to cut and weld materials by focusing the laser beam to a very small spot. This is like using a magnifying lens to focus the sun's light to a small spot that can burn things. As an engineer, you have designed a laser cutting device in which the material to be cut is placed 5.0 cm behind the lens. You have selected a high-power laser with a wavelength of 1.06 $\mu$m. Your calculations indicate that the laser must be focused to a 5.0-$\mu$m-diameter spot in order to have sufficient power to make the cut. What is the minimum diameter of the lens you must install?

39. ‖ BIO  Once dark adapted, the pupil of your eye is approximately 7 mm in diameter. The headlights of an oncoming car are 120 cm apart. If the lens of your eye is diffraction limited, at what distance are the two headlights marginally resolved? Assume a wavelength of 600 nm and that the index of refraction inside the eye is 1.33. (Your eye is not really good enough to resolve headlights at this distance, due both to aberrations in the lens and to the size of the receptors in your retina, but it comes reasonably close.)

40. ‖  The Hubble Space Telescope has a mirror diameter of 2.4 m. Suppose the telescope is used to photograph stars near the center of our galaxy, 30,000 light years away, using red light with a wavelength of 650 nm.
   a.  What's the distance (in km) between two stars that are marginally resolved? The resolution of a reflecting telescope is calculated exactly the same as for a refracting telescope.
   b.  For comparison, what is this distance as a multiple of the distance of Jupiter from the sun?

41. ‖  Alpha Centauri, the nearest star to our solar system, is 4.3 light years away. Assume that Alpha Centauri has a planet with an advanced civilization. Professor Dhg, at the planet's Astronomical Institute, wants to build a telescope with which he can find out whether any planets are orbiting our sun.
   a.  What is the minimum diameter for an objective lens that will just barely resolve Jupiter and the sun? The radius of Jupiter's orbit is 780 million km. Assume $\lambda = 600$ nm.
   b.  Building a telescope of the necessary size does not appear to be a major problem. What practical difficulties might prevent Professor Dhg's experiment from succeeding?

## Challenge Problems

42. In FIGURE CP24.42, what are the position, height, and orientation of the final image? Give the position as a distance to the right or left of the lens.
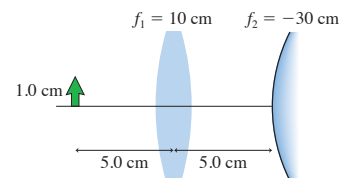


FIGURE CP24.42

43. Mars (6800 km diameter) is viewed through a telescope on a night when it is $1.1 \times 10^8$ km from the earth. Its angular size as seen through the eyepiece is 0.50°, the same size as the full moon seen by the naked eye. If the eyepiece focal length is 25 mm, how long is the telescope?

44. Your task in physics laboratory is to make a microscope from two lenses. One lens has a focal length of 2.0 cm, the other 1.0 cm. You plan to use the more powerful lens as the objective, and you want the eyepiece to be 16 cm from the objective.
   a. For viewing with a relaxed eye, how far should the sample be from the objective lens?
   b. What is the magnification of your microscope?

45. The lens shown in **FIGURE CP24.45** is called an *achromatic doublet,* meaning that it has no chromatic aberration. The left side is flat, and all other surfaces have radii of curvature $R$.

   
   $n_1$ $n_2$
   **FIGURE CP24.45**

   a. For parallel light rays coming from the left, show that the effective focal length of this two-lens system is $f = R/(2n_2 - n_1 - 1)$, where $n_1$ and $n_2$ are, respectively, the indices of refraction of the diverging and the converging lenses. Don't forget to make the thin-lens approximation.
   b. Because of dispersion, either lens alone would focus red rays and blue rays at different points. Define $\Delta n_1$ and $\Delta n_2$ as $n_{\text{blue}} - n_{\text{red}}$ for the two lenses. Find an expression for $\Delta n_2$ in terms of $\Delta n_1$ that makes $f_{\text{blue}} = f_{\text{red}}$ for the two-lens system. That is, the two-lens system does *not* exhibit chromatic aberration.
   c. Indices of refraction for two types of glass are given in the table. To make an achromatic doublet, which glass should you use for the converging lens and which for the diverging lens? Explain.

   |              | $n_{\text{blue}}$ | $n_{\text{red}}$ |
   |--------------|-------|-------|
   | Crown glass  | 1.525 | 1.517 |
   | Flint glass  | 1.632 | 1.616 |

   d. What value of $R$ gives a focal length of 10.0 cm?

46. **FIGURE CP24.46** shows a simple zoom lens in which the magnitudes of both focal lengths are $f$. If the spacing $d < f$, the image of the converging lens falls on the right side of the diverging lens. Our procedure of letting the image of the first lens act as the object of the second lens will continue to work in this case if we use a *negative* object distance for the second lens. This is called a *virtual object.* Consider a very distant object ($s \approx \infty$ for the first lens) and define the effective focal length as the distance from the midpoint between the lenses to the final image.
   a. Show that the effective focal length is
   $$f_{\text{eff}} = \frac{f^2 - fd + \frac{1}{2}d^2}{d}$$
   b. What is the zoom for a lens that can be adjusted from $d = \frac{1}{2}f$ to $d = \frac{1}{4}f$?
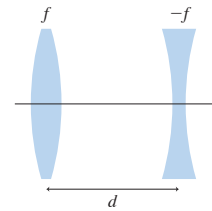
   
   $f$        $-f$
   $d$
   **FIGURE CP24.46**

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**STOP TO THINK ANSWERS**

**Stop to Think 24.1: b.** A diverging lens refracts rays away from the optical axis, so the rays will travel farther down the axis before converging.

**Stop to Think 24.2: a.** Because the shutter speed doesn't change, the $f$-number must remain unchanged. The $f$-number is $f/D$, so increasing $f$ requires increasing $D$.

**Stop to Think 24.3: a.** A magnifier is a converging lens. Converging lenses are used to correct hyperopia.

**Stop to Think 24.4: b.** If the objective magnification is halved, the eyepiece magnification must be doubled. $M_{\text{eye}} = 25\ \text{cm}/f_{\text{eye}}$, so doubling $M_{\text{eye}}$ requires halving $f_{\text{eye}}$.

**Stop to Think 24.5:** $w_a > w_d > w_b = w_c.$ The spot size is proportional to $f/D$.